



Faculty of Engineering

**MEP382: Design of Applied Measurement Systems**

# **Lecture 13:**

## **Oscillators**

# Agenda

- Introduction of Oscillator
- Linear Oscillator
  - Wien Bridge Oscillator
  - RC Phase-Shift Oscillator
  - LC Oscillator
- Stability

# Oscillators

**Oscillation:** an effect that repeatedly and regularly fluctuates about the mean value

**Oscillator:** circuit that produces oscillation

**Characteristics:** wave-shape, frequency, amplitude, distortion, stability

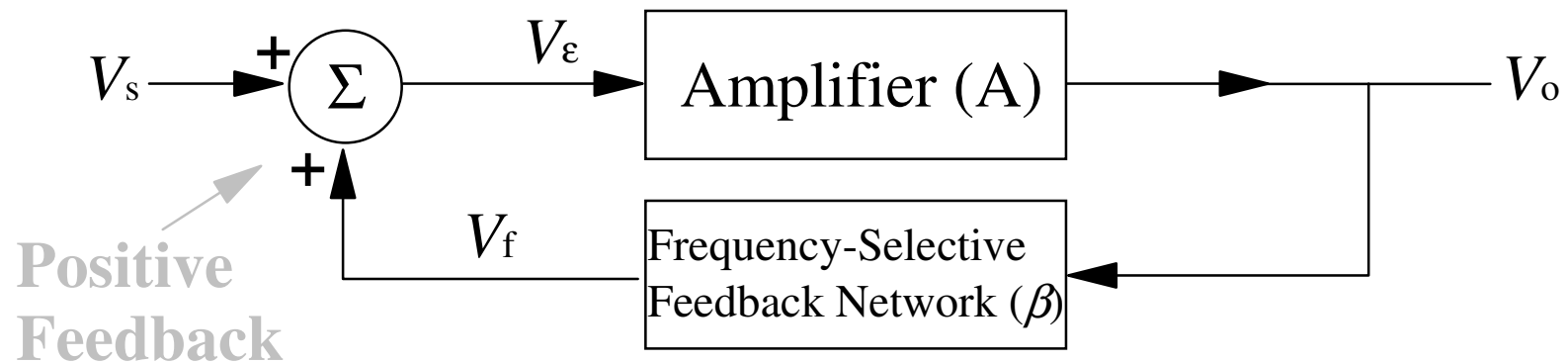
# Application of Oscillators

- Oscillators are used to generate signals, e.g.
  - Used as a local oscillator to transform the RF signals to IF signals in a receiver;
  - Used to generate RF carrier in a transmitter
  - Used to generate clocks in digital systems;
  - Used as sweep circuits in TV sets and CRO.

# Linear Oscillators

1. Wien Bridge Oscillators
2. RC Phase-Shift Oscillators
3. LC Oscillators
4. Stability

# Integrand of Linear Oscillators

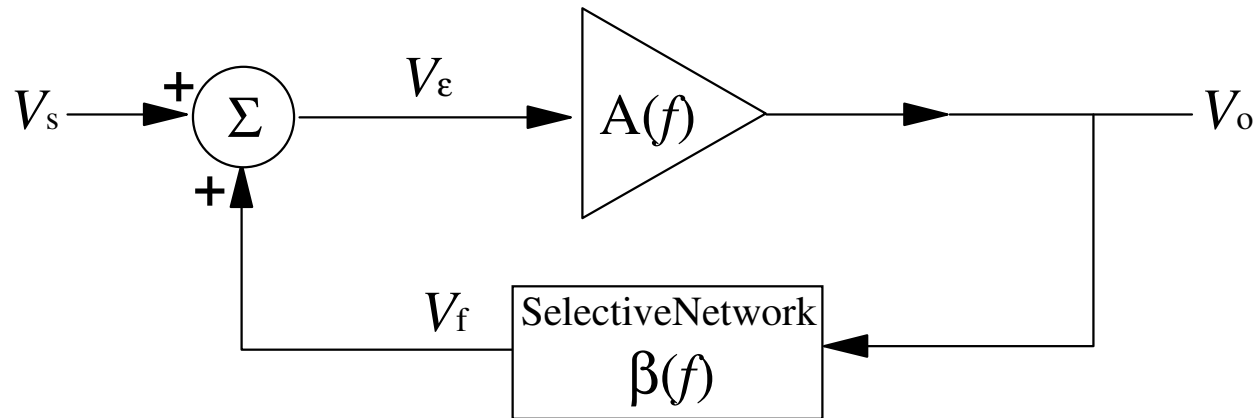


For sinusoidal input is connected  
“**Linear**” because the output is approximately sinusoidal

A linear oscillator contains:

- a frequency selection feedback network
- an amplifier to maintain the loop gain at **unity**

# Basic Linear Oscillator



$$V_o = AV_\epsilon = A(V_s + V_f) \quad \text{and} \quad V_f = \beta V_o$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{A}{1 - A\beta}$$

If  $V_s = 0$ , the only way that  $V_o$  can be nonzero is that **loop gain  $A\beta=1$**  which implies that

$$\begin{aligned} |A\beta| &= 1 \\ \angle A\beta &= 0 \end{aligned} \quad (\mathbf{Barkhausen\ Criterion})$$

