MCT321: Introduction to Nano-Mechatronics

Lectures #6&7: Simple Beam Theory

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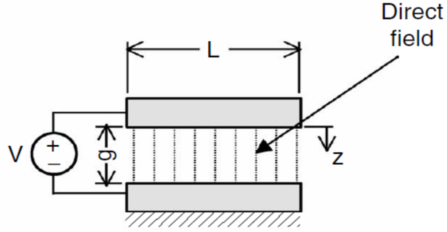
Outline

- Normal Electrostatic Forces
  - Parallel Plate Electrostatic Actuators
  - Static Analysis
- Tangential Electrostatic Forces
  - Comb Drive Actuators
  - Static Analysis
- Capacitance Sensing

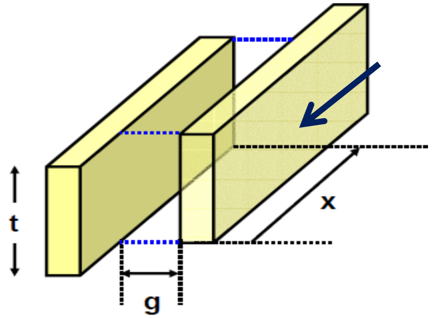
Electrostatic Forces

- As electrostatic forces scale better than electromagnetic force in the micro domain, electrostatic actuation is more frequently used in MEMS.
- Electrostatic force could be either **normal force** or **lateral (or tangential) force**.
- **Normal forces** are used to actuate (or drive) **closing gap** actuators



Parallel plate capacitor

- **Lateral forces** are used to actuate **overlapping electrode** actuators

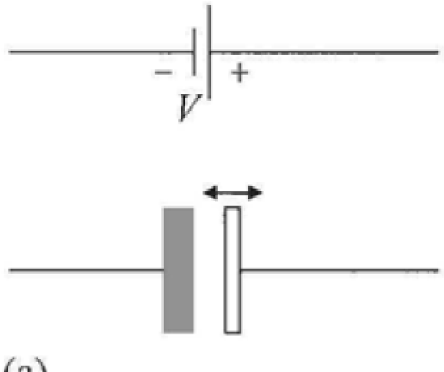


Electrostatic Forces

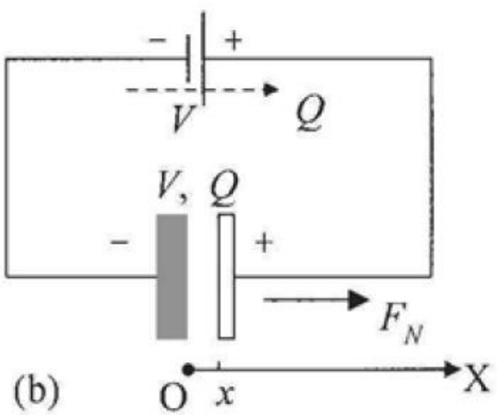
Normal Forces:

- Consider a capacitor as shown in figure.
- Dark plat is fixed and the other one is movable.
- The capacitance is:

$$C(x) = \frac{\varepsilon_o \varepsilon_r A}{x}$$



- where *A* is the plate's area, ε_o is the permittivity of vacuum, ε_r is relative permittivity of air (=1), and *x* is the separation between the two plates.



- Now, if the capacitor is connected to the battery as shown,
- The stored charge in the capacitor can be expressed as:

$$Q_c = C(x)V$$

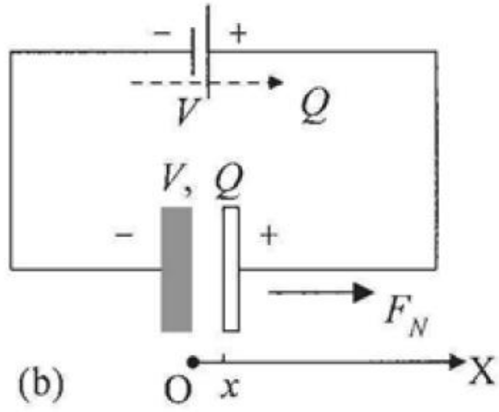
Electrostatic Forces

❑ Normal Forces:

- The energy stored in the capacitor can be expressed as

$$E_c = \frac{1}{2} Q_c V = \frac{1}{2} C(x) V^2$$

- The force applied on the movable plate is:



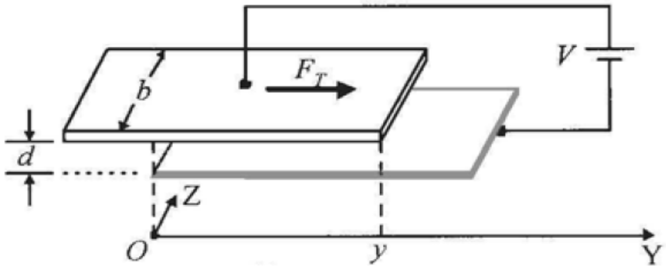
$$F_x = \frac{\partial E_c}{\partial x} = \frac{1}{2} V^2 \frac{\partial C(x)}{\partial x} = -\frac{\varepsilon_o \varepsilon_r A}{2x^2} V^2$$

- The force tries to increase the capacitance C(x).

