Faculty of Engineering
Computer and Systems
Engineering Department

CSE 372: Control Systems (2)

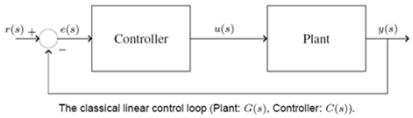
Topic# 2

Sampled Data Systems

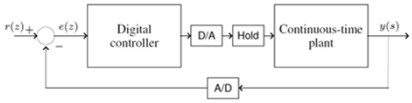
Outline

- Introduction
- Analog to digital converters (ADCs)
- Digital to analog converters (DACs)
- Selection of sampling frequency
- Summary

Introduction



The classical linear control loop (Plant: $G(s)$, Controller: $C(s)$).



A digital control loop (Plant: $G(s)$, Controller: $C(z)$).

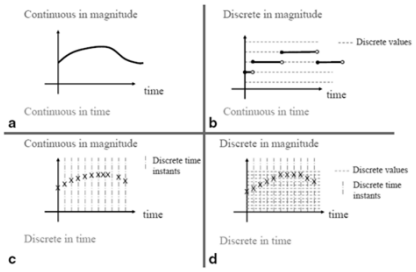
Introduction

Advantages of digital control

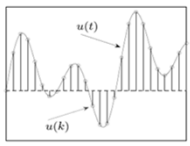
- Hardware is replaced by software, which is costly-effective
- Complex function can be implemented in software so easily rather then hardware
- Reliability in implementation, that means, you can simply modify the control function in software without extra cost.
- Computers can be used in data logging (monitoring), supervisory control and can control multiple loop simultaneously as the computers are well fast.

Introduction

Four types of signals



Introduction

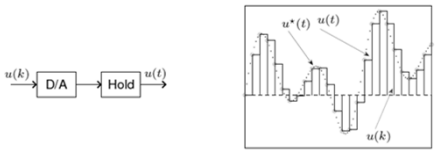


The A/D converter *transforms* a function of time $u(t)$ into a sequence $\{u(k)\}$. If the conversion is executed every T time instants then (with abuse of notation)

$$u(k) = u(kT),$$

for $k \in \mathbb{N}$ ($\mathbb{N}$ is the set of natural numbers, including zero). The time T is the sampling time. If the conversion is executed at times t_i, with $i \in \mathbb{N}$, then $u(k) = u(t_k)$.

Introduction



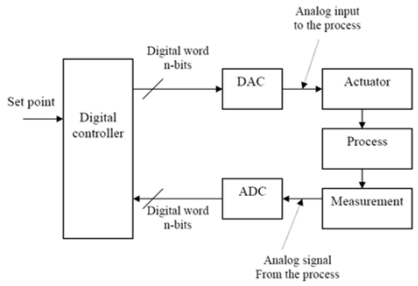
Other hold devices, i.e. with different profiles of the output, can be used.

The application of an A/D conversion, followed by a D/A conversion with Hold, to a signal $u(t)$ does not return the signal $u(t)$.

The A/D conversion associates the same sequence $u(k)$ to infinitely many signals $u(t)$.

The D/A and A/D conversions introduce other *distorsions*, such as quantization and delays,

Introduction



Introduction

Digital controllers could take one of the forms:

- ***A computer or simply microprocessor board***. Once they have developed and started to be manufactured commercially, digital controllers are developed.
- ***Microcontroller is*** a microprocessor system on chip as a single integrated circuit. It is used in embedded control applications such as TV, mobile phones, Air conditioner, Video Camera, Hard disk controllers, Robots, Smart car manufacturing, ...etc. It is used for a limited number of I/O signals in real time applications.
- ***Programmable logic controller (PLCs).*** PLC can handle a very large number of I/O signals (as hundreds or thousands) in industrial control applications. It has a standard interfaces with the field measurements. The PLC technology replaces the old hardwired control (relay logic control) cabinets in the industry.