# Wien Bridge Oscillator

Let  $X_{C1} = \frac{1}{\omega C_1}$  and  $X_{C2} = \frac{1}{\omega C_2}$

$$Z_1 = R_1 - jX_{C1}$$

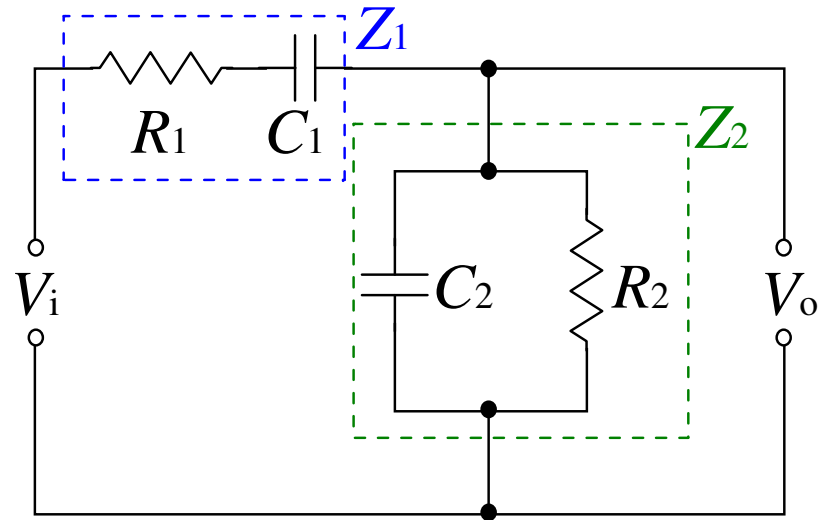
$$Z_2 = \left[ \frac{1}{R_2} + \frac{1}{-jX_{C2}} \right]^{-1} = \frac{-jR_2 X_{C2}}{R_2 - jX_{C2}}$$

Therefore, the feedback factor,

$$\beta = \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{(-jR_2 X_{C2} / R_2 - jX_{C2})}{(R_1 - jX_{C1}) + (-jR_2 X_{C2} / R_2 - jX_{C2})}$$

$$\beta = \frac{-jR_2 X_{C2}}{(R_1 - jX_{C1})(R_2 - jX_{C2}) - jR_2 X_{C2}}$$

Frequency Selection Network





$\beta$  can be rewritten as:

$$\beta = \frac{R_2 X_{C2}}{R_1 X_{C2} + R_2 X_{C1} + R_2 X_{C2} + j(R_1 R_2 - X_{C1} X_{C2})}$$

For **Barkhausen Criterion**, imaginary part = 0, i.e.,

$$R_1 R_2 - X_{C1} X_{C2} = 0$$

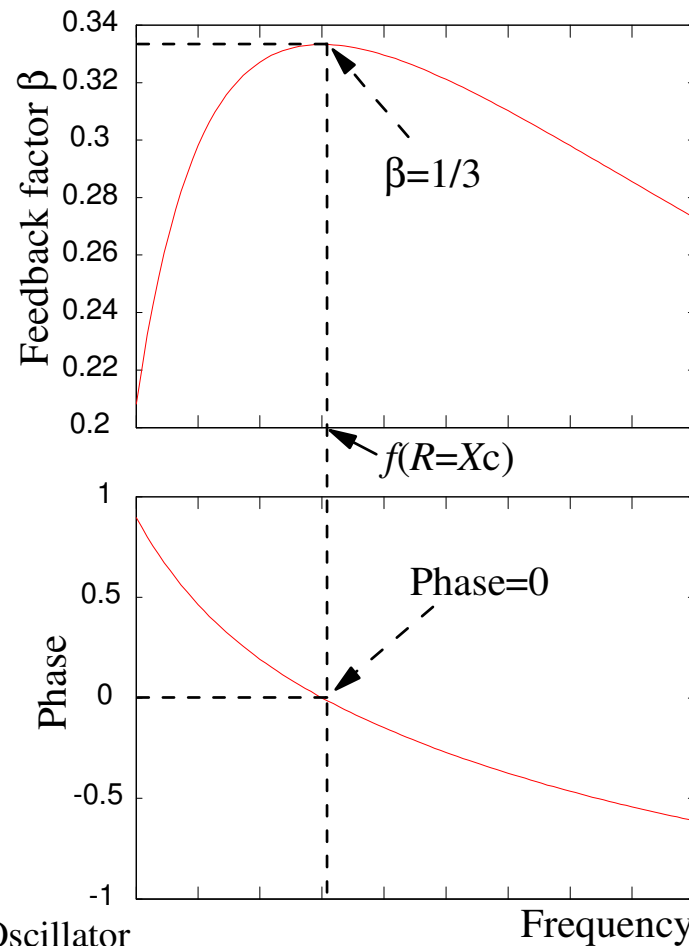
$$\text{or } R_1 R_2 = \frac{1}{\omega C_1} \frac{1}{\omega C_2}$$