Electrostatic Forces

Tangential Forces:

- Consider two parallel electrodes as shown in figure.



- The capacitance due to the overlapped electrodes is:

$$C(y) = \frac{\varepsilon_o \varepsilon_r A}{d} = \frac{\varepsilon_o \varepsilon_r}{d}(by)$$

- The force applied on the movable plate is:

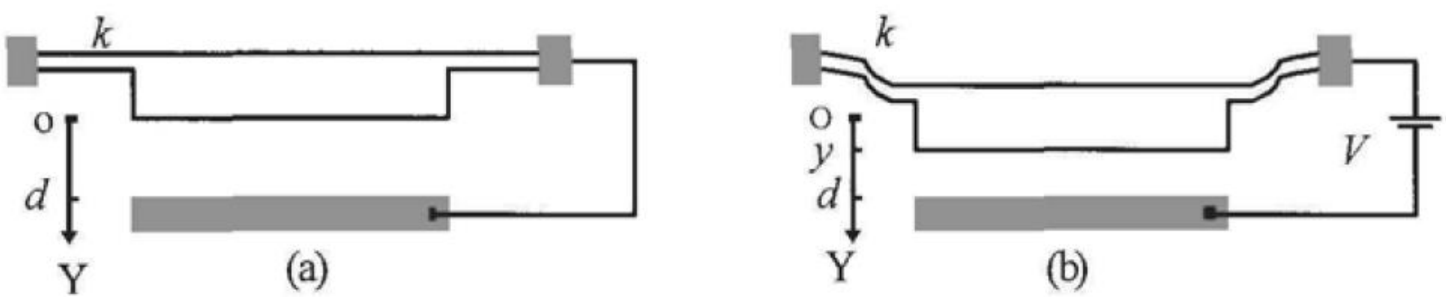
$$F_y = \frac{\partial E_c}{\partial y} = \frac{1}{2}V^2 \frac{\partial C(y)}{\partial y} = \frac{\varepsilon_o \varepsilon_r b}{2d}V^2$$

- This equation indicates that the force is constant and independent on the overlap y, but is in a direction which increases the capacitance.

Electrostatic Microactuators

❑ Parallel plate actuators:

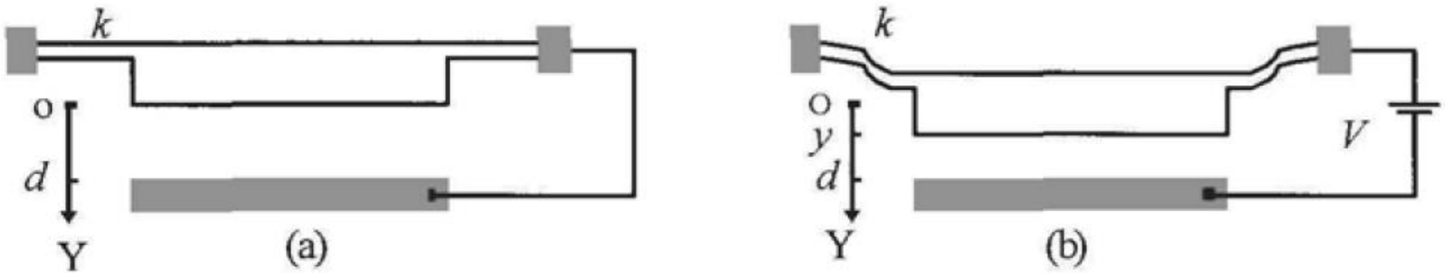
- Consider two parallel plate actuator as shown in figure below.



- The spring flexure allows the mass to move only in the normal direction.
- Original distance between fixed and movable electrodes is *d*.
- When a voltage *V* is applied between both electrodes, an electrostatic force, F_e, is applied to the central mass trying to pull it toward the fixed electrode (dark electrode).

Parallel Plate Actuators

Parallel plate actuators:



- Once the mass is displaced toward the fixed plate, a restoring force, F_k, will try to pull the mass towards its original position (upward).
- The final balanced (steady state) position at which the mass will displace is determined by the balance between both forces.

$$F_e = F_k$$

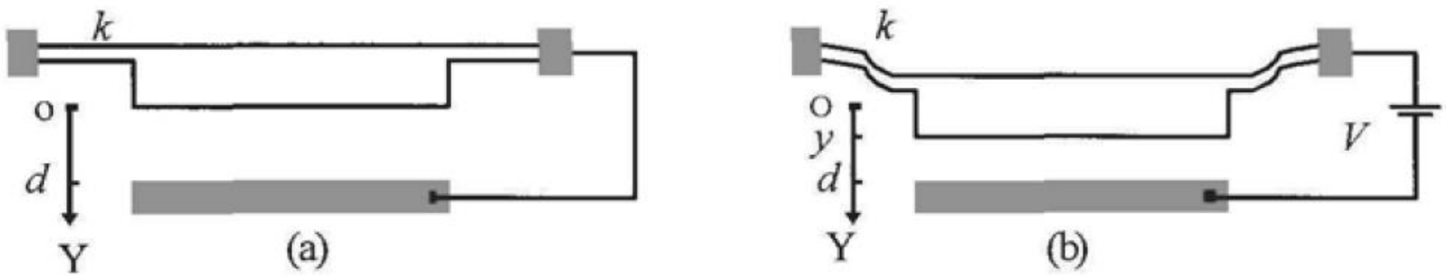
- The capacitance at any position *y* is:

$$C(y) = \frac{\varepsilon_o \varepsilon_r A}{(d-y)}$$

$$\therefore \frac{\partial C(y)}{\partial y} = \frac{\varepsilon_o \varepsilon_r A}{(d-y)^2}$$

