



Faculty of Engineering

MEP 382: Design of Applied Measurement Systems

Lecture 8: Filters

Overview

- Introduction to Passive and Active Filters
- Filter Categories
- Butterworth/Chebyshev/Bessel Filters
- Poles and Multiple Stages
- Transfer Function
- Bode Plot

Definition

- A *filter* is a system that processes a signal in some desired fashion.

Types of Filters

- There are two broad categories of filters:
 - An *analog filter* processes continuous-time signals
 - A *digital filter* processes discrete-time signals.
- They can also be categorized as
 - Passive filters
 - Active filters

Passive Filters

- Made up of passive components - resistors, capacitors and inductors
- No amplifying elements (- transistors, op-amps, etc)
- No signal gain
- 1st order - design is simple (just use standard equations to find resonant frequency of the circuit)
- 2nd order - complex equations
- Require no power supplies
- Not restricted by the bandwidth limitations of the op-amps
- Can be used at very high frequencies
- Can handle larger current or voltage levels than active devices
- Buffer amplifiers might be required

Passive elements : Inductor

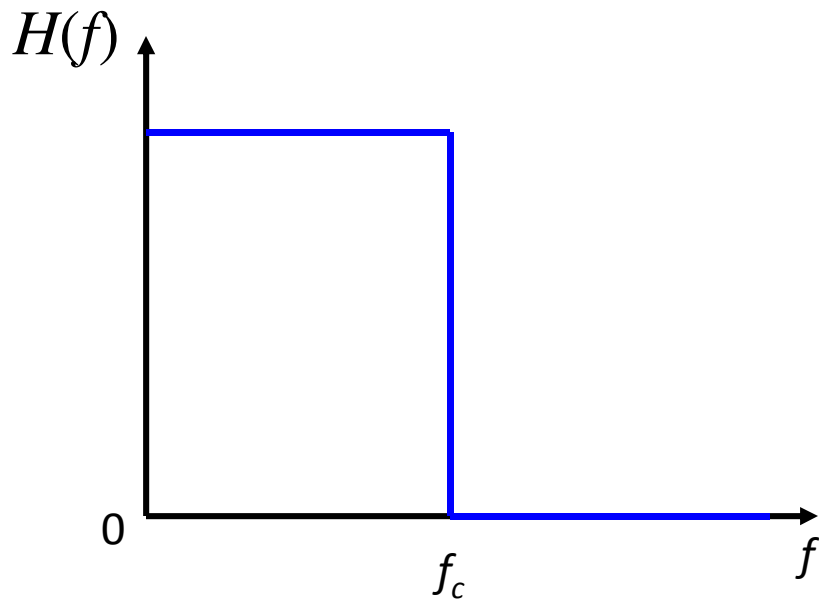
BIG PROBLEM!

- High accuracy (1% or 2%), small physical size, or large inductance values are required
- Standard values of inductors are not very closely spaced
- Difficult to find an off-the-shelf inductor within 10% of any arbitrary value
- Adjustable inductors are used
- Tuning such inductors to the required values is time-consuming and expensive for larger quantities of filters
- Inductors are often prohibitively expensive