Analog to Digital Converters

The ADC requires three operations in sequence:

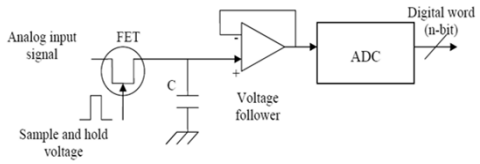
1- ***Sampling,*** we need to sample the analog signal at a constant rate. The sampler could be an electronic switch. The critical question is how to select the sampling frequency.

2- ***Holding,*** that holds the sample in during the sampling period until a new sample is captured. This is necessary to convert a constant value into digital word.

3- ***ADC,*** it is often sequential circuit that takes a considerable time to convert the holding sample into digital word.

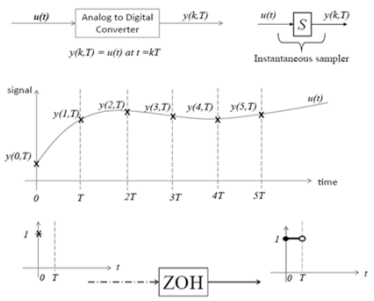
Analog to Digital Converters

Sampling and holding process



Digitized Value → $N = \frac{(V - V_{min})}{(V_{max} - V_{min})} 2^n$

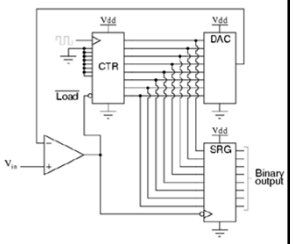
Analog to Digital Converters



Analog to Digital Converters

1-Digital ramp circuit

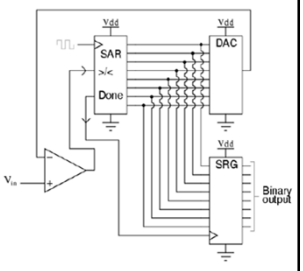
Also known as the ***stair-step ramp, or simply counter A/D converter, this is also fairly easy to*** understand but unfortunately suffers from several limitations. The basic idea is to connect the output of a free-running binary counter to the input of a DAC, then compare the analog output of the DAC with the analog input signal to be digitized and use the comparator's output to tell the counter when to stop counting and reset.



Analog to Digital Converters

2-Successive approximation circuit

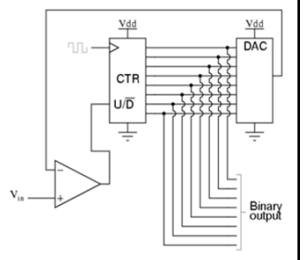
This circuit uses a very special counter known as a ***successive-approximation register (SAR).*** This register counts by trying all values of bits starting with the MSB and finishing at the LSB, instead of counting up in binary sequence. Throughout the count process, the register monitors the comparator's output to see if the binary count is less than or greater than the analog signal input, adjusting the bit values accordingly to 1 or 0 value.



Analog to Digital Converters

3-Tracking circuit

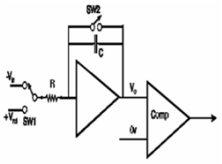
Instead of a regular "up" counter driving the DAC, this circuit uses an up/down counter. The counter is continuously clocked, and the up/down control line is driven by the output of the comparator. So, when the analog input signal exceeds the DAC output, the counter goes into the "count up" mode. When the DAC output exceeds the analog input, the counter switches into the "count down" mode. Either way, the DAC output always counts in the proper direction to ***track the input signal.***



Analog to Digital Converters

4-Dual slope circuit

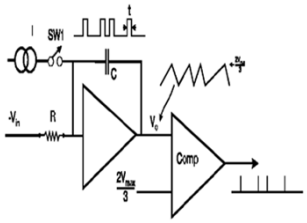
It is possible to avoid using a DAC if we substitute an analog ramping circuit and a digital counter with precise timing. This is the basic idea behind the so-called ***integrating ADC.*** Consider the following circuit, in operation the integrator is first zeroed (close SW2 for short time), then attached to the input (SW1 up) for a fixed time ***M counts of the clock (frequency 1/t). At the end of that time it is attached to the reference voltage (SW1 down), a digital*** counter has start to count (start of conversion), and the number of counts ***N which accumulate*** before the integrator reaches zero volts output and the comparator output changes are determined to signal to the logic circuit that is not shown in the figure, that end of conversion is reached.