Parallel Plate Actuators

- **Parallel plate actuators:**



- The electrostatic force can be calculated as:

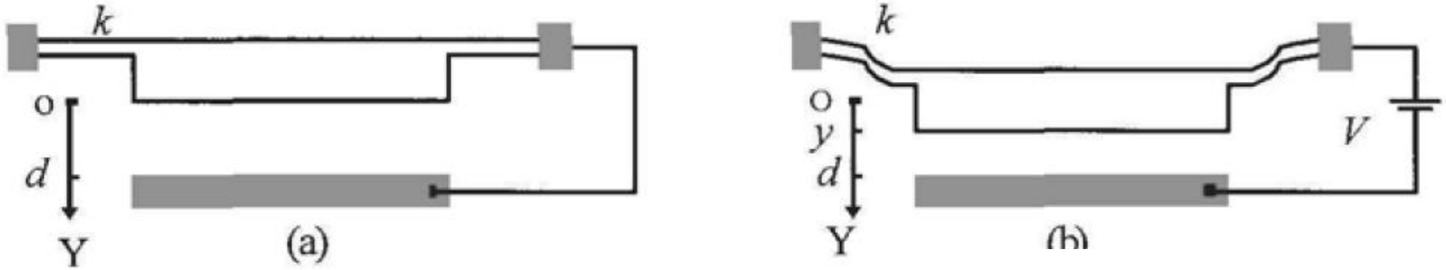
$$F_e = \frac{1}{2} V^2 \frac{\partial C(y)}{\partial y} = \frac{1}{2} \frac{\varepsilon_o \varepsilon_r A}{(d-y)^2} V^2$$

- Note that the electrostatic force is a **nonlinear** force and increases as y increases.
- The spring force can be expressed as: $F_k = Ky$
- At force balance, displacement, y, can be found by solving:

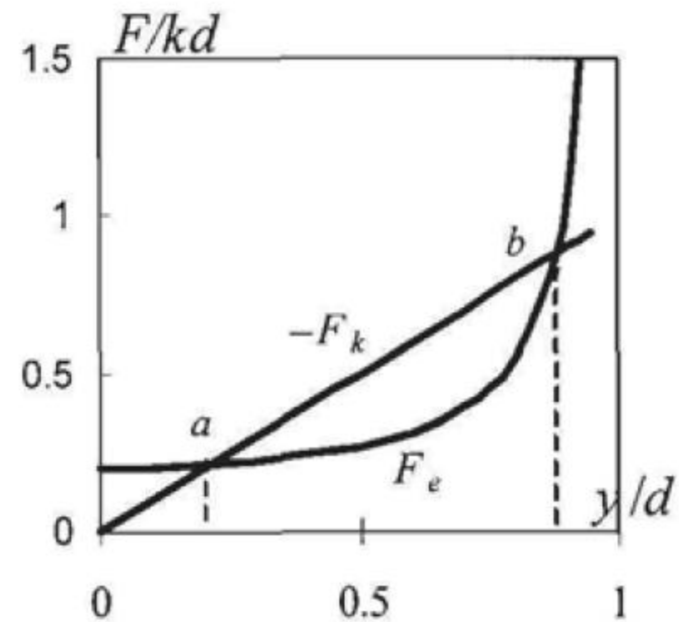
$$\frac{1}{2} \frac{\varepsilon_o \varepsilon_r A}{(d-y)^2} V^2 = Ky \qquad (1)$$

Parallel Plate Actuators

- **Parallel plate actuators:**



- Normalize F_e w.r.t. maximum spring restoring force, *Kd*.
- Normalize the traveled displacement, *y*, w.r.t. maximum displacement, *d*.
- We can solve for the balanced displacement graphically as follows.
- Point *a* is stable position, however, point *b* is unstable position.



