

Thermodynamics

ENGR360-MEP112

LECTURE 6

Objectives:

1. Moving boundary work.
2. Specific heat at constant volume (c_v) and the specific heat at constant pressure (c_p).
3. Relation between the specific heats (c_v & c_p) with the change in internal energy (ΔU) and enthalpy change (ΔH) of ideal gases, liquids and solids.

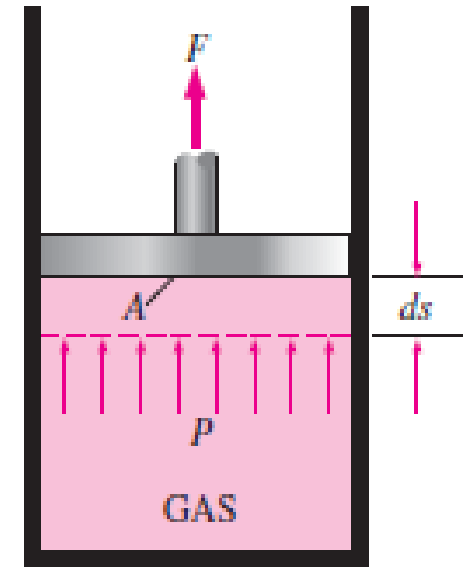
1. MOVING BOUNDARY WORK

□ One form of mechanical work frequently encountered in practice is associated with the expansion or compression of a gas in a piston–cylinder device. Therefore, the expansion and compression work is often called **moving boundary work**.

$$\delta W_b = F ds = pA ds = pdV$$

$$W_b = \int_1^2 pdV \quad (\text{J})$$

where **p** is the gas absolute pressure.



A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds .

$$W_b = \sum W_{\text{surrounding}} !!!$$

1. MOVING BOUNDARY WORK

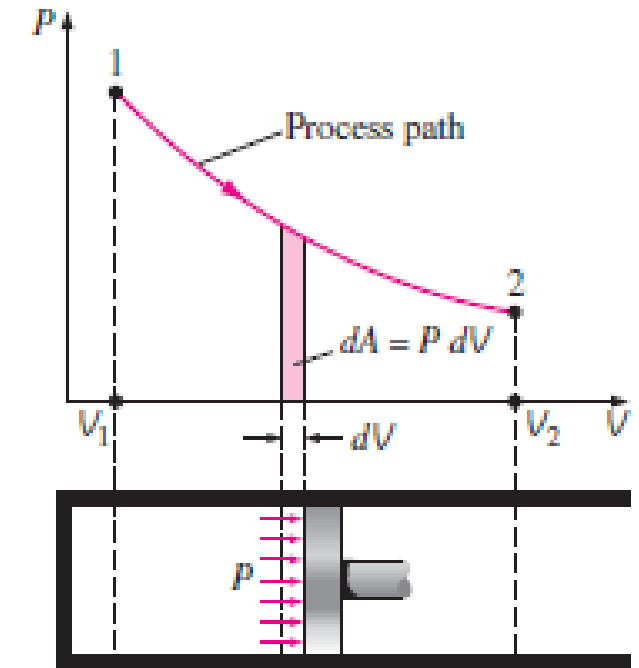
□ Quasi-equilibrium process

The area under the process curve on a p - V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.

○ On the p - v diagram, it represents the boundary work done per unit mass (J/kg).

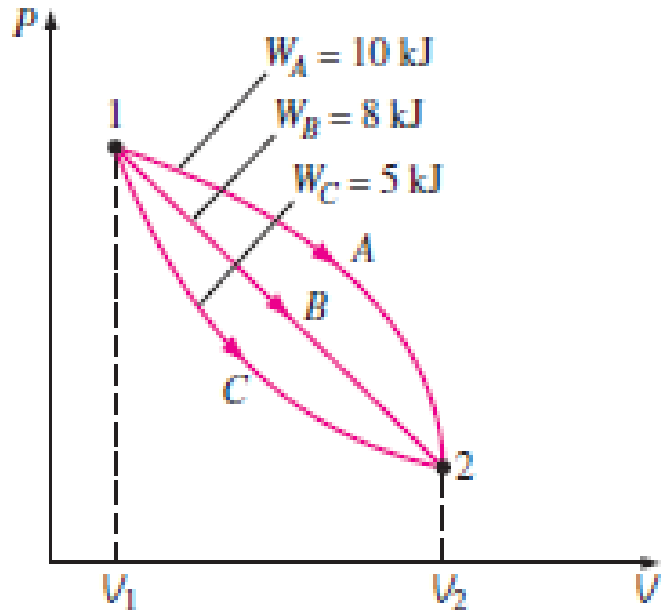
$$\text{Area}_{(p-V)} = A = \int_1^2 dA = \int_1^2 p dV \quad (\text{J})$$