$$\Rightarrow \omega = 1 / \sqrt{R_1 R_2 C_1 C_2}$$

Supposing,

$$R_1 = R_2 = R \text{ and } X_{C1} = X_{C2} = X_C,$$

$$\beta = \frac{R X_C}{3R X_C + j(R^2 - X_C^2)}$$



# Example

By setting  $\omega = \frac{1}{RC}$ , we get

Imaginary part = 0 and  $\beta = \frac{1}{3}$

Due to **Barkhausen Criterion**,

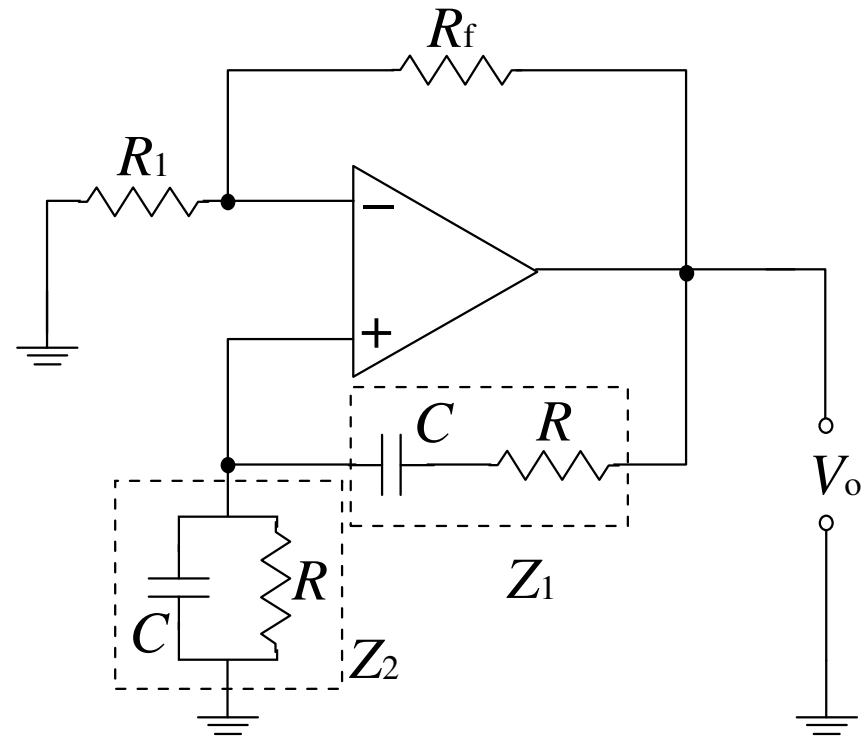
Loop gain  $A_v\beta=1$

where

$A_v$  : Gain of the amplifier

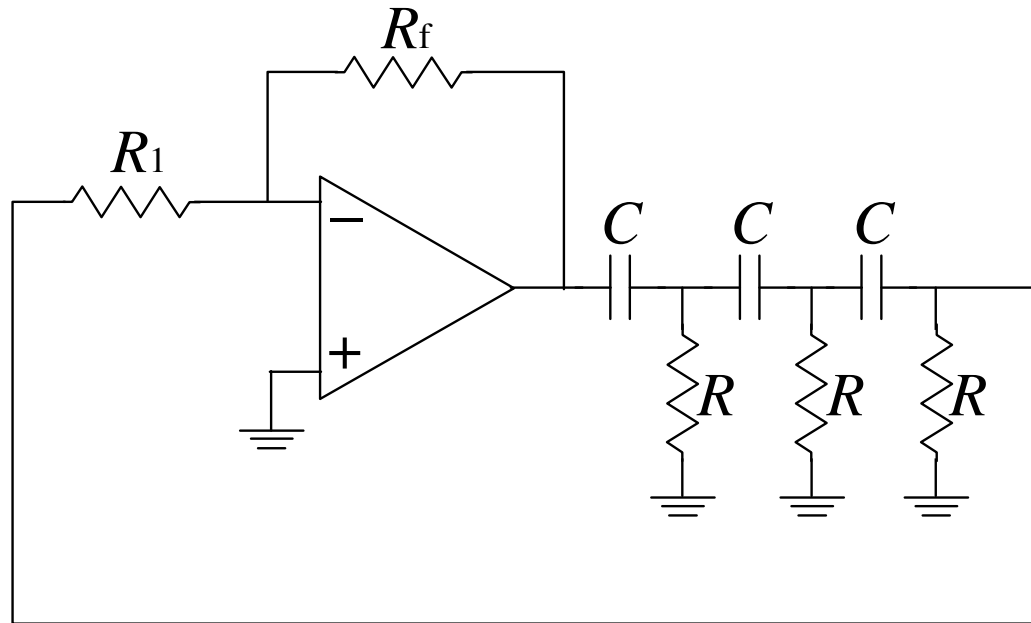
$$A_v\beta = 1 \Rightarrow A_v = 3 = 1 + \frac{R_f}{R_1}$$

$$\text{Therefore, } \frac{R_f}{R_1} = 2$$



Wien Bridge Oscillator

# RC Phase-Shift Oscillator



- Using an inverting amplifier
- The additional  $180^\circ$  phase shift is provided by an RC phase-shift network

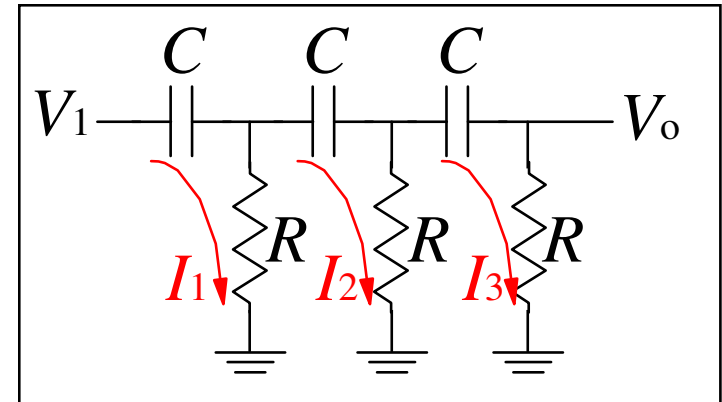
Applying KVL to the phase-shift network, we have

$$V_1 = I_1(R - jX_C) - I_2R$$

$$0 = -I_1R + I_2(2R - jX_C) - I_3R$$

$$0 = -I_2R + I_3(2R - jX_C)$$

Solve for  $I_3$ , we get



$$I_3 = \frac{\begin{vmatrix} R-jX_C & -R & V_1 \\ -R & 2R-jX_C & 0 \\ 0 & -R & 0 \end{vmatrix}}{\begin{vmatrix} R-jX_C & -R & 0 \\ -R & 2R-jX_C & -R \\ 0 & -R & 2R-jX_C \end{vmatrix}}$$

$$\text{Or } I_3 = \frac{V_1 R^2}{(R - jX_C)[(2R - jX_C)^2 - R^2] - R^2(2R - jX_C)}$$

The output voltage,

$$V_o = I_3 R = \frac{V_1 R^3}{(R - jX_C)[(2R - jX_C)^2 - R^2] - R^2(2R - jX_C)}$$