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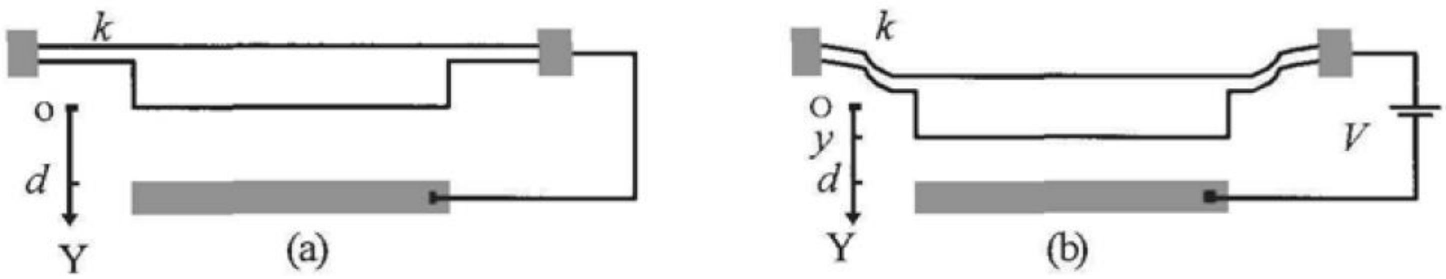
Parallel Plate Actuators

- **<u>Parallel plate actuators:</u>**

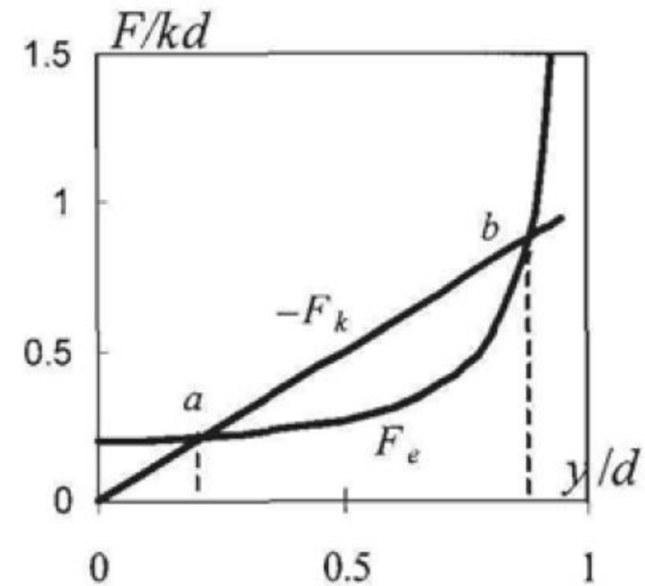


- Maximum stable (steady state) displacement (points *a* and *b* merge) can be found from:

$$\frac{\partial F_k}{\partial y} > \frac{\partial F_e}{\partial y} \qquad ==> K > \frac{\varepsilon_o \varepsilon_r A V^2}{(d-y)^3} \qquad (2)$$

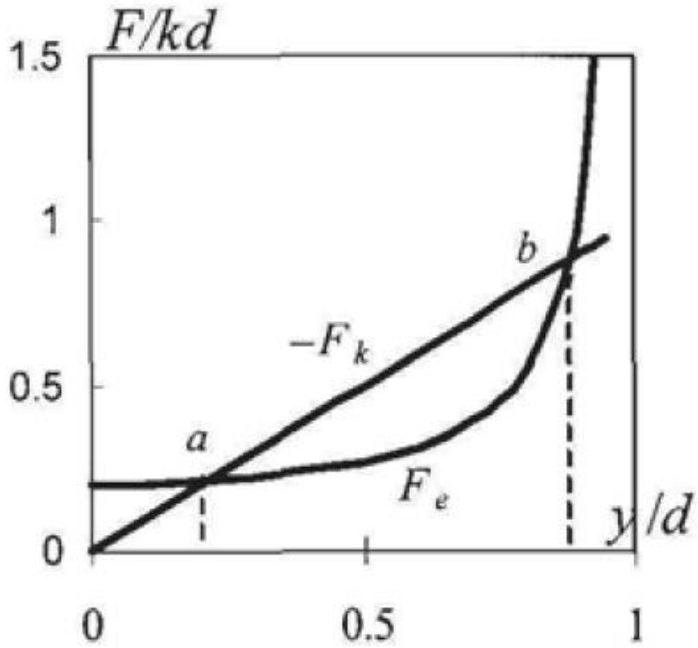
- From equation (1), $K = \dfrac{\varepsilon_o \varepsilon_r A V^2}{2y(d-y)^2}$ (3)

- From (3) in (2) ➔ $y \leq \dfrac{d}{3}$



Parallel Plate Actuators

- **Parallel plate actuators:**
  - **Pull-in phenomenon:**



- When points *a* and *b* merge together, any increase in the voltage means that F_e and F_k will not intersect, and no steady state solution can be found.
- For this situation, the central mass will move towards the fixed plate until sticking occurs.
- If the voltage decreases, the plates will still stuck until the voltage is reduced to ZERO.
- Pull-in voltage, V_{pi} can be found by substituting y=d/3 in equation (1).

$$V_{pi} = \sqrt{\frac{8Kd^3}{27A\varepsilon_o\varepsilon_r}}$$

Comb Drive Actuators

- Comb drive actuators are very important MEMS structures that are widely used in many applications.
- They are currently used in resonant accelerometers, resonant gyroscopes, and micro-grippers, …etc.
- In addition, they are extensively used in opto-mechanical subsystems such as switches and variable optical attenuators.