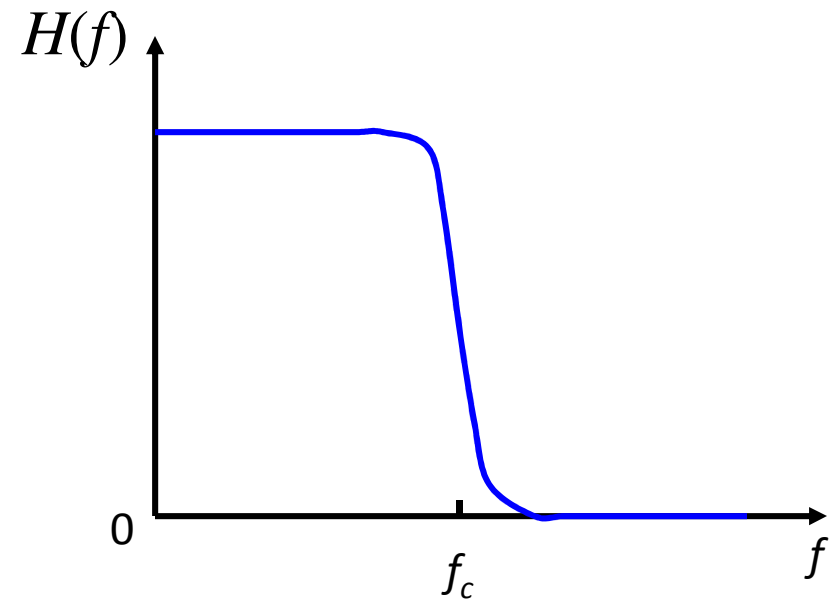
Active Filters

- No inductors
- Made up of op-amps, resistors and capacitors
- Provides virtually any arbitrary gain
- Generally easier to design
- High input impedance prevents excessive loading of the driving source
- Low output impedance prevents the filter from being affected by the load
- At high frequencies is limited by the gain-bandwidth of the op-amps
- Easy to adjust over a wide frequency range without altering the desired response

Analog Filter Response (Bode Plot)



Ideal "brick wall" filter

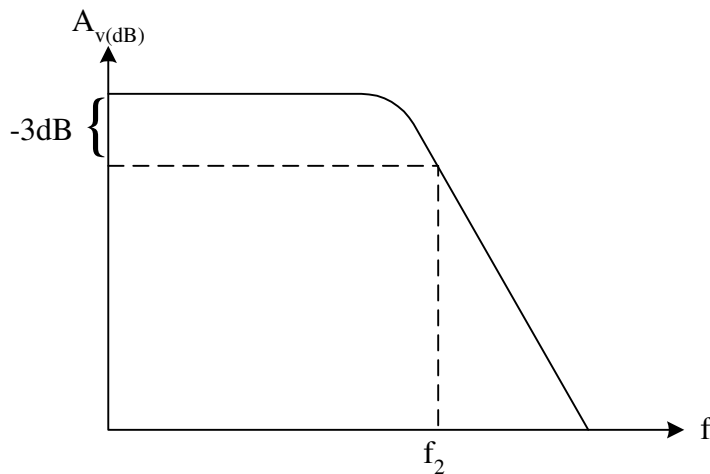


Practical filter

Categories of Filters

Low Pass Filters (LPF):

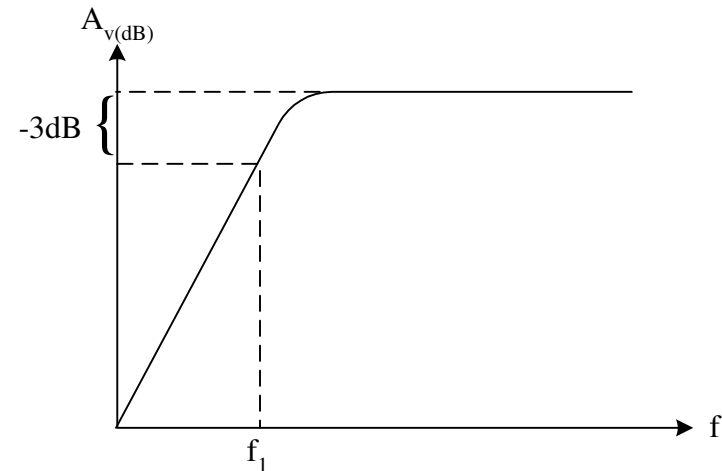
pass all frequencies from dc up to the upper cutoff frequency.



Low-pass response

High Pass Filters (HPF):

pass all frequencies that are above its lower cutoff frequency

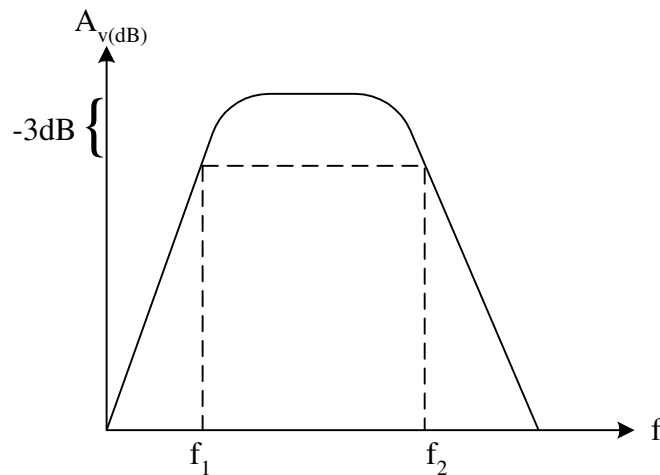


High-pass response

Categories of Filters

Band Pass Filters:

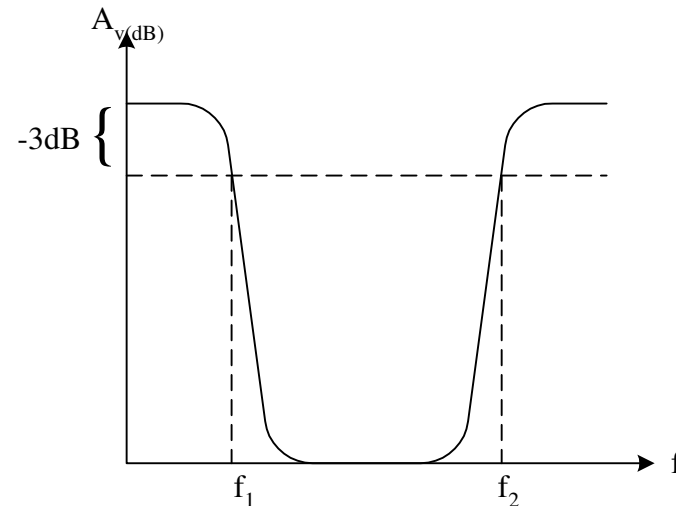
pass only the frequencies that fall between its values of the lower and upper cutoff frequencies.



Band Pass Response

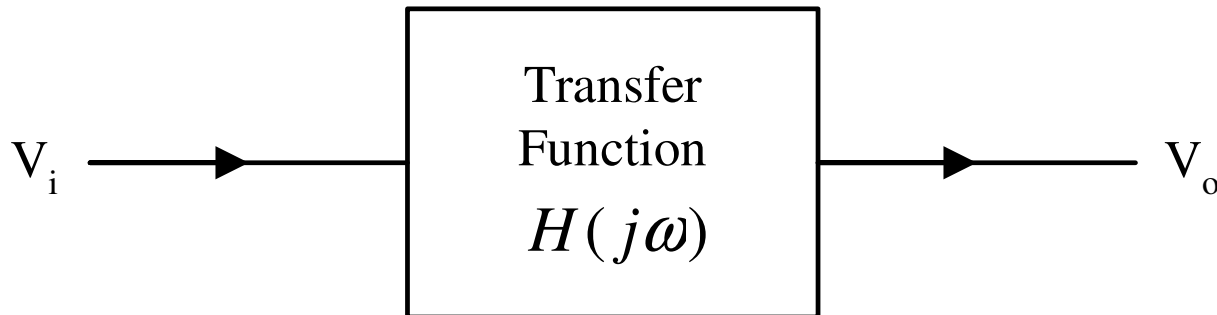
Band Stop (Notch) Filters:

eliminate all signals within the stop band while passing all frequencies outside this band.



Band Stop Response

Transfer function $H(j\omega)$



$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

$$|H| = \sqrt{\text{Re}(H)^2 + \text{Im}(H)^2}$$

$$H = \text{Re}(H) + j \text{Im}(H)$$

$$\angle H = \tan^{-1} \left(\frac{\text{Im}(H)}{\text{Re}(H)} \right) \quad \text{Re}(H) > 0$$

$$\angle H = 180^\circ + \tan^{-1} \left(\frac{\text{Im}(H)}{\text{Re}(H)} \right) \quad \text{Re}(H) < 0$$