$$\text{Area}_{(p-v)} = A = \int_1^2 dA = \int_1^2 p dv \quad (\text{J/kg})$$



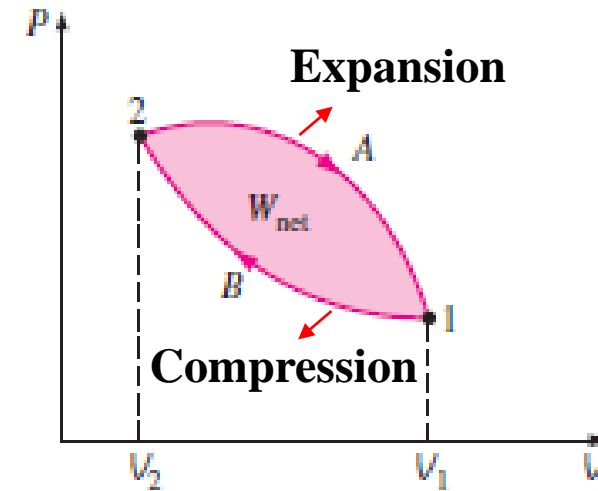
The area under the process curve on a P - V diagram represents the boundary work.

1. MOVING BOUNDARY WORK



The boundary work done during a process depends on the path followed as well as the end states.

$$W_A = +10 \text{ kJ} > W_B = +8 \text{ kJ} > W_C = +5 \text{ kJ}$$



The net work done during a cycle is the difference between the work done by the system and the work done on the system.

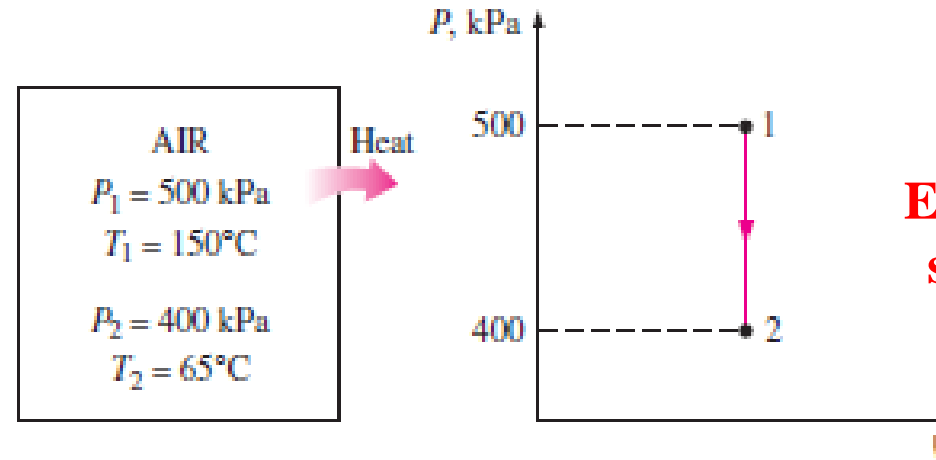
$$W_{\text{net}} = \int_1^2 p dV = \int_1^2 p dV + \int_2^1 p dV$$

$$W_{\text{net}} = W_B + W_A$$

-ve +ve

1. MOVING BOUNDARY WORK

a. Constant volume process (Isochoric):