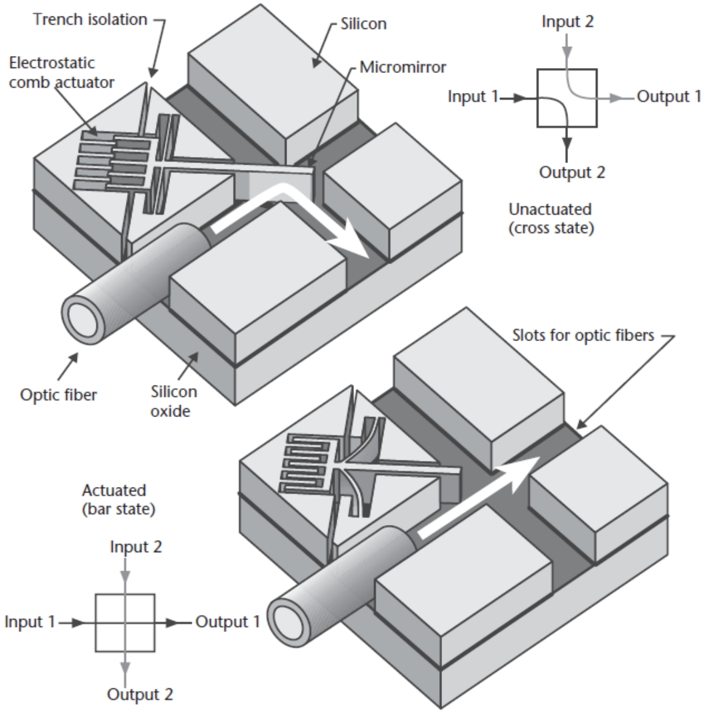
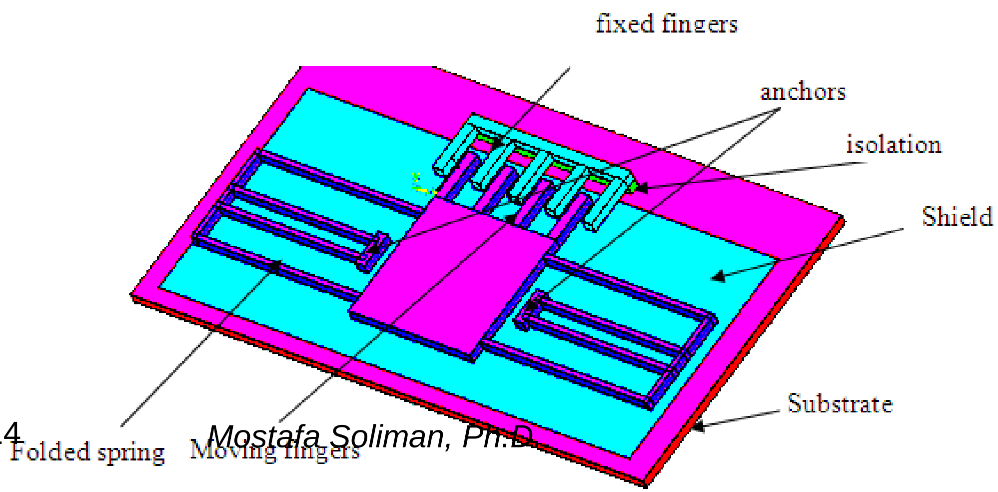


Figure 5.14 Illustration of a 2 × 2 binary reflective optical switch fabricated using SOI wafers and DRIE. An electrostatic comb actuator controls the position of a micromirror. In the cross state, light from an input fiber is deflected by 90°. In the bar state, the light from that fiber travels unobstructed through the switch. Side schematics illustrate the signal path for each state.

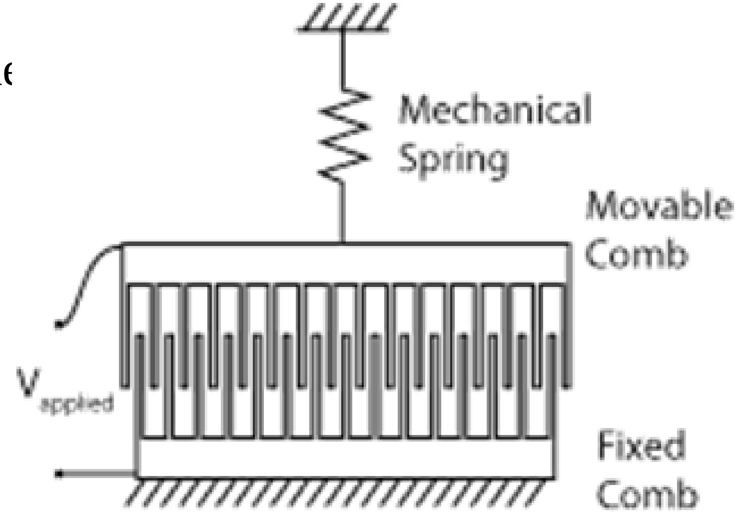
Comb Drive Actuators

- The comb drive is simply an electrostatic actuator in which the capacitor plates, creating the electrostatic forces, take the form of a comb structure to increase the effective capacitor area and thus the resultant electrostatic force.
- Comb drive is a type of capacitive electrostatic actuators that creates a force between its inter-digitated fingers.
- An electrostatic comb drive requires two comb structures, one comb remains stationary and the other comb is free to move, but it is suspended by a suspension mean.
- The force that causes the free structure to move is created by the capacitance between the two structures.
- **The generated force is almost linear with the travelled displacement.**