Frequency transfer function of filter $H(j\omega)$

(I) Low - Pass Filter

$$|H(j\omega)| = 1 \quad f < f_o$$

$$|H(j\omega)| = 0 \quad f > f_o$$

(II) High - Pass Filter

$$|H(j\omega)| = 0 \quad f < f_o$$

$$|H(j\omega)| = 1 \quad f > f_o$$

(III) Band - Pass Filter

$$|H(j\omega)| = 1 \quad f_L < f < f_H$$

$$|H(j\omega)| = 0 \quad f < f_L \text{ and } f > f_H$$

(IV) Band - Stop (Notch) Filter

$$|H(j\omega)| = 0 \quad f_L < f < f_H$$

$$|H(j\omega)| = 1 \quad f < f_L \text{ and } f > f_H$$

(V) All - Pass (or phase - shift) Filter

$$|H(j\omega)| = 1 \quad \text{for all } f$$

has a specific phase response

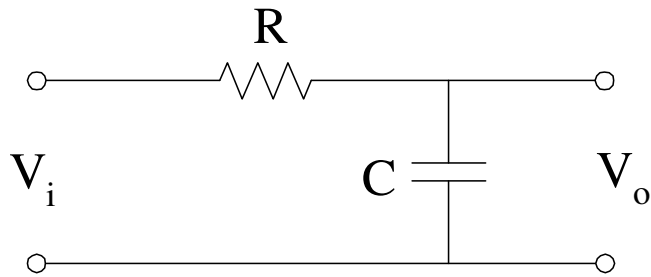
Bode Plots

- Bode plots are important when considering the frequency response characteristics of amplifiers. They plot the magnitude or phase of a transfer function in dB versus frequency.

Poles & Zeros of the transfer function

- pole—value of s where the denominator goes to zero.
- zero—value of s where the numerator goes to zero.

Passive Single-Pole LPF



$$V_o = \frac{X_C}{X_C + R} V_i$$

$$V_o = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i = \frac{1}{1 + j\omega CR} V_i$$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}} \quad \text{where} \quad \omega_0 = \frac{1}{RC}$$

or

$$H(s) = \frac{\omega_0}{s + \omega_0}$$

where

$$s = j\omega$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$\omega \rightarrow 0 \Rightarrow |V_o| = |V_i| \leftarrow \text{max. value}$$

$$\omega \rightarrow \infty \Rightarrow |V_o| = 0 \leftarrow \text{min. value}$$

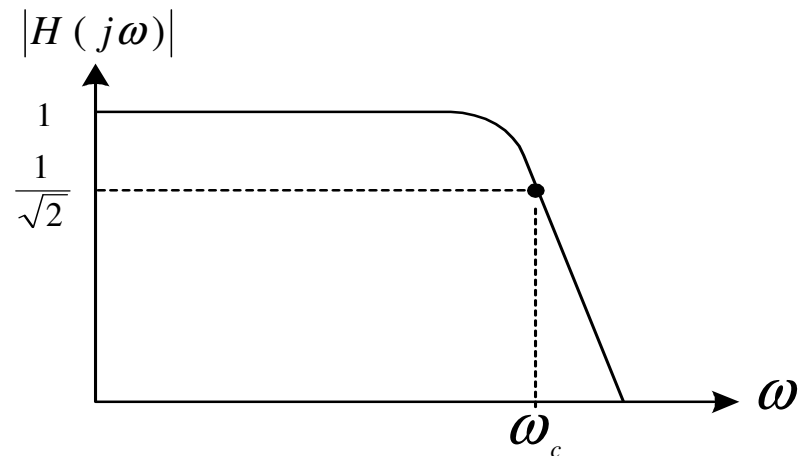
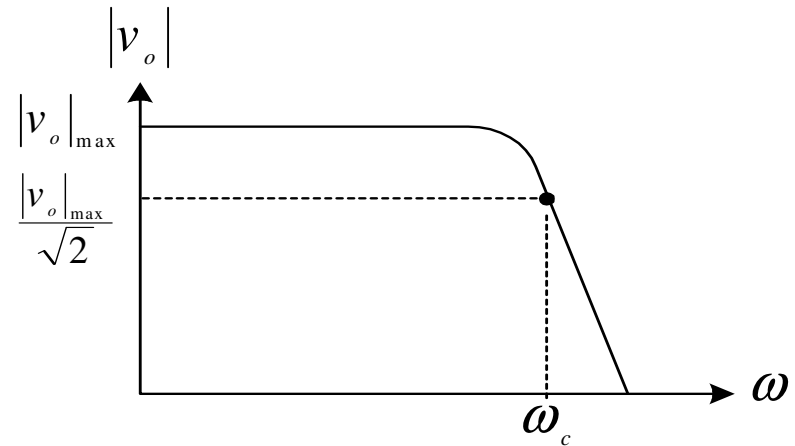
$$V_o = \frac{1}{1 + j\omega CR} V_i$$

$$\omega = \frac{1}{RC} \Rightarrow |V_o| = ??$$

$$V_o = \frac{1}{1 + j} V_i$$

$$|V_o| = \frac{1}{\sqrt{1^2 + 1^2}} |V_i| = \frac{1}{\sqrt{2}} |V_i|$$

$$\omega_c = \omega_o = \frac{1}{RC} \text{ (cut - off frequency)}$$



Bode Plot (single pole)