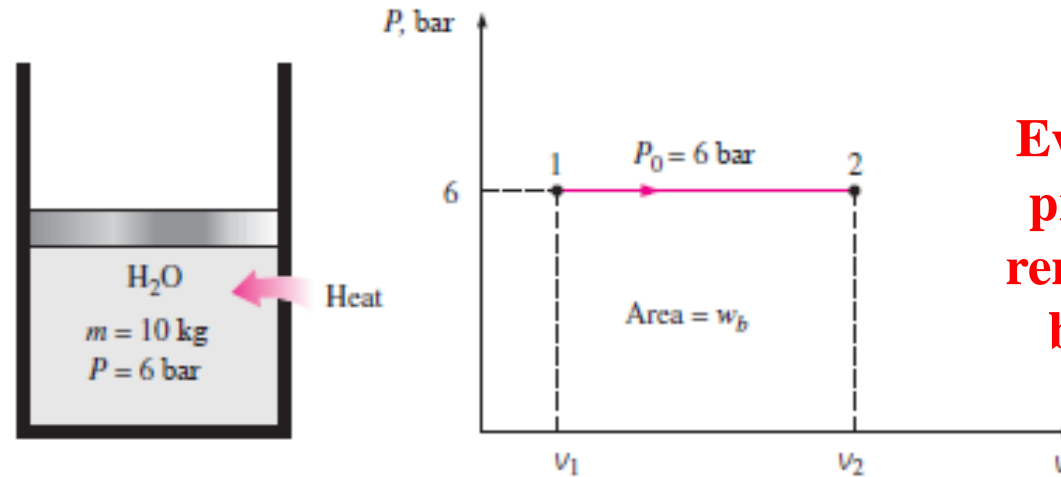


Energy interaction between the system and its surrounding is due to heat only

$$W_b = \int_1^2 p dV = 0$$

1. MOVING BOUNDARY WORK

b. Constant pressure process (Isobaric):



Even though it is not explicitly stated, the pressure of the steam inside the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant.

$$W_b = \int_1^2 p dV = p_o \int_1^2 dV$$

$$W_b = p_o (V_2 - V_1) = m p_o (v_2 - v_1)$$

1. MOVING BOUNDARY WORK

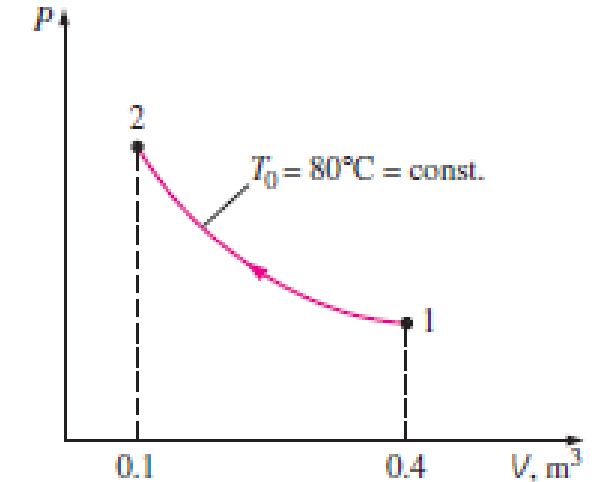
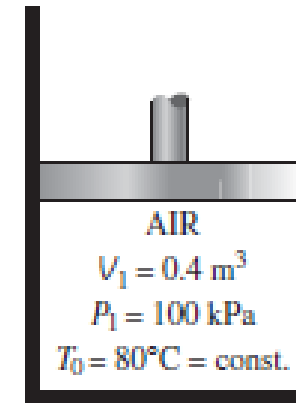
c. Constant temperature process (Isothermal):

○ For ideal gases:

$$pV = mRT_0, \quad T \text{ is constant} = T_0$$

Therefore; $p_1 V_1 = p_2 V_2 = mRT_0 = \text{constant}$

$$W_b = \int_1^2 p dV = \int_1^2 \frac{mRT_0}{V} dV = mRT_0 \int_1^2 \frac{dV}{V}$$



$$W_b = \begin{cases} mRT_0 \ln \left(\frac{V_2}{V_1} \right) = p_1 V_1 \ln \left(\frac{V_2}{V_1} \right) = p_2 V_2 \ln \left(\frac{V_2}{V_1} \right) \\ mRT_0 \ln \left(\frac{p_1}{p_2} \right) = p_1 V_1 \ln \left(\frac{p_1}{p_2} \right) = p_2 V_2 \ln \left(\frac{p_1}{p_2} \right) \end{cases}$$

1. MOVING BOUNDARY WORK

c. Polytropic process:

○ In a polytropic process:

$pV^n = \text{constant (C)}$, Therefore; $p_1 V_1^n = p_2 V_2^n = C$

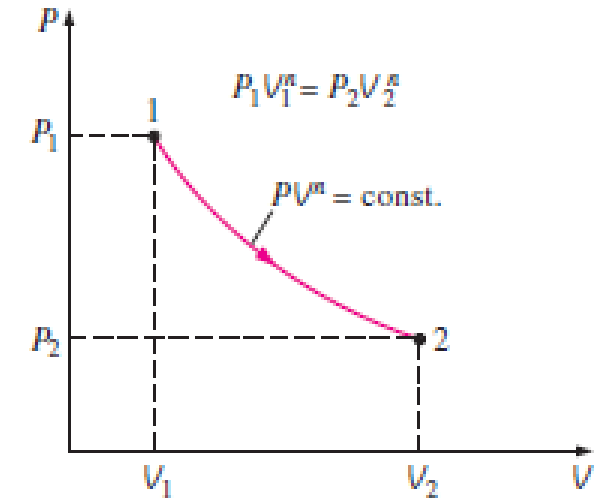
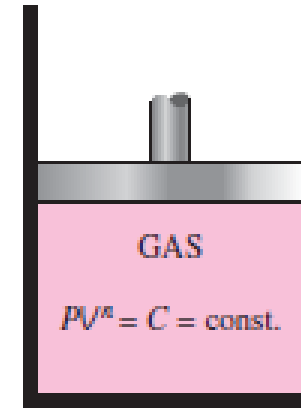
n is called the **polytropic index**.

○ For ideal gases:

$pV = mRT$

$$W_b = \int_1^2 p dV = C \int_1^2 \frac{dV}{V^n} = C \left[\frac{V_2^{1-n} - V_1^{1-n}}{1-n} \right] = \left[\frac{(p_2 V_2^n) V_2^{1-n} - (p_1 V_1^n) V_1^{1-n}}{1-n} \right]$$

$$W_b = \frac{p_2 V_2 - p_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

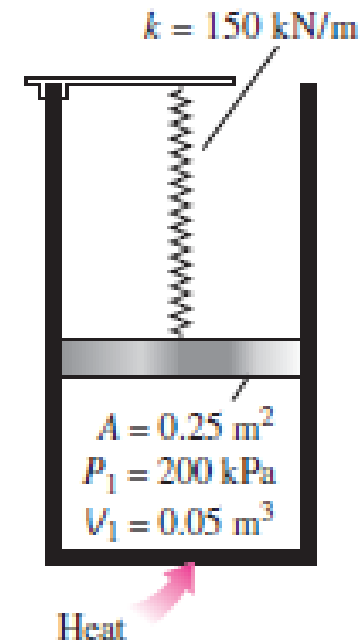


1. MOVING BOUNDARY WORK

□ Example:

A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa . At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2 , when there is no friction between the piston and the cylinder wall during its motion and the atmospheric pressure is 1 bar , determine:

- The mass of the piston.
- The final pressure inside the cylinder.
- The total work done by the gas.
- The work done against the spring to compress it.
- The work done to lift the mass of the piston.
- The work done against the atmosphere.



1. MOVING BOUNDARY WORK

(a) The mass of the piston

$$F_p = F_{atm} + F_W$$

$$2 * 10^5 * 0.25 = 10^5 * 0.25 + M_p * 9.81$$

$$M_p = 2548 \text{ kg} \cong 2.55 \text{ ton}$$

(b) The final pressure inside the cylinder

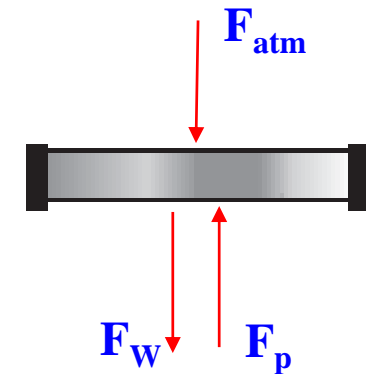
Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising.

