<u>Model answer - Quiz #1</u>

1. Water flows in a constant diameter pipe of 10 cm in diameter. At point (1), pressure and elevation were measured and found to be 5 bar and 10 m respectively. At another point (2), the pressure is 10 bar. If the mass flow rate through the pipe is 100 kg/s. Assume that the water flow is frictionless. ***<u>Find the energy potential of the water and the elevation at point (2).</u>***

$$v_{water} = \frac{\dot{m}}{\rho . A_{pipe}} = \frac{\dot{m}}{\rho . \left(\frac{\pi}{4} d^2\right)} = \frac{100}{1000. \left(\frac{\pi}{4} 0.1^2\right)} = 12.7 \frac{m}{s}$$

$$\text{Energy potential} = e_{mech(1)} = e_{mech(2)} = e_{mech} = \left[\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1\right]$$

$$= \left[\frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2\right]$$

$$\text{Energy potential} = \left[\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1\right] = \left[\frac{5 * 10^5}{1000} + \frac{12.7^2}{2} + 9.81 * 10\right] = \mathbf{679\ J}$$

$$\left[\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1\right] = \left[\frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2\right]$$

$$679 = \left[\frac{10 * 10^5}{1000} + \frac{12.7^2}{2} + 9.81 * z_2\right]$$

$$z_2 = \mathbf{-41\ m}$$

2. Water flows in a constant diameter pipe of 10 cm in diameter. At point (1), pressure and elevation were measured and found to be 5 bar and 10 m respectively. At another point (2), pressure and elevation are 10 bar and 20 m. If the mass flow rate through the pipe is 100 kg/s. Pump is installed between the two points (1) and (2) and assume that the water flow is frictionless. ***Find the velocity through the pipe and the rate of energy potential given to water.***

$$v_{water} = \frac{\dot{m}}{\rho . A_{pipe}} = \frac{\dot{m}}{\rho . \left(\frac{\pi}{4} d^2\right)} = \frac{100}{1000 . \left(\frac{\pi}{4} 0.1^2\right)} = \mathbf{12.7 \frac{m}{s}}$$

$$e_{mech(1)} = \left[\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1\right] = \left[\frac{5 * 10^5}{1000} + \frac{12.7^2}{2} + 9.81 * 10\right] = 679 \text{ J}$$

$$e_{mech(2)} = \left[\frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2\right] = \left[\frac{10 * 10^5}{1000} + \frac{12.7^2}{2} + 9.81 * 20\right] = 1277 \text{J}$$

$$\text{Energy potential given to water} = \Delta E_{mech} = e_{mech(2)} - e_{mech(1)}$$

$$\Delta E_{mech} = \mathbf{598\ J}$$

3. Water flows in a constant diameter pipe of 10 cm in diameter. At point (1), pressure and elevation were measured and found to be 5 bar and 10 m respectively. At another point (2), the elevation is 5 m. If the mass flow rate through the pipe is 100 kg/s. Pump is installed between the two points (1) and (2) which is driven by an electric motor that consumes 10 kW of electric power. Assume that the water flow is frictionless. If the combined pump-motor efficiency is 90%, ***find the pressure at point (2).***

$$v_{water} = \frac{\dot{m}}{\rho . A_{pipe}} = \frac{\dot{m}}{\rho . \left(\frac{\pi}{4} d^2\right)} = \frac{100}{1000. \left(\frac{\pi}{4} 0.1^2\right)} = 12.7 \frac{m}{s}$$

$$\eta_{pump-motor} = \frac{\Delta \dot{E}_{mech}}{\dot{W}_{mech}} = 0.9$$

$$\eta_{pump-motor} = 0.9 = \frac{\dot{m}\left[\left(\frac{p_2 - p_1}{\rho}\right) + \left(\frac{v_2^2 - v_1^2}{2}\right) + g(z_2 - z_1)\right]}{10 * 10^3}$$

$$\eta_{pump-motor} = 0.9 = \frac{100 * \left[\left(\frac{p_2 - 5 * 10^5}{1000}\right) + 0 + 9.81 * (5 - 10)\right]}{10 * 10^3}$$

$$9000 = 100 * \left[\left(\frac{p_2 - 5 * 10^5}{1000}\right) + 0 + 9.81 * (5 - 10)\right]$$

$$p_2 = \mathbf{639\ kPa}$$

4. Water flows in a constant diameter pipe of 10 cm in diameter. At point (1), pressure and elevation were measured and found to be 5 bar and 10 m respectively. At another point (2), the pressure is 1 bar. If the mass flow rate through the pipe is 100 kg/s. A generator that generates 4 kW of electric power, is driven by a turbine which is installed between the two points (1) and (2). Assume that the water flow is frictionless. If the turbine efficiency is 95% and the generator efficiency is 98%, ***find the elevation of point (2).***

$$v_{water} = \frac{\dot{m}}{\rho . A_{pipe}} = \frac{\dot{m}}{\rho . \left(\frac{\pi}{4} d^2\right)} = \frac{100}{1000. \left(\frac{\pi}{4} 0.1^2\right)} = 12.7 \frac{m}{s}$$

$$\eta_{turbine-generator} = \eta_{turbine} * \eta_{generator} = 0.95 * 0.98 = 0.93 = \frac{\dot{W}_{mech}}{\Delta \dot{E}_{mech}}$$

$$\eta_{turbine-generator} = 0.93 = \frac{4 * 10^3}{\dot{m}\left[\left(\frac{p_2 - p_1}{\rho}\right) + \left(\frac{v_2^2 - v_1^2}{2}\right) + g(z_2 - z_1)\right]}$$

$$\eta_{turbine-generator} = 0.93$$

$$= \frac{4 * 10^3}{100 * \left[\left(\frac{1 * 10^5 - 5 * 10^5}{1000}\right) + 0 + 9.81 * (z_2 - 10)\right]}$$

$$4301 = 100 * \left[\left(\frac{-4 * 10^5}{1000}\right) + 0 + 9.81 * (z_2 - 10)\right]$$

$$z_2 = \mathbf{55\ m}$$