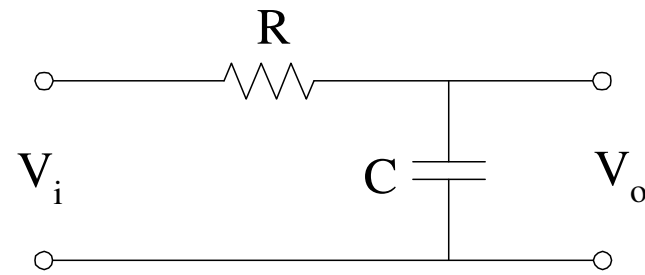
$$H(j\omega) = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j\left(\frac{\omega}{\omega_o}\right)}$$

$$\Rightarrow |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}}$$

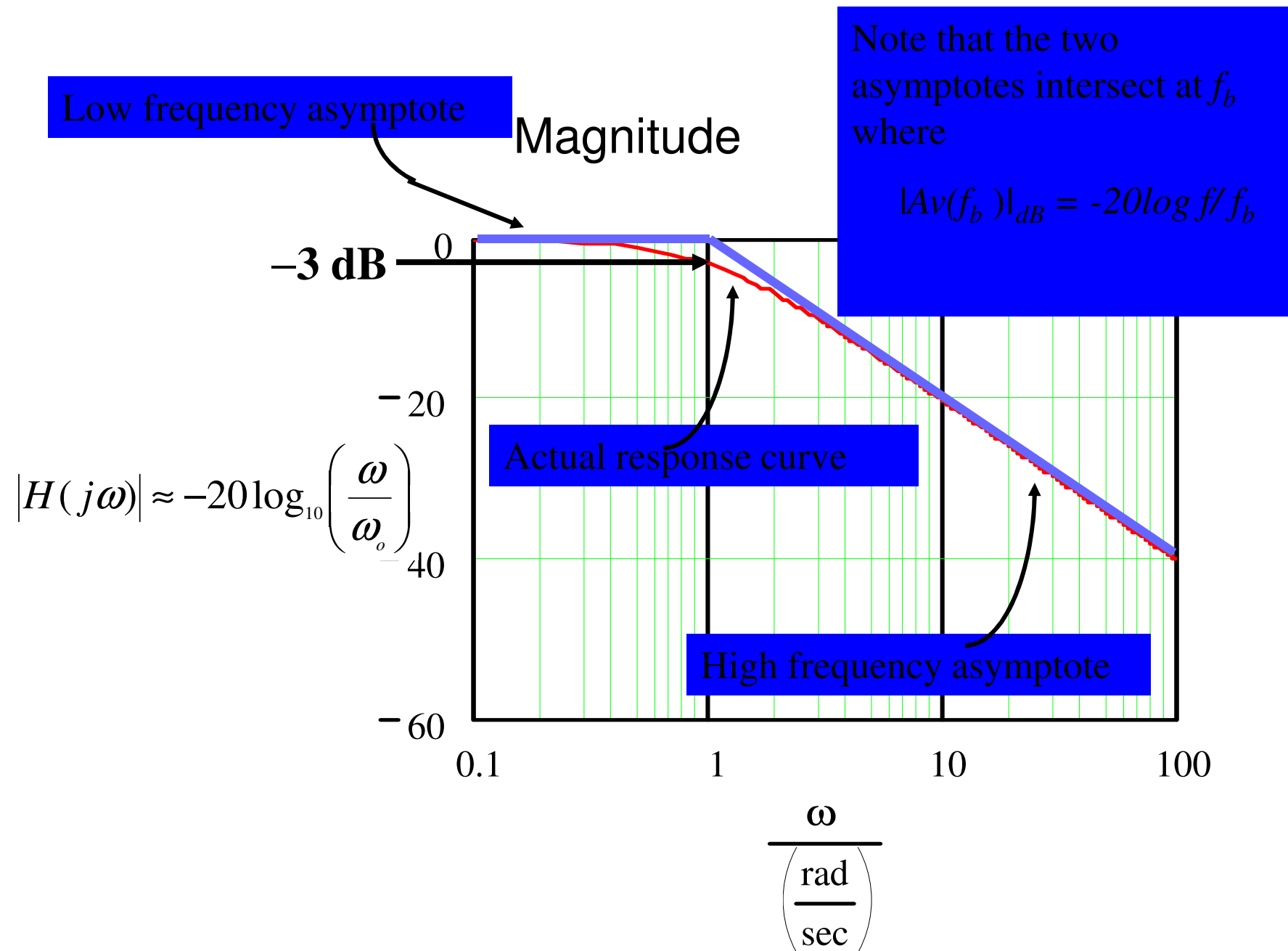
$$|H(j\omega)|_{dB} = 20\log_{10}|H(j\omega)| = 20\log_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}}\right)$$