Analog to Digital Converters

5-Voltage to frequency circuit

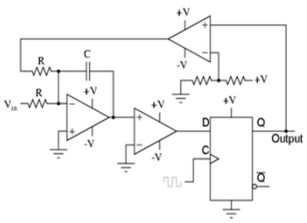
The voltage-to frequency converter uses a linear voltage controlled oscillator to produce a frequency output.



Analog to Digital Converters

6-Delta-Sigma (ΔΣ) circuit

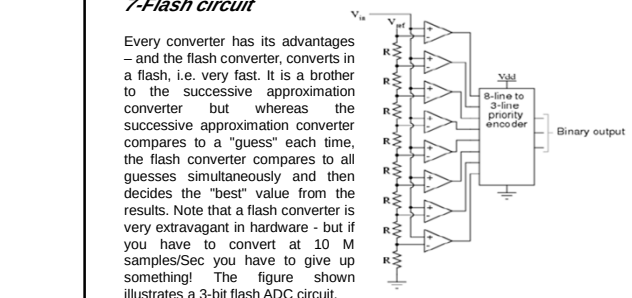
One of the more advanced ADC technologies is the so-called delta-sigma, or ΔΣ (using the proper Greek letter notation). In mathematics and physics, the capital Greek letter delta (Δ) represents ***difference or change, while the capital letter sigma (Σ) represents summation: the*** adding of multiple terms together.



Analog to Digital Converters

7-Flash circuit

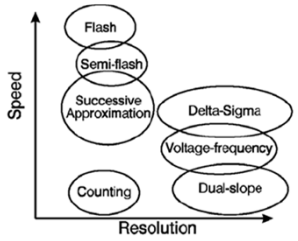
Every converter has its advantages – and the flash converter, converts in a flash, i.e. very fast. It is a brother to the successive approximation converter but whereas the successive approximation converter compares to a "guess" each time, the flash converter compares to all guesses simultaneously and then decides the "best" value from the results. Note that a flash converter is very extravagant in hardware - but if you have to convert at 10 M samples/Sec you have to give up something! The figure shown illustrates a 3-bit flash ADC circuit.



Analog to Digital Converters

Practical considerations of ADC circuits

The most important considerations of an ADC is its ***resolution and the speed of conversion. Resolution is proportional to the*** number of binary bits output by the converter.



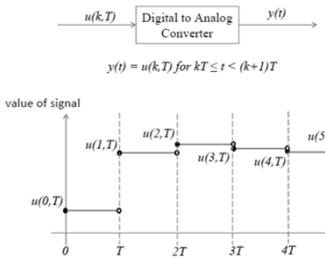
Digital to Analog Converters

DAC requires two operations in sequence:

1- ***DAC,*** in general is faster than ADC ones and easier in implementation.

2- ***Holding,*** it is very difficult to apply the discrete signal that outputs from DAC directly to an analog process. It will excite the system and fatigue the actuator. Therefore, holding these samples makes them in a continuous form (stepping levels).