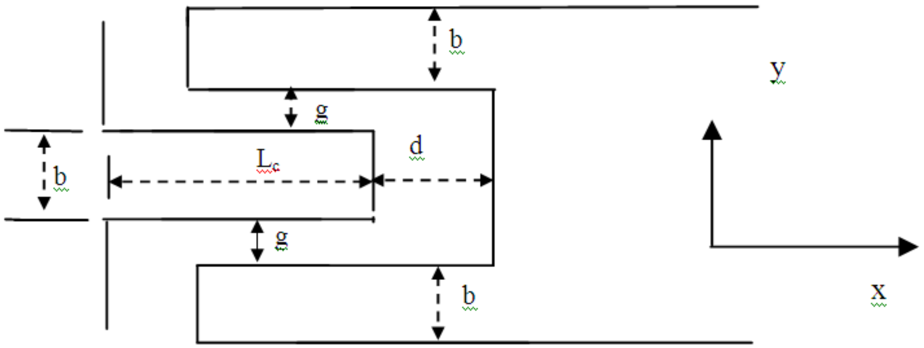
Comb Drive Actuators

- **Force Analysis**
- If there is no load applied to the linear comb shuttle, the only forces acting on the shuttle are, the electrostatic forces created by the capacitive combs, the spring restoring force created by the folded beam spring, and the damping arising from air resistance to the moving comb structure.
- Comb drive actuator can be modeled as a second orde



Comb Drive Actuators

- **Electrostatic Force Analysis**



Geometrical details of comb drive fingers.

- Where b : finger width, L_c: comb finger length, d: maximum allowed displacement

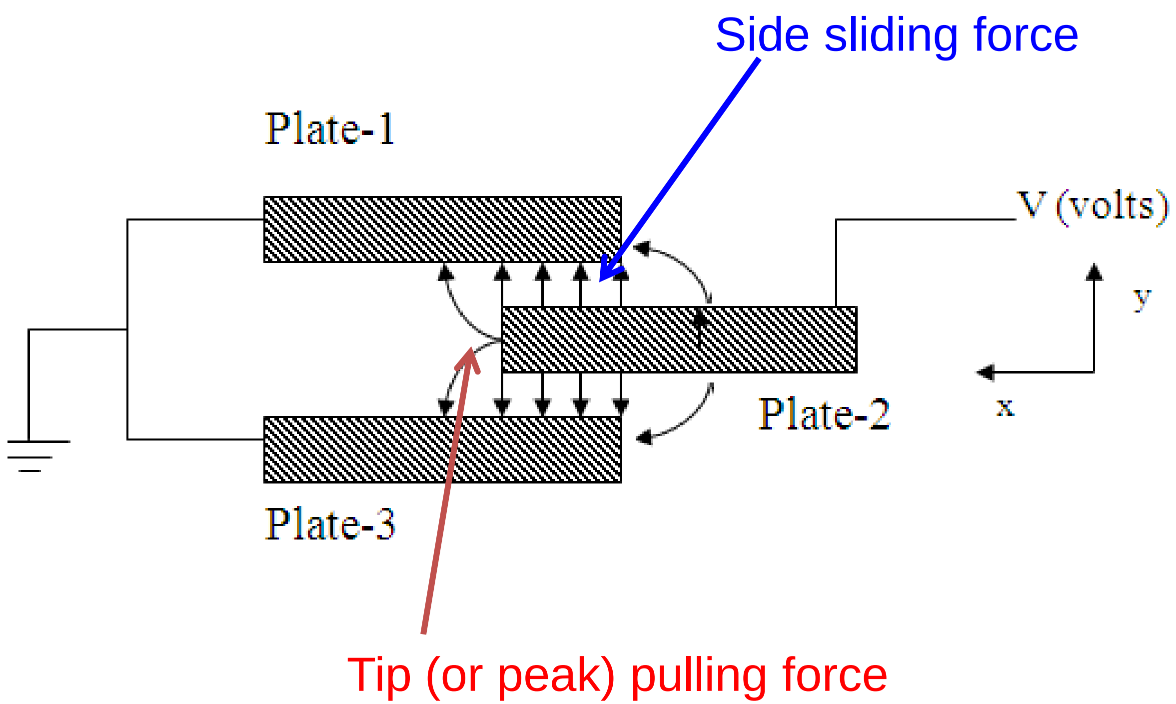
  g: air gap between fixed and moving fingers

- Forces in both x and y directions can be calculated as:

$$F_x = \frac{\partial W}{\partial x} = \frac{1}{2} V^2 \frac{\partial C_t}{\partial x}, \quad F_y = \frac{\partial W}{\partial y} = \frac{1}{2} V^2 \frac{\partial C_t}{\partial y}$$

Comb Drive Actuators

- **Electrostatic Force Analysis (F_x)**
- F_x is in a direction which increases the capacitance. That will lead to a force trying to insert (or to pull) the moving finger in between two fixed fingers.