For $\omega \gg \omega_o$

$$|H(j\omega)|_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_o}\right)$$



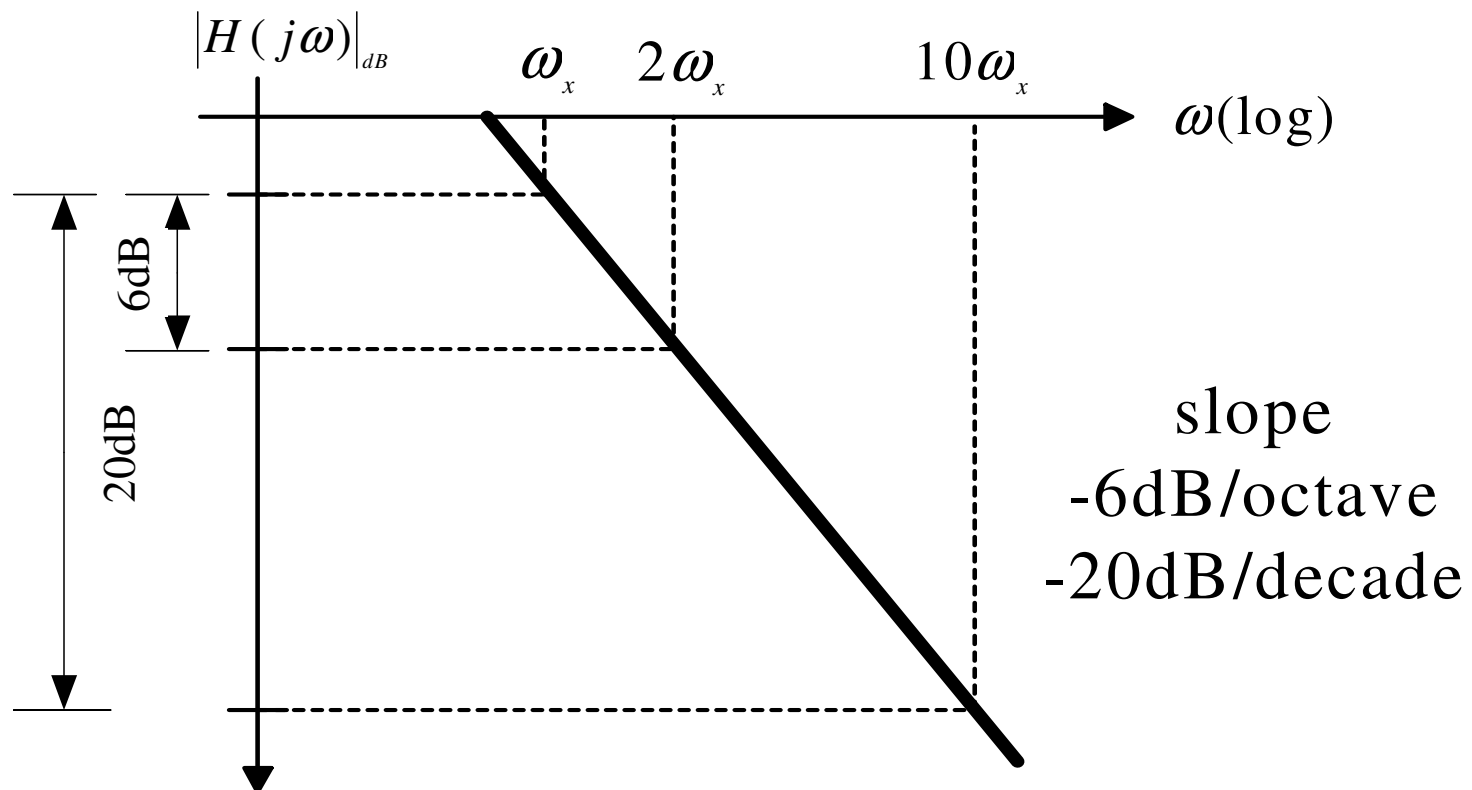
Single pole low-pass filter



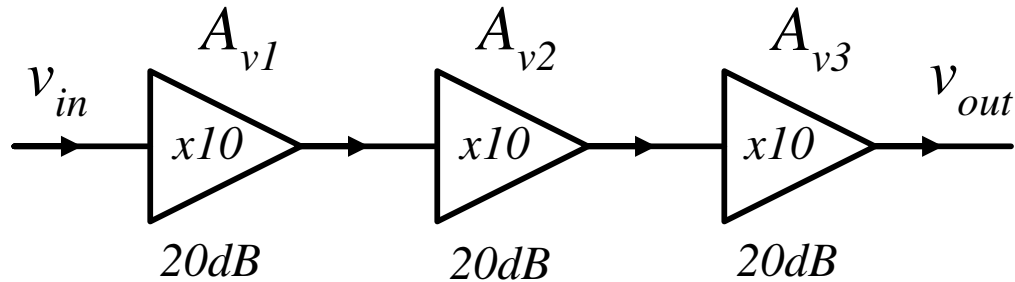
$$|H(j\omega)| \approx -20 \log_{10} \left(\frac{\omega}{\omega_o} \right)$$

For octave apart, $\frac{\omega}{\omega_o} = \frac{2}{1}$ $|H(j\omega)| \approx -6dB$

For decade apart, $\frac{\omega}{\omega_o} = \frac{10}{1}$ $|H(j\omega)| \approx -20dB$



Cascaded System



$$A_v = A_{v1} \times A_{v2} \times A_{v3}$$

$$A_v = 10 \times 10 \times 10 = 10^3$$

$$A_v (dB) = 20 \log_{10} (A_{v1} \times A_{v2} \times A_{v3})$$

$$A_v (dB) = 20 \log_{10} (A_{v1}) + 20 \log_{10} (A_{v2}) + 20 \log_{10} (A_{v3})$$