$$V_2 = 2V_1 = 2 * 0.05 = 0.1 \text{ m}^3$$

$$\text{The displacement of the piston is then: } \Delta X = \frac{\Delta V}{A_p} = \frac{0.1 - 0.05}{0.25} = 0.2 \text{ m}$$

$$\text{The pressure rise due to the compression of the spring: } \Delta p_s = \frac{F_{spring}}{A_p} = \frac{K * \Delta X}{A_p} = \frac{150 * 10^3 * 0.2}{0.25} = 120 \text{ kPa}$$

$$p_{final} = p_{initial} + \Delta p = 200 + 120 = 320 \text{ kPa} = 3.2 \text{ bar}$$



1. MOVING BOUNDARY WORK

(c) The total work done by the gas (Boundary work)

$$W_b = \int_1^2 p dV = \int_1^2 (p_{\text{initial}} + p_{\text{spring}}) * A_p dx = A_p \int_1^2 \left[p_{\text{initial}} + \left(\frac{K * X}{A_p} \right) \right] dx$$

$$W_b = \int_1^2 [A_p * p_{\text{initial}} + K * x] dx = A_p * p_{\text{initial}} * (x_2 - x_1) + \frac{1}{2} K * (x_2^2 - x_1^2)$$

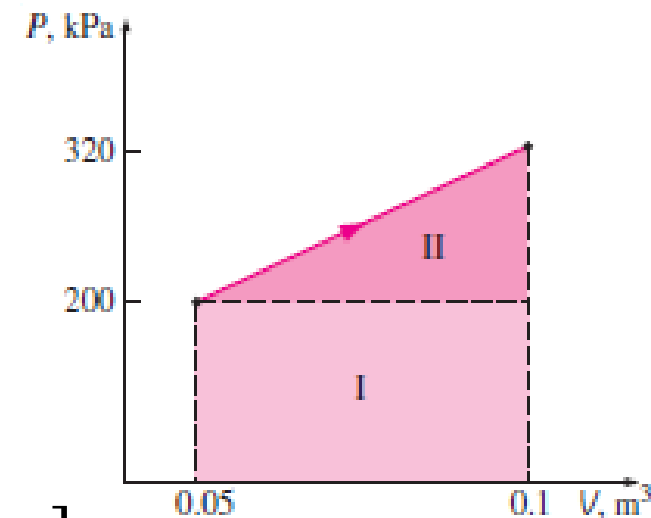
$$W_b = 0.25 * 200 * 10^3 (0.2 - 0) + \frac{1}{2} * 150 * 10^3 (0.2^2 - 0)$$

$$W_b = 13,000 \text{ J} = 13 \text{ kJ}$$

Or

$W_b = \text{Area under } p - V \text{ diagram (I + II)}$

$$W_b = \int_1^2 p dV = (0.1 - 0.05) * \left[200 * 10^3 + \frac{1}{2} (320 * 10^3 - 200 * 10^3) \right] = 13,000 \text{ J} = 13 \text{ kJ}$$



1. MOVING BOUNDARY WORK

(d) The work done against the spring to compress it

$$W_{\text{spring}} = \text{Area under } p - V \text{ diagram (II)} = \frac{1}{2} K(x_2^2 - x_1^2)$$

$$W_{\text{spring}} = (0.1 - 0.05) * \left[\frac{1}{2} (320 * 10^3 - 200 * 10^3) \right] = \frac{1}{2} * 150 * 10^3 * (0.2^2 - 0)$$

$$W_{\text{spring}} = 3,000 \text{ J} = 3 \text{ kJ}$$

(e) The work done to lift the mass of the piston

$$W_{\text{mass}} = M_p * g * \Delta x = 2548 * 9.81 * 0.2 = 5,000 \text{ J} = 5 \text{ kJ}$$

(f) The work done against the atmosphere

$$W_{\text{atm}} = p_{\text{atm}} * \Delta V = 100 * 10^3 * (0.1 - 0.05) = 5,000 \text{ J} = 5 \text{ kJ}$$

$$W_b = W_{\text{spring}} + W_{\text{mass}} + W_{\text{atm}}$$
$$13 = 3 + 5 + 5$$

1. MOVING BOUNDARY WORK

○ Relation between boundary work and enthalpy change:

Consider a stationary piston-cylinder system with the piston is free to move.
When heat is interact with the system, there will be boundary work at a constant pressure as a result.