Hence the transfer function of the phase-shift network is given by,

$$\beta = \frac{V_o}{V_1} = \frac{R^3}{(R^3 - 5RX_C^2) + j(X_C^3 - 6R^2X_C)}$$

For 180° phase shift, the imaginary part = 0, i.e.,

$$X_C^3 - 6R^2X_C = 0 \quad \text{or} \quad X_C = 0 \text{ (Rejected)}$$

$$\Rightarrow X_C^2 = 6R^2$$

$$\omega = \frac{1}{\sqrt{6RC}}$$

and,

$$\beta = -\frac{1}{29}$$

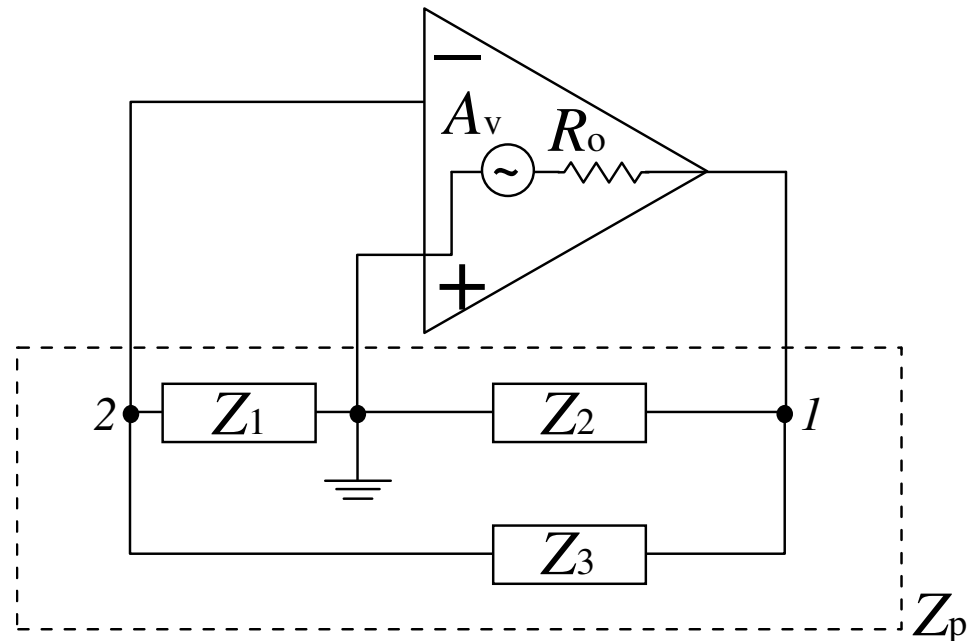
Note: The -ve sign mean the phase inversion from the voltage

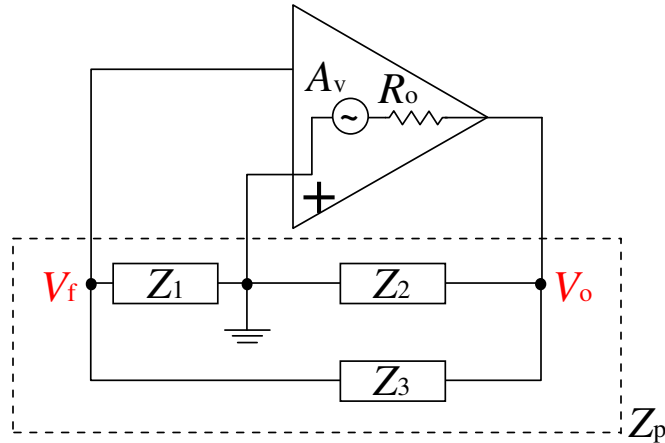
# LC Oscillators

- The frequency selection network ( $Z_1$ ,  $Z_2$  and  $Z_3$ ) provides a phase shift of  $180^\circ$
- The amplifier provides an additional shift of  $180^\circ$

Two well-known Oscillators:

- Colpitts Oscillator
- Harley Oscillator

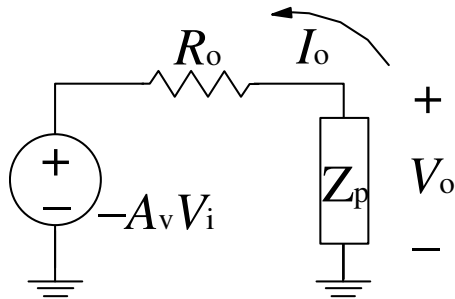




$$V_f = \beta V_o = \frac{Z_1}{Z_1 + Z_3} V_o$$

$$Z_p = Z_2 \parallel (Z_1 + Z_3) = \frac{Z_2(Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}$$

For the equivalent circuit from the output



$$\frac{-A_v V_i}{R_o + Z_p} = \frac{V_o}{Z_p} \quad \text{or} \quad \frac{V_o}{V_i} = \frac{-A_v Z_p}{R_o + Z_p}$$

Therefore, the amplifier gain is obtained,

$$A = \frac{V_o}{V_i} = \frac{-A_v Z_2 (Z_1 + Z_3)}{R_o (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$$

The loop gain,

$$A\beta = \frac{-A_v Z_1 Z_2}{R_o(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

If the impedance are all pure reactances, i.e.,

$$Z_1 = jX_1, \quad Z_2 = jX_2 \text{ and } Z_3 = jX_3$$

The loop gain becomes,  $A\beta = \frac{A_v X_1 X_2}{jR_o(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$