Comb Drive Actuators

- **<u>Electrostatic Force Analysis (F_x)</u>**
- C_u: Upper plate capacitance,

  C_l: Lower plate capacitance

  C_p: Peak end (or tip) capacitance

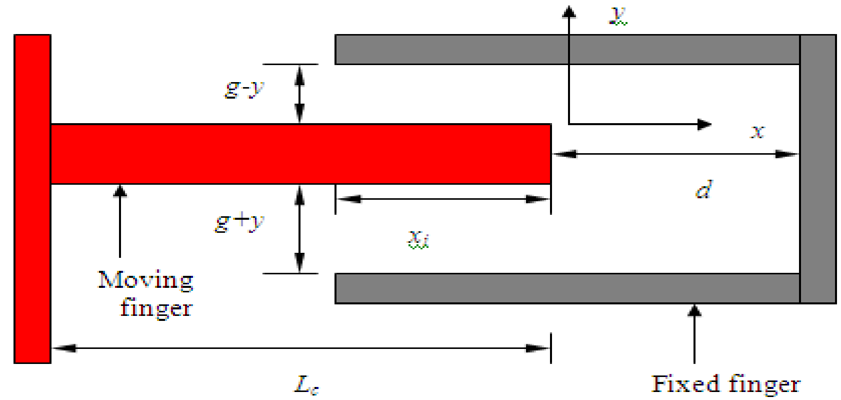
  x_i: initial overlap.

➢ Assuming thickness of '*t*' (inside the paper)

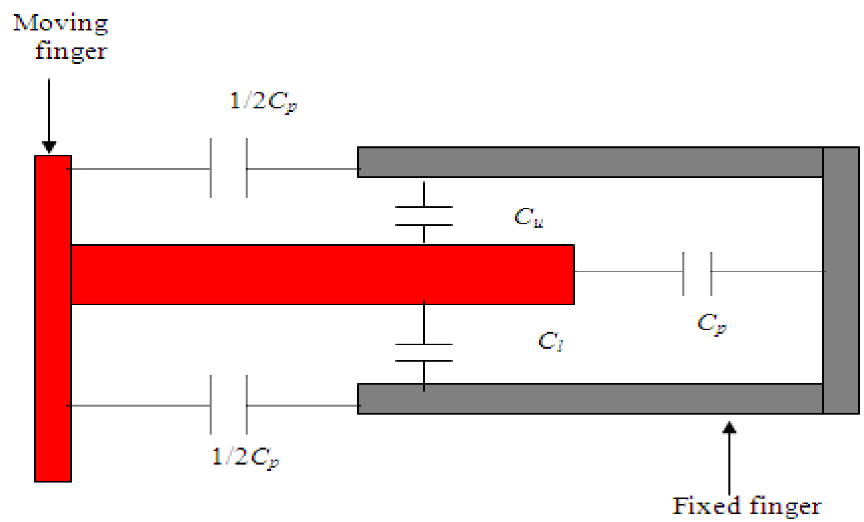
$$C_u = n\varepsilon_o\varepsilon_r t \frac{x_i + x}{g - y}$$

$$C_l = n\varepsilon_o\varepsilon_r t \frac{x_i + x}{g + y}$$

$$C_p = n\varepsilon_o\varepsilon_r \frac{tb}{(d - x)}$$



General position of moving finger between the two fixed fingers.



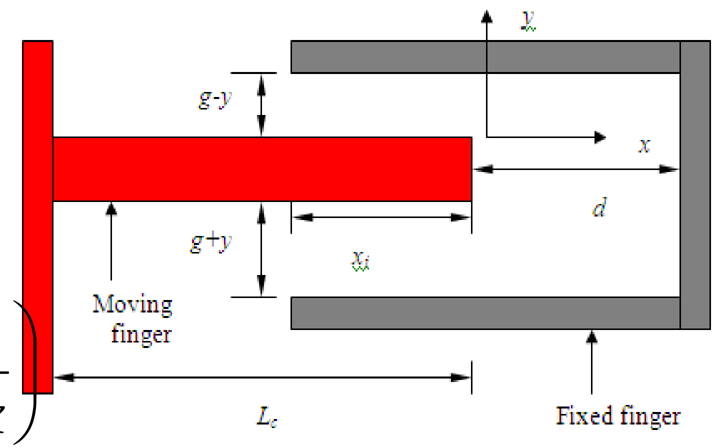
Capacitance representation of one finger set.