The first-law of thermodynamics:

$$(Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) = \Delta U + \Delta \text{KE} + \Delta \text{PE}$$

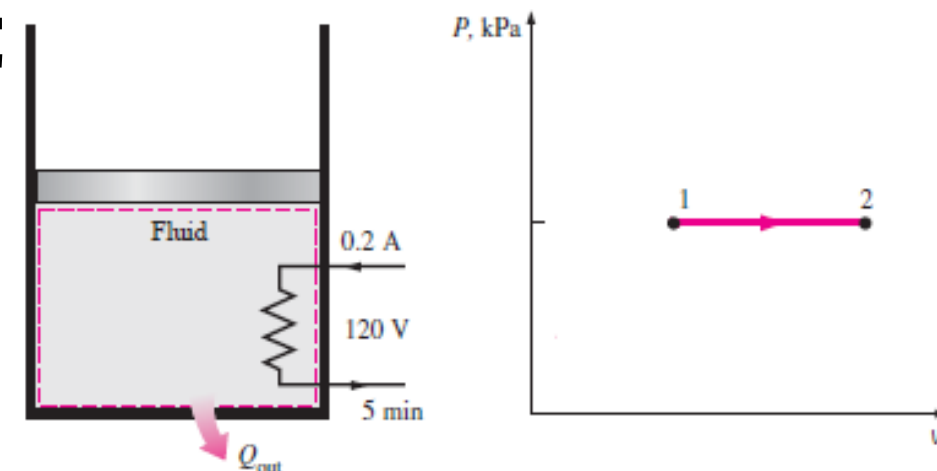
$$(0 - Q_{\text{out}}) + (W_{\text{electric}} - W_{\text{b}}) = \Delta U + 0 + 0$$

$$W_{\text{electric}} - Q_{\text{out}} = \Delta U + W_{\text{b}}$$

$$W_{\text{electric}} - Q_{\text{out}} = U_2 - U_1 + p_o(V_2 - V_1)$$

$$W_{\text{electric}} - Q_{\text{out}} = (U_2 + p_o V_2) - (U_1 + p_o V_1)$$

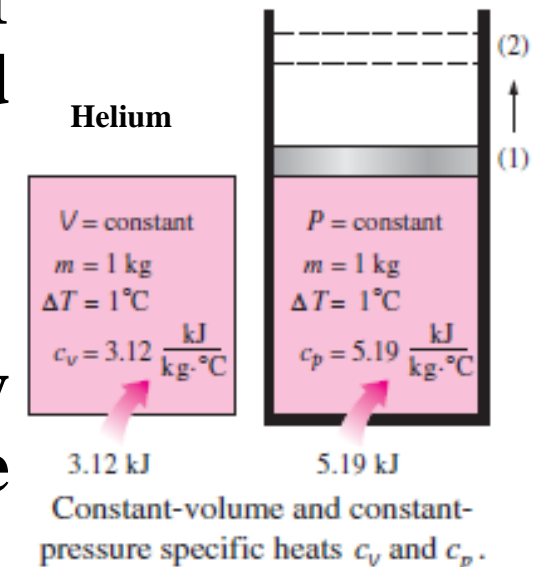
$$\boxed{W_{\text{electric}} - Q_{\text{out}} = H_2 - H_1}$$



2. SPECIFIC HEATS

The specific heat (c) is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.

- In thermodynamics, we are interested in two kinds of specific heats: **specific heat at constant volume (c_v)** and **specific heat at constant pressure (c_p)**.
- The units of specific heat is $\text{J/kg}\cdot\text{K} = \text{J/kg}\cdot^\circ\text{C}$.
- The specific heat at constant volume (c_v) is the energy required to raise the temperature of unit mass of a substance by one degree as *the volume is maintained constant*.
- The specific heat at constant pressure (c_p) can is the energy required to raise the temperature of unit mass of a substance by one degree as *the pressure is maintained constant*.