The imaginary part = 0 only when  $X_1 + X_2 + X_3 = 0$

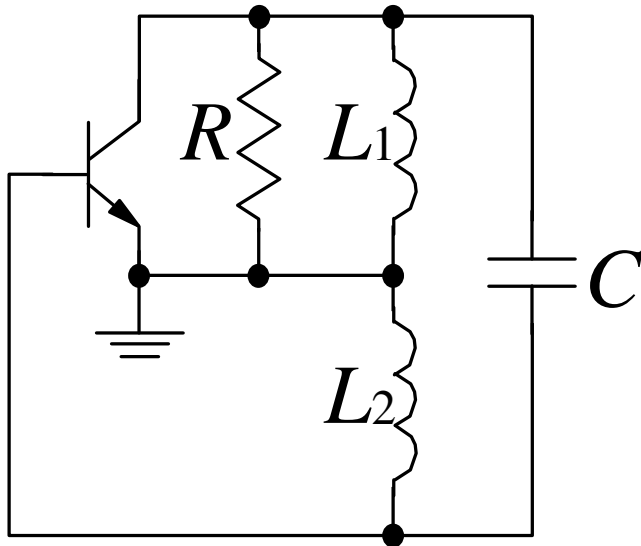
- It indicates that at least one reactance must be -ve (capacitor)
- $X_1$  and  $X_2$  must be of same type and  $X_3$  must be of opposite type

With imaginary part = 0,  $A\beta = \frac{-A_v X_1}{X_1 + X_3} = \frac{A_v X_1}{X_2}$

For Unit Gain & 180° Phase-shift,  $A\beta = 1 \Rightarrow A_v = \frac{X_2}{X_1}$



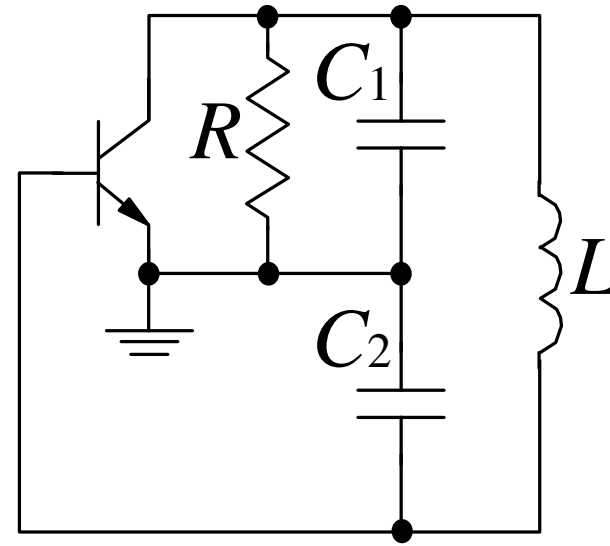
# Hartley Oscillator



$$\omega_o = \frac{1}{\sqrt{(L_1 + L_2)C}}$$

$$g_m = \frac{L_1}{RL_2}$$

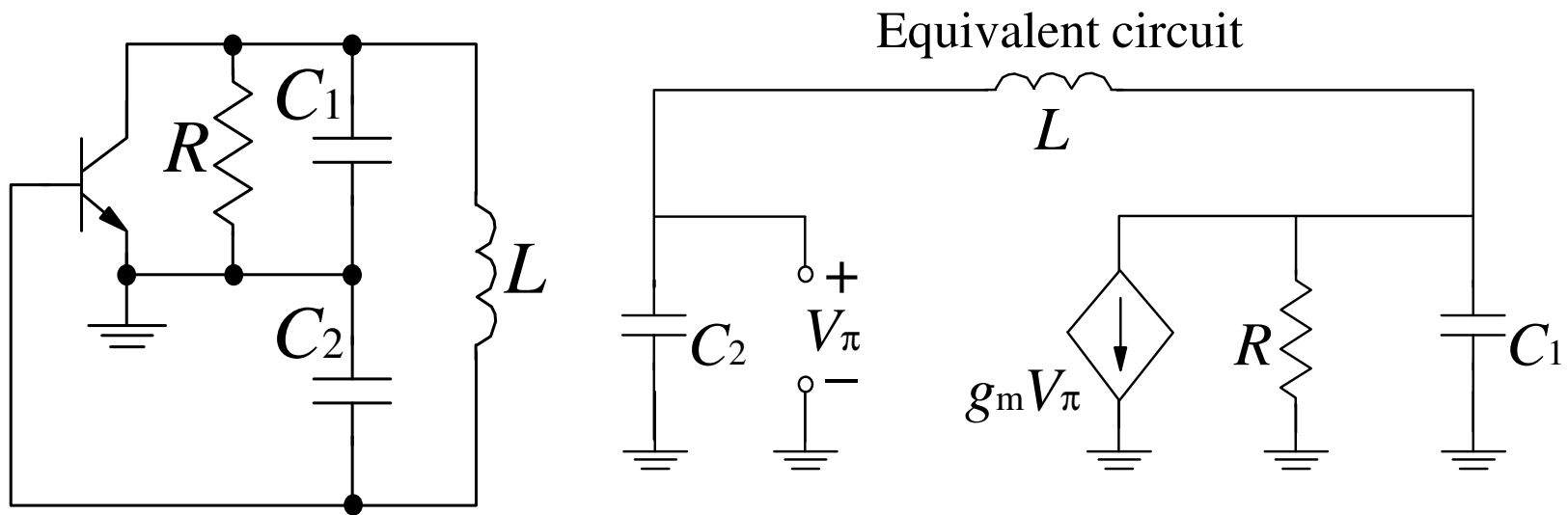
# Colpitts Oscillator



$$\omega_o = \frac{1}{\sqrt{LC_T}} \quad C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$g_m = \frac{C_2}{RC_1}$$

# Colpitts Oscillator



In the equivalent circuit, it is assumed that:

- Linear small signal model of transistor is used
- The transistor capacitances are neglected
- Input resistance of the transistor is large enough