$$A_v (dB) = A_{v1} (dB) + A_{v2} (dB) + A_{v3} (dB)$$

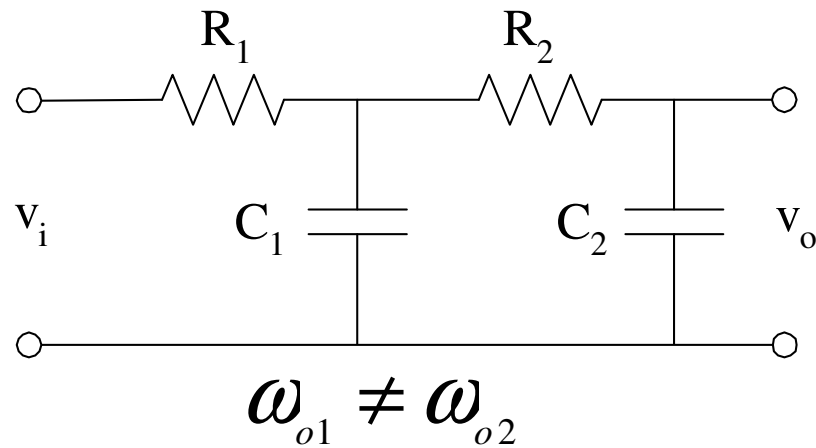
$$A_v (dB) = 20dB + 20dB + 20dB$$

$$A_v (dB) = 60dB$$

$$20 \log_{10} (10) = 20dB$$

$$20 \log_{10} (10^3) = 60dB$$

Bode plot (Two-pole)



$$|H(j\omega)| = 20 \log_{10} \left\{ 1 / \left(\sqrt{1 + \left(\frac{\omega}{\omega_{o1}} \right)^2} \sqrt{1 + \left(\frac{\omega}{\omega_{o2}} \right)^2} \right) \right\}$$

- The technique for approximating a filter function based on Bode plots is useful for low order, simple filter designs
- More complex filter characteristics are more easily approximated by using some well-described rational functions, the roots of which have already been tabulated and are well-known.