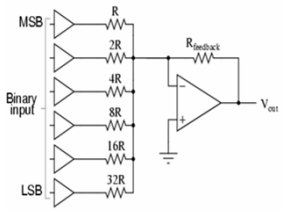
Digital to Analog Converters



Digital to Analog Converters

1-The R/2ⁿR Circuit

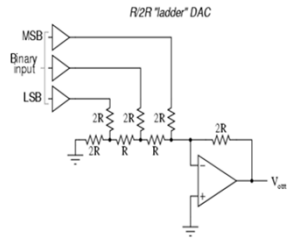
This DAC circuit is known as the ***binary-weighted input DAC. The circuit*** uses the classic inverting summer which is an operational amplifier using negative feedback for controlled gain, with several voltage inputs and one voltage output. The output voltage is the inverted (opposite polarity) sum of all input voltages.



Digital to Analog Converters

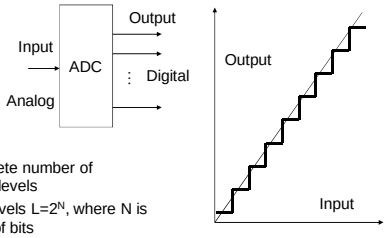
2-The R-2R Circuit

A disadvantage of the binary-weighted DAC was its requirement of several different precise input resistor values: one unique value per binary input bit. There is, however, a more efficient design methodology. By constructing a different kind of resistor network on the input of our summing circuit, we can achieve the same kind of binary weighting with only two kinds of resistor values.



Digital to Analog Converters

Quantization error in conversion



- Have a discrete number of quantization levels
- Number of levels $L=2^N$, where N is the number of bits
- e.g. N=3 bits, $L=2^3=8$ levels

Digital to Analog Converters

Bits	Levels	Analog Signal	Error
1	2	5	5/2=2.5
2	4	5	5/4=1.25
3	8	5	5/8=0.625
4	16	5	5/16=0.3125

Error = input signal / (2^N)

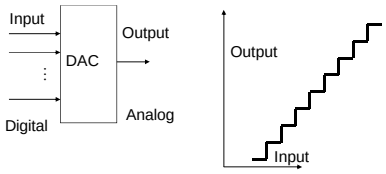
Digital to Analog Converters

More bits more accuracy.

The commonly used ADC has

- 8-bits: $L=2^8=256$ (coarse)
- 10-bits: $L=2^{10}=1024$ (adequate)
- 12-bits: $L=2^{12}=4096$ (works well)
- 16-bits: $L=2^{16}=65536$ (almost overkill)

Digital to Analog Converters



Example: For N=8 and the signal is from 0 to 5 volt, find the output value for the number 145.

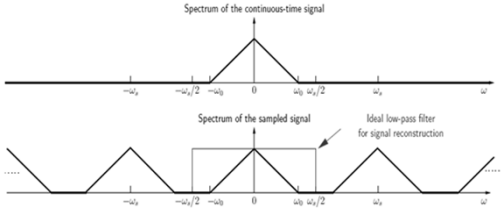
Solution: 5/255=x/145,
x=5*145/255=2.8431=2.84 volt

Selection of Sampling Frequency

It is imperative that an ADC's sample time is fast enough to capture essential changes in the analog waveform. In data acquisition terminology, the highest-frequency waveform that an ADC can theoretically capture is called ***Nyquist frequency***, *which equals to one-half of the ADC's sample* frequency. Therefore, if an ADC circuit has a sample frequency of 5000 Hz, the highest frequency waveform will be the Nyquist frequency of 2500 Hz. If an ADC is subjected to an analog input signal whose frequency exceeds the Nyquist frequency for that ADC, the converter will output a digitized signal of falsely low frequency.This phenomenon is known as ***aliasing effect***.

Selection of Sampling Frequency

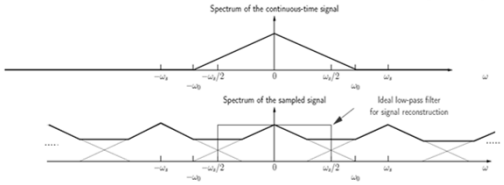
Sprectra of continuous-time band-limited signal and sampled signal for $\omega_s > 2\omega_0$ $(\omega_N > \omega_0)$.