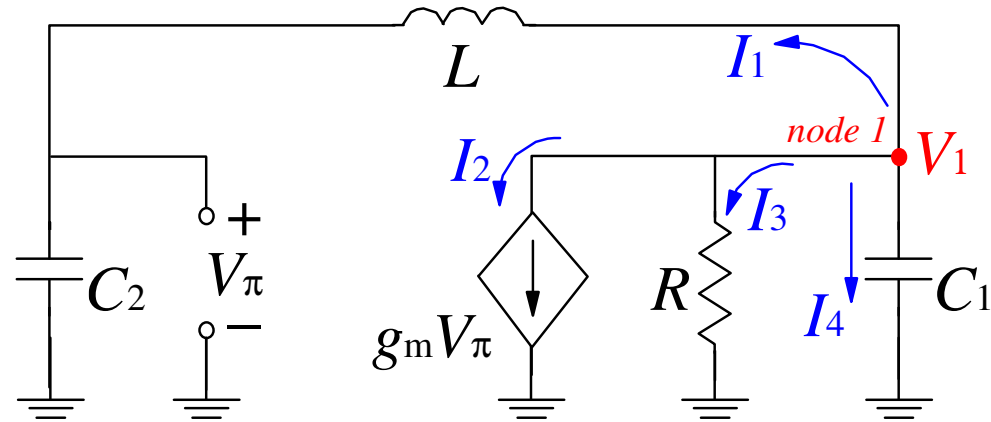
At node 1,

$$V_1 = V_\pi + i_1(j\omega L)$$

where,

$$i_1 = j\omega C_2 V_\pi$$

$$\Rightarrow V_1 = V_\pi (1 - \omega^2 LC_2)$$



Apply KCL at node 1, we have

$$j\omega C_2 V_\pi + g_m V_\pi + \frac{V_1}{R} + j\omega C_1 V_1 = 0$$

$$j\omega C_2 V_\pi + g_m V_\pi + V_\pi (1 - \omega^2 LC_2) \left( \frac{1}{R} + j\omega C_1 \right) = 0$$

For Oscillator  $V_\pi$  must not be zero, therefore it enforces,

$$\left( g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R} \right) + j[\omega(C_1 + C_2) - \omega^3 LC_1 C_2] = 0$$

$$\left( g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R} \right) + j[\omega(C_1 + C_2) - \omega^3 LC_1 C_2] = 0$$

Imaginary part = 0, we have

$$\omega_o = \frac{1}{\sqrt{LC_T}} \quad C_T = \frac{C_1 C_2}{C_1 + C_2}$$

Real part = 0, yields

$$g_m = \frac{C_2}{RC_1}$$

# Frequency Stability

- The frequency stability of an oscillator is defined as

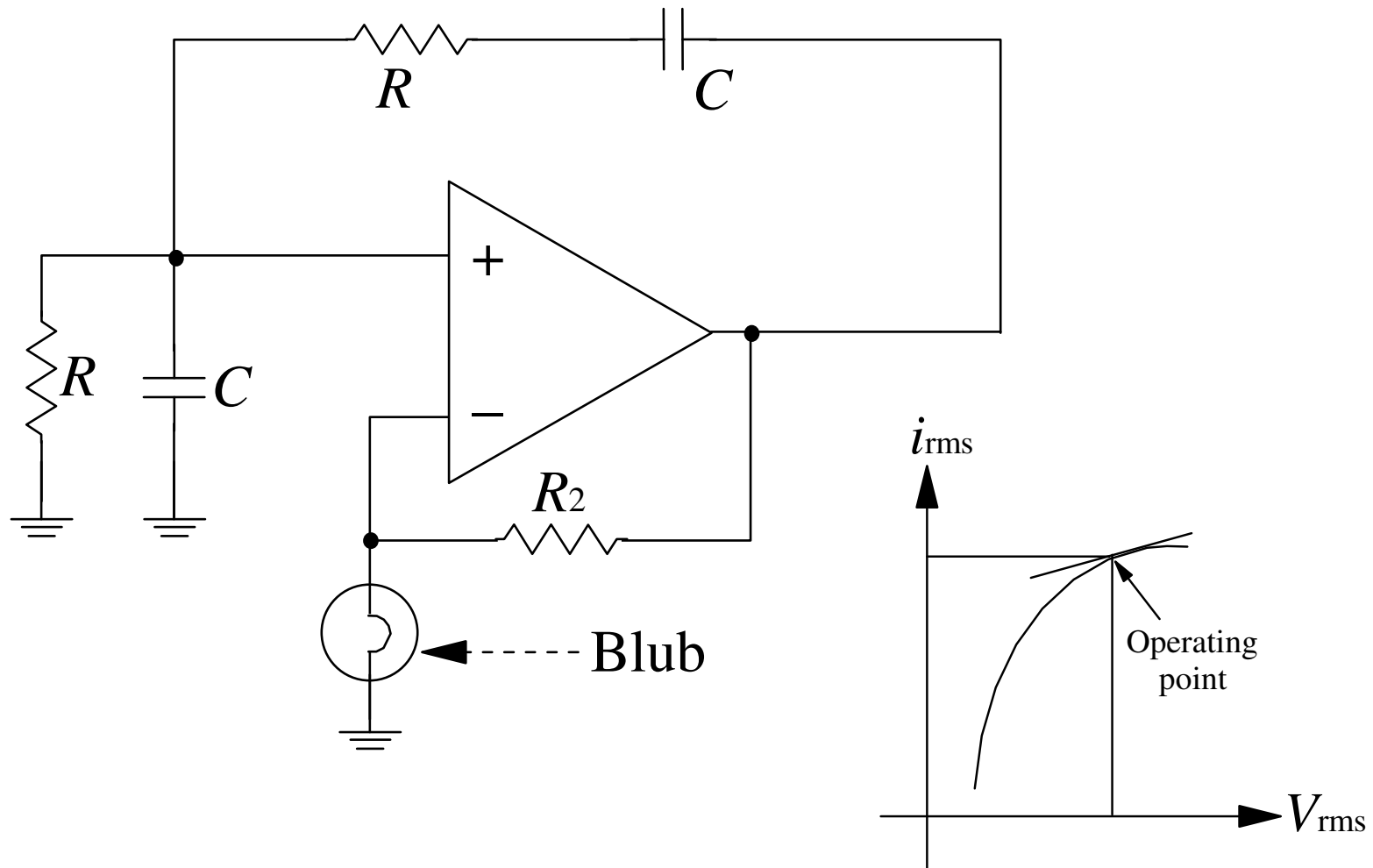
$$\frac{1}{\omega_o} \cdot \left( \frac{d\omega}{dT} \right)_{\omega = \omega_o} \quad \text{ppm/}^\circ\text{C}$$

- Use high stability capacitors, e.g. silver mica, polystyrene, or teflon capacitors and low temperature coefficient inductors for high stable oscillators.

