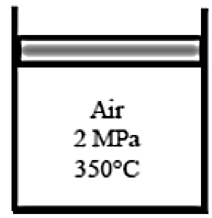
QUIZ 3

Problem #1:

A piston–cylinder device contains 0.15 kg of air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.

Properties The properties of air are $R = 0.287$ kJ/kg.K , $k = 1.4$ (Table A-2a).

Analysis For the isothermal expansion process:



$$V_1 = \frac{mRT}{P_1} = \frac{(0.15\,\text{kg})(0.287\,\text{kJ/kg.K})(350+273\,\text{K})}{(2000\,\text{kPa})} = 0.01341\,\text{m}^3$$

$$V_2 = \frac{mRT}{P_2} = \frac{(0.15\,\text{kg})(0.287\,\text{kJ/kg.K})(350+273\,\text{K})}{(500\,\text{kPa})} = 0.05364\,\text{m}^3$$

$$W_{b,1-2} = P_1V_1\ln\left(\frac{V_2}{V_1}\right) = (2000\,\text{kPa})(0.01341\,\text{m}^3)\ln\left(\frac{0.05364\,\text{m}^3}{0.01341\,\text{m}^3}\right) = \mathbf{37.18\,kJ}$$

For the polytropic compression process:

$$P_2V_2^n = P_3V_3^n \longrightarrow (500\,\text{kPa})(0.05364\,\text{m}^3)^{1.2} = (2000\,\text{kPa})V_3^{1.2} \longrightarrow V_3 = 0.01690\,\text{m}^3$$

$$W_{b,2-3} = \frac{P_3V_3 - P_2V_2}{1-n} = \frac{(2000\,\text{kPa})(0.01690\,\text{m}^3) - (500\,\text{kPa})(0.05364\,\text{m}^3)}{1-1.2} = \mathbf{-34.86\,kJ}$$

For the constant pressure compression process:

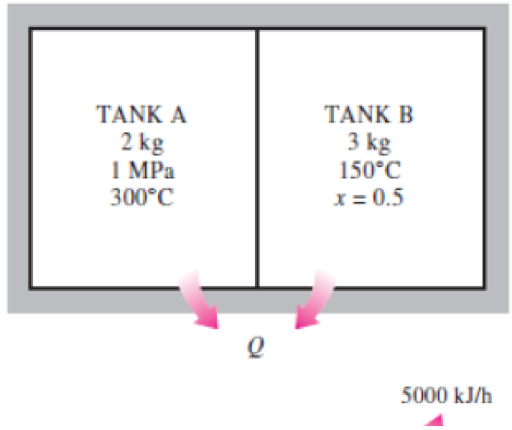
$$W_{b,3-1} = P_3(V_1 - V_3) = (2000\,\text{kPa})(0.01341 - 0.01690)\text{m}^3 = \mathbf{-6.97\,kJ}$$

The net work for the cycle is the sum of the works for each process

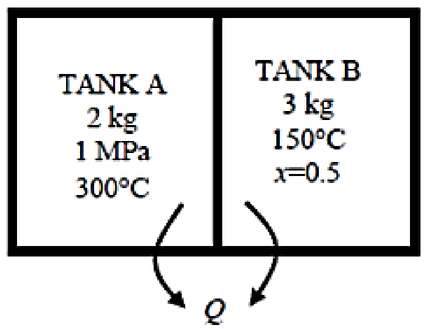
$$W_{\text{net}} = W_{b,1-2} + W_{b,2-3} + W_{b,3-1} = 37.18 + (-34.86) + (-6.97) = \mathbf{-4.65\,kJ}$$

Problem #2:

Two tanks (Tank A and Tank B) are separated by a partition. Initially Tank A contains 2-kg steam at 1 MPa and 300°C while Tank B contains 3-kg saturated liquid–vapor mixture with a vapor mass fraction of 50 percent. Now the partition is removed and the two sides are allowed to mix until the mechanical and thermal equilibrium are established. If the pressure at the final state is 300 kPa, determine (a) the temperature and quality of the steam (if mixture) at the final state and (b) the amount of heat lost from the tanks.



Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.



Analysis (*a*) We take the contents of both tanks as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} = \Delta U_A + \Delta U_B = [m(u_2 - u_1)]_A + [m(u_2 - u_1)]_B \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

The properties of steam in both tanks at the initial state are (Tables A-4 through A-6)

$$\left.\begin{array}{l} P_{1,A} = 1000 \text{ kPa} \\ T_{1,A} = 300°\text{C} \end{array}\right\} \begin{array}{l} v_{1,A} = 0.25799 \text{ m}^3/\text{kg} \\ u_{1,A} = 2793.7 \text{ kJ/kg} \end{array}$$

$$\left.\begin{array}{l} T_{1,B} = 150°\text{C} \\ x_1 = 0.50 \end{array}\right\} \begin{array}{ll} v_f = 0.001091, & v_g = 0.39248 \text{ m}^3/\text{kg} \\ u_f = 631.66, & u_{fg} = 1927.4 \text{ kJ/kg} \end{array}$$

$$v_{1,B} = v_f + x_1 v_{fg} = 0.001091 + [0.50 \times (0.39248 - 0.001091)] = 0.19679 \text{ m}^3/\text{kg}$$

$$u_{1,B} = u_f + x_1 u_{fg} = 631.66 + (0.50 \times 1927.4) = 1595.4 \text{ kJ/kg}$$