- Original signal could be reconstructed by ideal low-pass filter.

Selection of Sampling Frequency

Sprectra of continuous-time band-limited signal and sampled signal for $\omega_s < 2\omega_0$ $(\omega_N < \omega_0)$.



- Original signal cannot be reconstructed filter due to aliasing.
- A signal with frequency $\omega_d > \omega_N$ appears as signal with the lower frequency $(\omega_s - \omega_d)$ in the sampled signal.

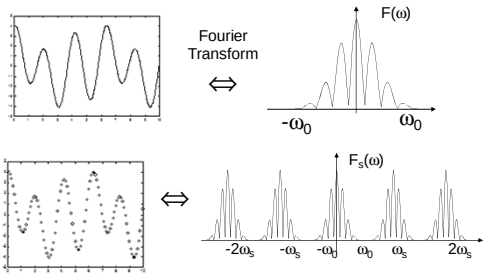
Selection of Sampling Frequency

The system bandwidth frequency is not the only limit to select the sampling frequency, there is also other constraints due to time considerations in ADC, DAC, and microprocessor to execute the control program. In general, the sampling period Ts to control a single loop can be selected using the following relationship:

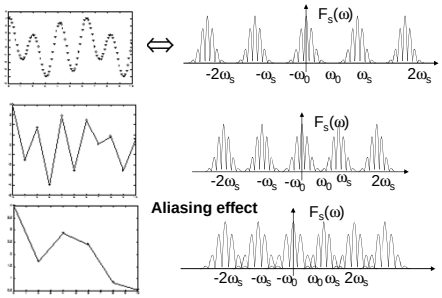
$$1/(2\ f_B) > Ts > (T_{ADC} + T_{\mu p} + T_{DAC})$$

Where f_B = frequency bandwidth of the analog signal
T_{ADC} = conversion time of ADC
T_{DAC} = conversion time of DAC(can be ignored)
$T_{\mu p}$ = Execution time of the control program in microprocessor, it depends the speed of microprocessor

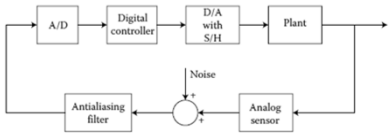
Selection of Sampling Frequency



Selection of Sampling Frequency



Selection of Sampling Frequency



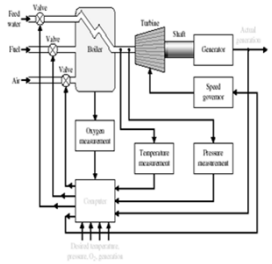
Preventing aliases

- Make sure your sampling frequency is greater than twice of the highest frequency component of the signal. In practice, take it ten times the highest frequency component.
- Pre-filtering of the analog signal
- Set your sampling frequency to the maximum if possible

Selection of Sampling Frequency

$T_{upper} > Ts > T_{Lower}$

In case of multi-channel control (multivariable) system, The **upper limit** of the sampling period depends the fastest dynamics which is the dominant one, while the **lower limit** depends the number of channels multiplied by the maximum delay in each channel. In this application, multiplexing and demultiplexing technique is used with a single computer, ADC, and DAC.



Coordinated control system for a boiler-generator.

Summary

- Modern hardware and software technologies increases the use of digital control systems to replace analog ones
- In spite of the great advantages of digital industrial controllers, the finite word-length (number of bits) and limitations on the sampling rate are practical challenges
- The selection of ADC has to compromise between: accuracy, speed, and cost for real time applications
- The zero order hold is added between the DAC and the process to limit the frequency bandwidth of the control signal.

Summary

In real time digital control applications, the sampling period is bounded by lower and upper limits

- The lower sampling limit depends on the computational and conversion delays
- The upper sampling limit depends on the frequency bandwidth of the sampled signals
- Noise rejection in the measured signals by using low pass filter is necessary to avoid aliasing effect in the frequency spectrum