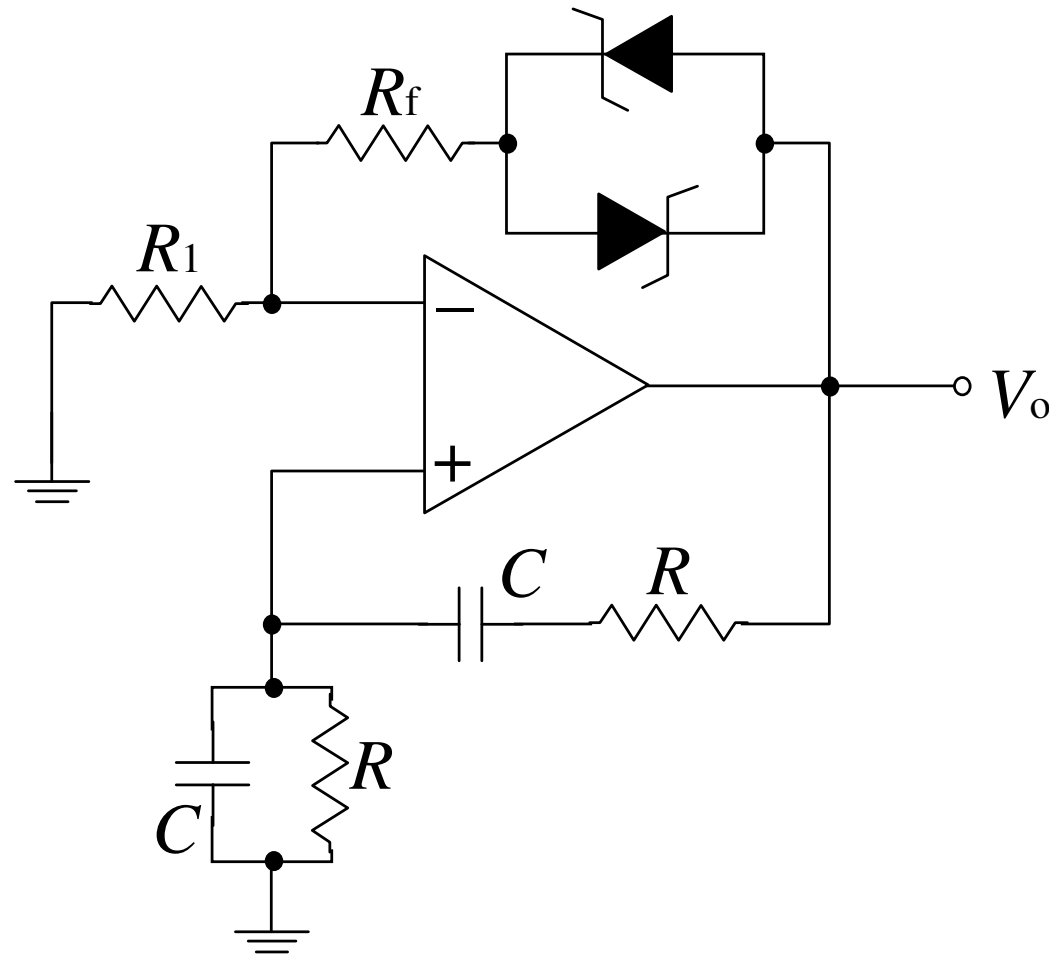
# Amplitude Stability

- In order to start the oscillation, the loop gain is usually slightly greater than unity.
- LC oscillators in general do not require amplitude stabilization circuits because of the selectivity of the LC circuits.
- In RC oscillators, some non-linear devices, e.g. NTC/PTC resistors, FET or zener diodes can be used to stabilize the amplitude