2. SPECIFIC HEATS

□ Specific heat at constant volume (c_v):

Apply the first-law of thermodynamics:

$$(Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) = \Delta U + \Delta \text{KE} + \Delta \text{PE}$$

$$(Q_{\text{in}} - 0) + (0 - 0) = \Delta U + 0 + 0$$

$$Q_{\text{in}} = \Delta U$$

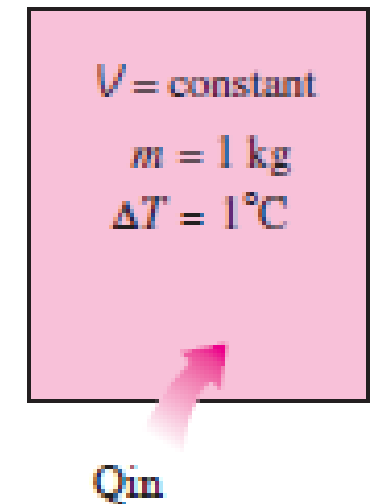
From the definition of c_v :

$$Q_{\text{in}} = m * c_v * \Delta T$$

$$\therefore Q_{\text{in}} = \Delta U = m * c_v * \Delta T$$

$$\frac{\Delta U}{m} = c_v * \Delta T = \Delta u$$

$$c_v = \left. \frac{\partial u}{\partial T} \right|_v$$



2. SPECIFIC HEATS

□ Specific heat at constant pressure (c_p):

Apply the first-law of thermodynamics:

$$(Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) = \Delta U + \Delta \text{KE} + \Delta \text{PE}$$

$$(Q_{\text{in}} - 0) + (0 - W_b) = \Delta U + 0 + 0$$

$$Q_{\text{in}} = \Delta U + W_b = \Delta H$$

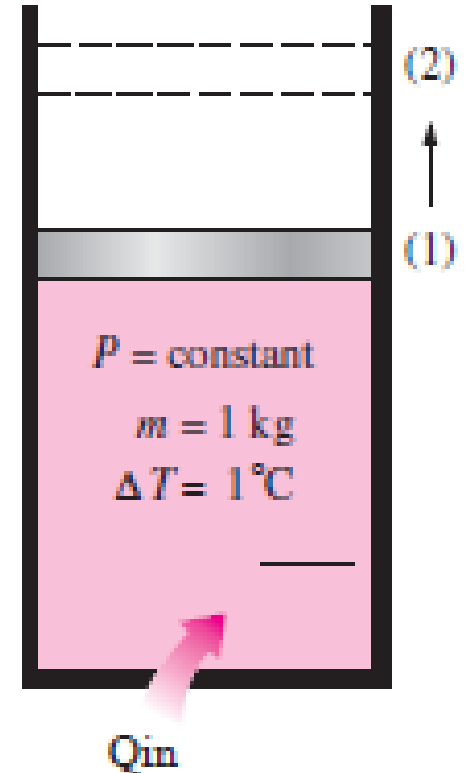
From the definition of c_p :

$$Q_{\text{in}} = m * c_p * \Delta T$$

$$\therefore Q_{\text{in}} = \Delta H = m * c_p * \Delta T$$

$$\frac{\Delta H}{m} = c_p * \Delta T = \Delta h$$

$$c_p = \left. \frac{\partial h}{\partial T} \right|_p$$



3. INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

For ideal gas:

$$\begin{aligned} pV &= mRT \\ pv &= RT \end{aligned}$$

By definition enthalpy is:

$$\begin{aligned} h &= u + pv \\ \therefore h &= u + RT \end{aligned}$$

Differentiating with respect to T:

$$\frac{\partial h}{\partial T} = \frac{\partial u}{\partial T} + R$$

Or

$$\boxed{c_p = c_v + R} \rightarrow 1$$

It is obvious that $c_p > c_v$

Define the specific heat ratio (k):

$$\boxed{k = \frac{c_p}{c_v}} \rightarrow 2$$

From 1 & 2:

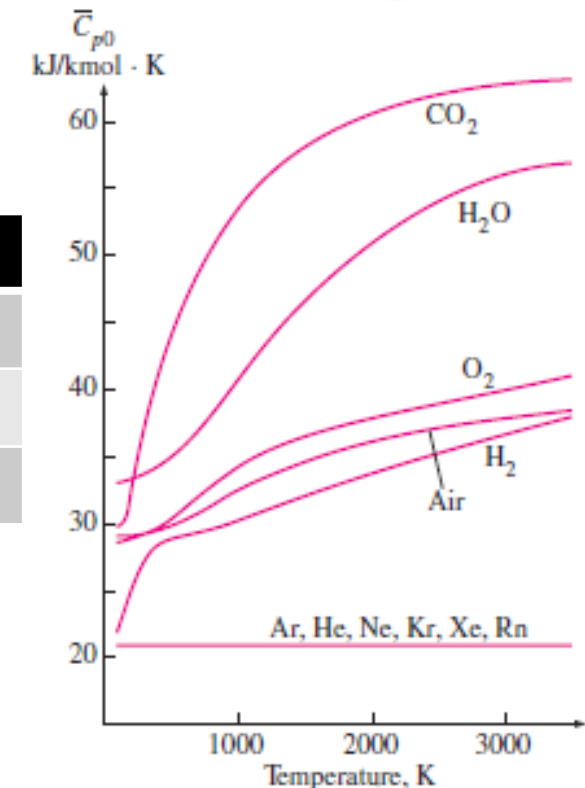
$$\boxed{c_p = \frac{kR}{k-1}} \quad \boxed{c_v = \frac{R}{k-1}}$$

3. INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

Specific heats (c_v & c_p) vary with temperature but we consider their values constant over a small range of temperature variation.

Specific heats of noble gases (inert gases) are temperature independent.

Description	Specific heat ratio (k)	Gas
Monoatomic	$5/3 \cong 1.67$	Helium, Argon, Neon, ...
Diatomic	$7/5 = 1.4$	Oxygen, Nitrogen, Air, ...
Polyatomic	$4/3 \cong 1.33$	Carbon dioxide, Methane, ...



3. INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

For ideal gas, with the assumption of constant specific heats:

$$\Delta u = u_2 - u_1 = \int_1^2 c_v dT = c_v \Delta T$$

$$\Delta u = u_2 - u_1 = c_v (T_2 - T_1)$$

Similarly

$$\Delta h = h_2 - h_1 = \int_1^2 c_p dT = c_p \Delta T$$

$$\Delta h = h_2 - h_1 = c_p (T_2 - T_1)$$

4. INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLID & LIQUIDS

- A substance whose **specific volume** (or **density**) is constant is called an **incompressible** substance.
- The **specific volumes** of **solids** and **liquids** essentially remain **constant** during a process. Therefore, **liquids** and **solids** can be considered as **incompressible** substances.

$$c_p = c_v = c$$

$$\Delta u = c(T_2 - T_1)$$

By definition enthalpy is:

$$h = u + pv$$

$$dh = du + p\cancel{dv} + vdp$$

$$\therefore dh = du + vdp$$

4. INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLID & LIQUIDS**A. Constant pressure process:**

$$dh = du + vdp^0$$

$$\Delta h = du = c\Delta T = c(T_2 - T_1)$$

B. Constant temperature process:

$$dh = du + vdp$$

$$\Delta h = c\Delta T^0 + v(p_2 - p_1)$$

$$\Delta h = h_2 - h_1 = v(p_2 - p_1)$$

4. INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLID & LIQUIDS□ Example:

A 50-kg iron block at 80°C is dropped into an insulated tank that contains 0.5 m³ of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.

$$c_{\text{water}} = 4.18 \text{ kJ/kg.K} \quad \& \quad c_{\text{iron}} = 0.45 \text{ kJ/kg.K}$$

$$\begin{aligned} \cancel{E_{\text{in}}} - \cancel{E_{\text{out}}} &= \Delta E_{\text{system}} \\ \Delta E_{\text{system}} &= 0 \\ \Delta U_{\text{system}} + \cancel{\Delta KE} + \cancel{\Delta PE} &= 0 \\ \Delta U_{\text{system}} &= 0 \end{aligned}$$



$$m_{\text{water}} \cdot c_{\text{water}} \cdot (T_{\text{final}} - T_{\text{initial}}) \Big|_{\text{water}} + m_{\text{iron}} \cdot c_{\text{iron}} \cdot (T_{\text{final}} - T_{\text{initial}}) \Big|_{\text{iron}} = 0$$

$$(1000 * 0.5) * 4.18 * 10^3 * (T_{\text{final}} - 25) \Big|_{\text{water}} + 50 * 0.45 * 10^3 * (T_{\text{final}} - 80) \Big|_{\text{iron}} = 0$$

$$T_{\text{final}} = 25.6 \text{ } ^\circ\text{C}$$