The total volume and total mass of the system are

$$V = V_A + V_B = m_A v_{1,A} + m_B v_{1,B} = (2 \text{ kg})(0.25799 \text{ m}^3/\text{kg}) + (3 \text{ kg})(0.19679 \text{ m}^3/\text{kg}) = 1.106 \text{ m}^3$$

$$m = m_A + m_B = 3 + 2 = 5 \text{ kg}$$

Now, the specific volume at the final state may be determined

$$v_2 = \frac{V}{m} = \frac{1.106\ \text{m}^3}{5\ \text{kg}} = 0.22127\ \text{m}^3/\text{kg}$$

which fixes the final state and we can determine other properties

$$\left.\begin{array}{l} P_2 = 300\ \text{kPa} \\ v_2 = 0.22127\ \text{m}^3/\text{kg} \end{array}\right\} \begin{array}{l} T_2 = T_{\text{sat @ 300 kPa}} = \mathbf{133.5°C} \\ x_2 = \dfrac{v_2 - v_f}{v_g - v_f} = \dfrac{0.22127 - 0.001073}{0.60582 - 0.001073} = \mathbf{0.3641} \\ u_2 = u_f + x_2 u_{fg} = 561.11 + (0.3641 \times 1982.1) = 1282.8\ \text{kJ/kg} \end{array}$$

(*b*) Substituting,

$$-Q_{\text{out}} = \Delta U_A + \Delta U_B = [m(u_2 - u_1)]_A + [m(u_2 - u_1)]_B$$
$$= (2\ \text{kg})(1282.8 - 2793.7)\text{kJ/kg} + (3\ \text{kg})(1282.8 - 1595.4)\text{kJ/kg} = -3959\ \text{kJ}$$

or $$Q_{\text{out}} = \mathbf{3959\ kJ}$$

Problem #3:

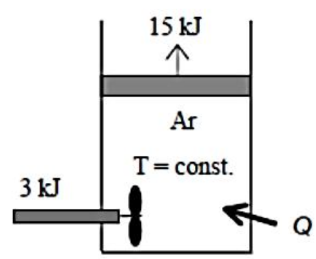
A piston–cylinder device contains 4 kg of argon at 250 kPa and 35°C. During a quasi-equilibrium, isothermal expansion process, 15 kJ of boundary work is done by the system, and 3 kJ of paddle-wheel work is done on the system. Determine the heat transfer for this process.

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{\text{in}} + W_{\text{pw,in}} - W_{\text{b,out}} = \Delta U = m(u_2 - u_1) = mc_v(T_2 - T_1) = 0$$

since $u = u(T)$ for ideal gases, and thus $u_2 = u_1$ when $T_1 = T_2$. Therefore,

$$Q_{\text{in}} = W_{\text{b,out}} - W_{\text{pw,in}} = 15 - 3 = \mathbf{12\ kJ}$$



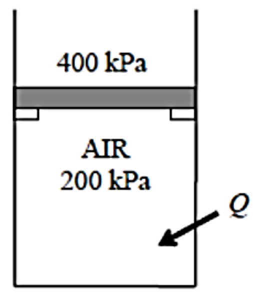
Problem #4:

A piston–cylinder device, whose piston is resting on a set of stops, initially contains 3 kg of air at 200 kPa and 27°C. The mass of the piston is such that a pressure of 400 kPa is required to move it. Heat is now transferred to the air until its volume doubles. Determine the work done by the air and the total heat transferred to the air during this process. Also show the process on a P-v diagram.

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_3 - u_1)$$

$$Q_{in} = m(u_3 - u_1) + W_{b,out}$$

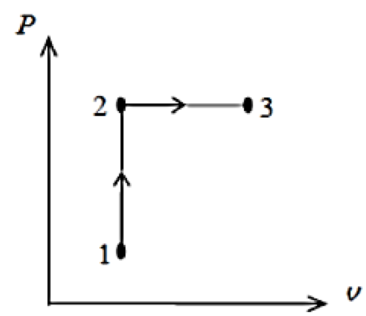


The initial and the final volumes and the final temperature of air are

$$V_1 = \frac{mRT_1}{P_1} = \frac{(3\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{ K})}{200\text{ kPa}} = 1.29\text{ m}^3$$

$$V_3 = 2V_1 = 2\times 1.29 = 2.58\text{ m}^3$$

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \longrightarrow T_3 = \frac{P_3}{P_1}\frac{V_3}{V_1}T_1 = \frac{400\text{ kPa}}{200\text{ kPa}}\times 2\times(300\text{ K}) = 1200\text{ K}$$



No work is done during process 1-2 since $V_1 = V_2$. The pressure remains constant during process 2-3 and the work done during this process is

$$W_{b,out} = \int_1^2 P\,dV = P_2(V_3 - V_2) = (400\text{ kPa})(2.58 - 1.29)\text{m}^3 = \mathbf{516\ \text{kJ}}$$

The initial and final internal energies of air are (Table A-17)

$$u_1 = u_{@300\text{ K}} = 214.07\text{ kJ/kg}$$

$$u_3 = u_{@1200\text{ K}} = 933.33\text{ kJ/kg}$$

Then from the energy balance,

$$Q_{in} = (3\text{ kg})(933.33 - 214.07)\text{kJ/kg} + 516\text{ kJ} = \mathbf{2674\text{ kJ}}$$

Alternative solution The specific heat of air at the average temperature of $T_{avg} = (300 + 1200)/2 = 750$ K is, from Table A-2b, $c_{v,avg} = 0.800$ kJ/kg.K. Substituting,

$$Q_{in} = m(u_3 - u_1) + W_{b,out} \cong mc_v(T_3 - T_1) + W_{b,out}$$

$$Q_{in} = (3\text{ kg})(0.800\text{ kJ/kg.K})(1200 - 300)\text{ K} + 516\text{ kJ} = \mathbf{2676\text{ kJ}}$$