# Wien-bridge oscillator with bulb stabilization

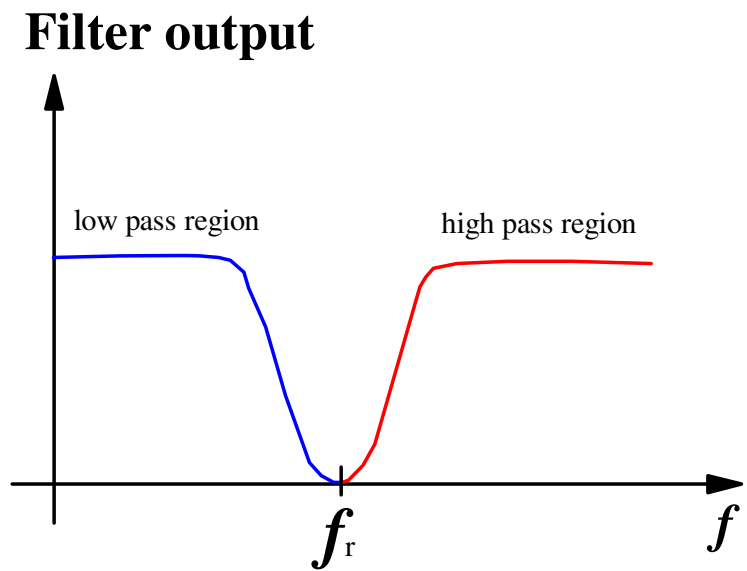
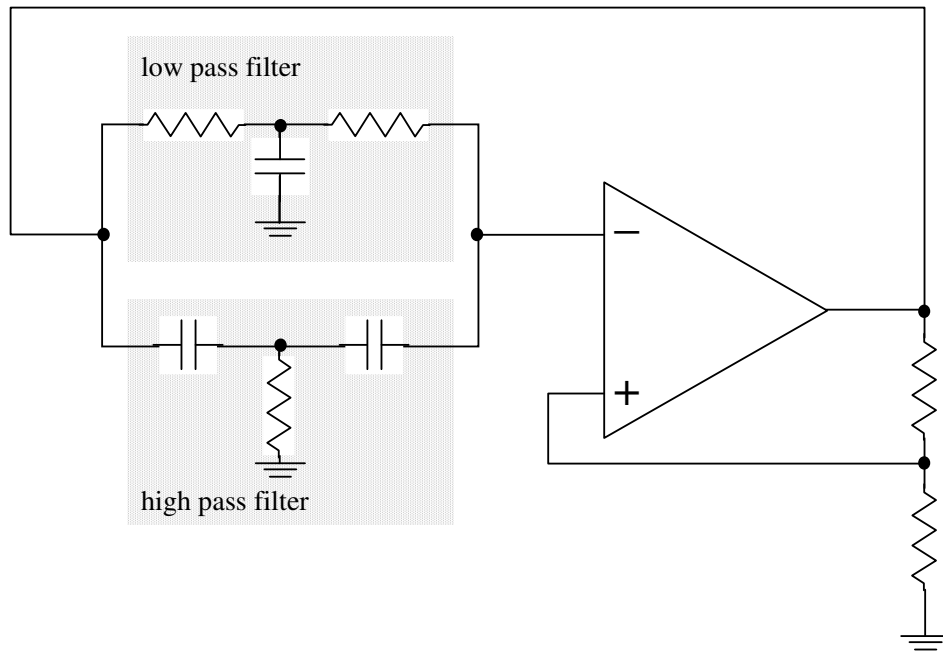


# Wien-bridge oscillator with diode stabilization

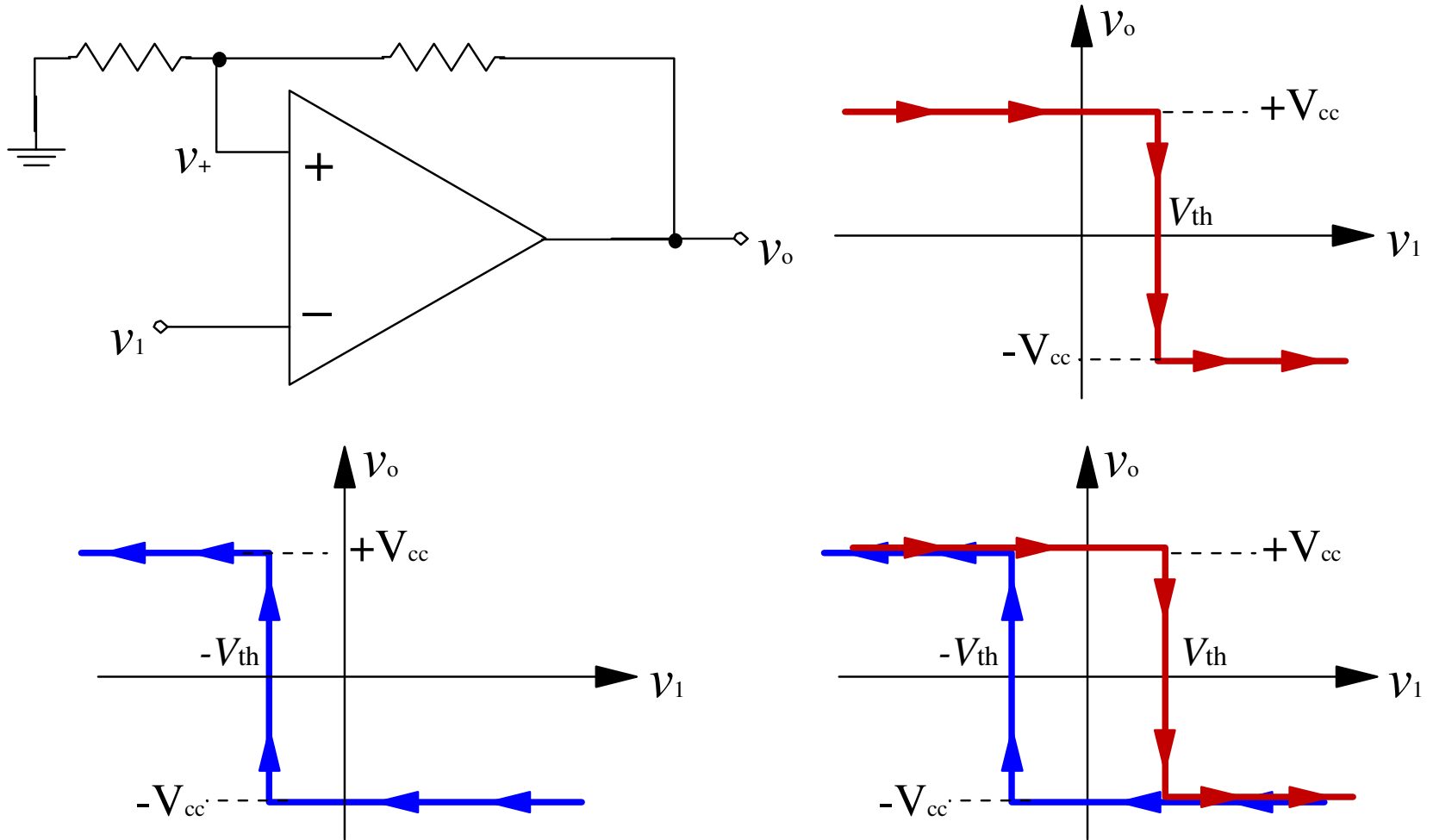




# Twin-T Oscillator



# Bistable Circuit



# A Square-wave Oscillator

