Name:	Program ID:

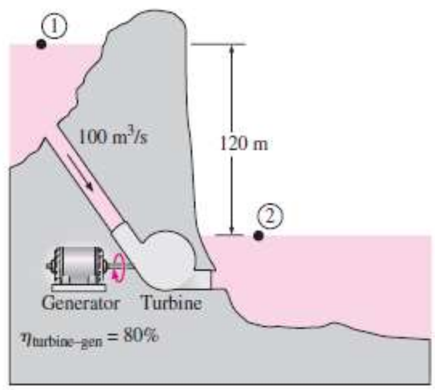
Model answer - Midterm Exam # 1

Fall 2014

Time allowed: one hour

Round-off all numbers of your answer into reasonable digits

1. (10 points)



In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The overall efficiency of the turbine–generator is 80 percent. Disregarding frictional losses in piping, determine:

a. **The energy potential of the water required per year. (5 points)**
b. **The electric power output of this plant. (5 points)**

a)

The total mass of water required per year $= \rho\dot{V} * \text{time}$

$$= 1000 * 100 * 365 * 24 * 60 * 60 = \mathbf{3.15\ Gton}$$

The energy potential of the water required per year $= \text{PE} = \text{mgz} = \mathbf{3712\ TJ}$

$$e_{mech} = pe = gz = (9.81\,\text{m/s}^2)(120\,\text{m})\left(\frac{1\,\text{kJ/kg}}{1000\,\text{m}^2/\text{s}^2}\right) = 1.177\,\text{kJ/kg}$$

The mass flow rate is

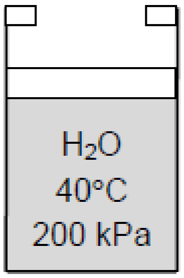
$$\dot{m} = \rho\dot{V} = (1000\,\text{kg/m}^3)(100\,\text{m}^3/\text{s}) = 100{,}000\,\text{kg/s}$$

Then the maximum and actual electric power generation become

$$\dot{W}_{max} = \dot{E}_{mech} = \dot{m}e_{mech} = (100{,}000\,\text{kg/s})(1.177\,\text{kJ/kg})\left(\frac{1\,\text{MW}}{1000\,\text{kJ/s}}\right) = 117.7\,\text{MW}$$

$$\dot{W}_{electric} = \eta_{overall}\dot{W}_{max} = 0.80(117.7\,\text{MW}) = \mathbf{94.2\ MW}$$

2. (15 points)



A piston–cylinder device initially contains 50 L of liquid water at 40°C and 200 kPa. The piston has a surface area of 0.1 m^2. Heat is transferred to the water at constant pressure until the entire liquid is vaporized, at this point the piston is touching a set of two stops. Now more heat is added to the water till a pressure of 5 bar is maintained. (the atmospheric pressure is 1 bar)

- **a. What is the mass of the piston? (3 points)**
- **b. What is the mass of the water? (3 points)**
- **c. What is the volume while the piston is touching the stops? (3 points)**
- **d. What is the final temperature? (3 points)**
- **e. Show the process on a T-v diagram with respect to saturation lines. (3 points)**

The piston is moving freely under constant pressure before it reaches the stops. Therefore, the force balance on the piston leads to:

$$p_{atm} * A_{piston} + M_{piston} * g = p_{h2o} * A_{piston}$$

$$100{,}000 * 0.1 + M_{piston} * 9.81 = 200{,}000 * 0.1$$

$$\boxed{\mathbf{M_{piston} = 1019\ kg}}$$

At 40°C and 200 kPa, the water is compressed liquid.

$$v_1 \approx v_{f@40°C} = 0.001008\ m^3/kg$$

$$\boxed{\mathbf{M_{water} = \frac{V_1}{v_1} = \frac{0.05}{0.001008} = 49.6\ kg}}$$

When the entire liquid vaporizes, the water is saturated vapor at the same pressure of 200 kPa:

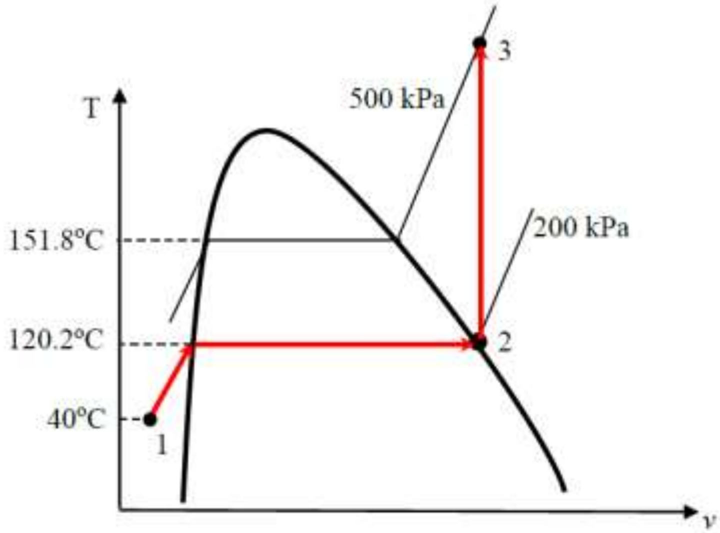
$$v_2 = v_{g@200\ kPa} = 0.88578\ m^3/kg$$

$$\boxed{\mathbf{V_2 = v_2 * M_{water} = 0.88578 * 49.6 = 44\ m^3}}$$

When the water pressure is 5 bar, the specific volume is 0.88578 m^3/kg

From table A-6, the temperature is:

$$\boxed{\mathbf{T_3 = 688\ °C}}$$



Name:	Program ID:

Model answer - Midterm Exam # 1

Fall 2014

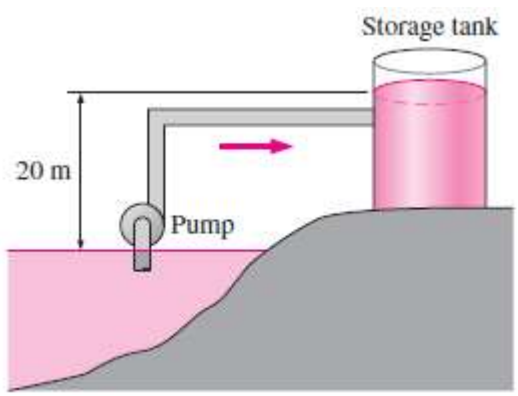
Time allowed: one hour

Round-off all numbers of your answer into reasonable digits

1. (10 points)

Water is pumped from a lake to a storage tank 20 m above at a rate of 70 L/s while consuming 20.4 kW of electric power. Disregarding any frictional losses in the pipes and any changes in kinetic energy, determine:

- **a.** The overall efficiency of the pump–motor unit. **(5 points)**
- **b.** The pressure difference between the inlet and the exit of the pump. **(5 points)**



$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.070\ \text{m}^3/\text{s}) = 70\ \text{kg/s}$$

$$pe_2 = gz_2 = (9.81\ \text{m/s}^2)(20\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.196\ \text{kJ/kg}$$

Then the rate of increase of the mechanical energy of water becomes

$$\Delta\dot{E}_{\text{mech,fluid}} = \dot{m}(e_{\text{mech,out}} - e_{\text{mech,in}}) = \dot{m}(pe_2 - 0) = \dot{m}pe_2 = (70\ \text{kg/s})(0.196\ \text{kJ/kg}) = 13.7\ \text{kW}$$

The overall efficiency of the combined pump-motor unit is determined from its definition,

$$\eta_{\text{pump-motor}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}} = \frac{13.7\ \text{kW}}{20.4\ \text{kW}} = 0.672 \quad \text{or} \quad \mathbf{67.2\%}$$

(*b*) Now we consider the pump. The change in the mechanical energy of water as it flows through the pump consists of the change in the flow energy only since the elevation difference across the pump and the change in the kinetic energy are negligible. Also, this change must be equal to the useful mechanical energy supplied by the pump, which is 13.7 kW:

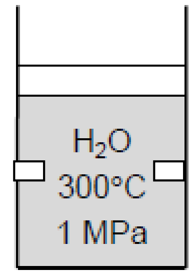
$$\Delta\dot{E}_{\text{mech,fluid}} = \dot{m}(e_{\text{mech,out}} - e_{\text{mech,in}}) = \dot{m}\frac{P_2 - P_1}{\rho} = \dot{V}\Delta P$$

Solving for ΔP and substituting,

$$\Delta P = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{V}} = \frac{13.7\ \text{kJ/s}}{0.070\ \text{m}^3/\text{s}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3}{1\ \text{kJ}}\right) = \mathbf{196\ kPa}$$

Therefore, the pump must boost the pressure of water by 196 kPa in order to raise its elevation by 20 m.

2. (15 points)



A piston–cylinder device contains 8 kg of steam at 300°C and 1 MPa. Steam is cooled at constant pressure until one-half of the mass condenses, at this point the piston is touching a set of two stops. Now more heat is rejected from the water till a temperature of 165°C is achieved.

a. **What is the initial volume of the water? (3 points)**
b. **What is the volume of the saturated liquid when the piston touches the stops? (3 points)**
c. **What is the final pressure of the water? (3 points)**
d. **What is the final quality of the water? (3 points)**
e. **Show the process on a p-v diagram with respect to saturation lines. (3 points)**

From table A-6, at 300°C and 1 MPa:

$$v_1 = 0.25799 \text{ m}^3/\text{kg}$$

$$\boxed{\mathbf{V_1 = v_1 * M_{water} = 0.25799 * 8 = 2.1\ m^3}}$$

The piston is moving freely under constant pressure before it reaches the stops:

$$v_2 = v_{f2} + x_2(v_{g2} - v_{f2}) = 0.001127 + 0.5 * (0.19436 - 0.001127) = 0.0977 \text{ m}^3/\text{kg}$$

$$\boxed{\mathbf{V_{f2} = v_{f2} * M_{water} = 0.001127 \ * (0.5 * 8) = 0.0045\ m^3 = 4.5\ L}}$$

When the water temperature is 165°C, the specific volume is 0.0977 m^3/kg ($v_2 = v_3$)
From table A-4, the pressure is:

$$\boxed{\mathbf{p_3 = 701\ kPa}}$$

$$v_f = 0.001108 \text{ m}^3/\text{kg}, \qquad v_g = 0.27244 \text{ m}^3/\text{kg}$$

$$v_3 = v_{f3} + x_3(v_{g3} - v_{f3}) = 0.001108 + x_3 * (0.27244 - 0.001108) = 0.0977 \text{ m}^3/\text{kg}$$

$$\boxed{\mathbf{x_3 = 35.6\%}}$$