Comb Drive Actuators

- **Electrostatic Force Analysis (F_x)**
- Total capacitance of the systems is C_t

$$C_t = C_u \,//\, C_l \,//\, 2C_p$$

$$\therefore C_t = n\varepsilon_o\varepsilon_r t(x_i + x)\left[\frac{1}{g-y} + \frac{1}{g+y}\right] + \left(2n\varepsilon_o\varepsilon_r \frac{tb}{d-x}\right)$$



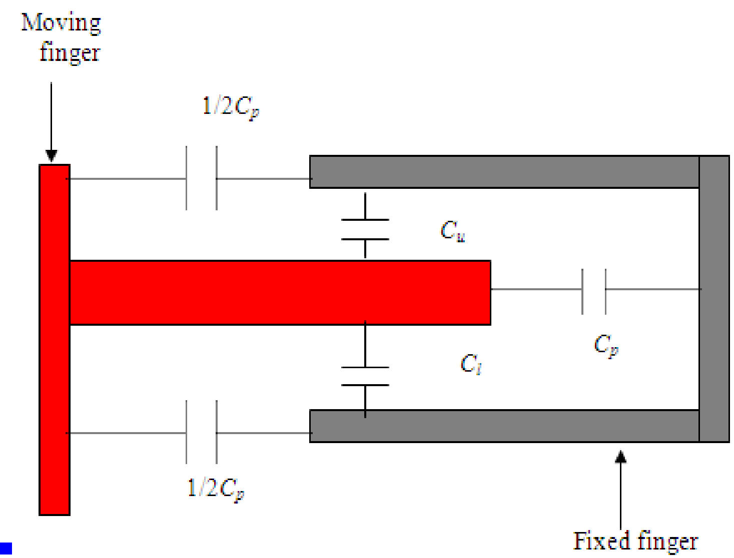
General position of moving finger between the two fixed fingers.

- If the moving finger is perfectly centered between the two fixed fingers, then $y = 0$.

$$\therefore C_t = n\frac{2\varepsilon_o\varepsilon_r t(x_i + x)}{g} + n\varepsilon_o\varepsilon_r \frac{2tb}{d-x}$$

- F_x can be calculated as:

$$F_x = \frac{1}{2}V^2\frac{\partial C_t}{\partial x} = n\varepsilon_o\varepsilon_r t\left(\frac{1}{g} + \frac{b}{(d-x)^2}\right)V^2$$



Capacitance representation of one finger set.

Comb Drive Actuators

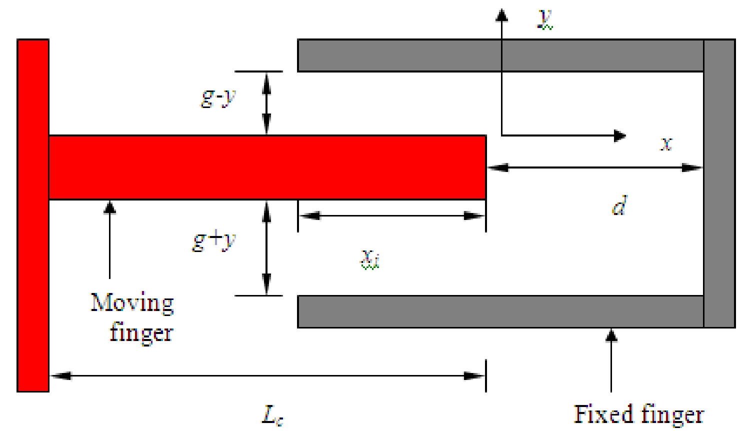
- **<u>Electrostatic Force Analysis (F_x)</u>**
- F_x can be broken to two forces:
- F_d which is a constant force term and depends on the finger depth (height t), air gap g, and the applied voltage V.

$$F_d = n\frac{\varepsilon_o \varepsilon_r t V^2}{g}$$

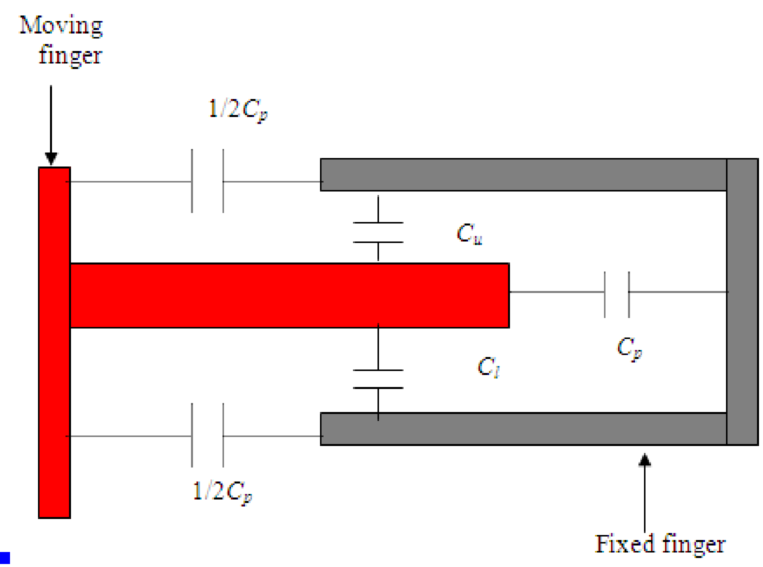
- F_p which is nonlinear force term and depends on the finger depth (height, t), air gap, g, and the applied voltage, V.

$$F_p = n\frac{\varepsilon_o \varepsilon_r t V^2 b}{(d-x)^2}$$

- F_p can cause the front instability (front sticking)



General position of moving finger between the two fixed fingers.



Capacitance representation of one finger set.

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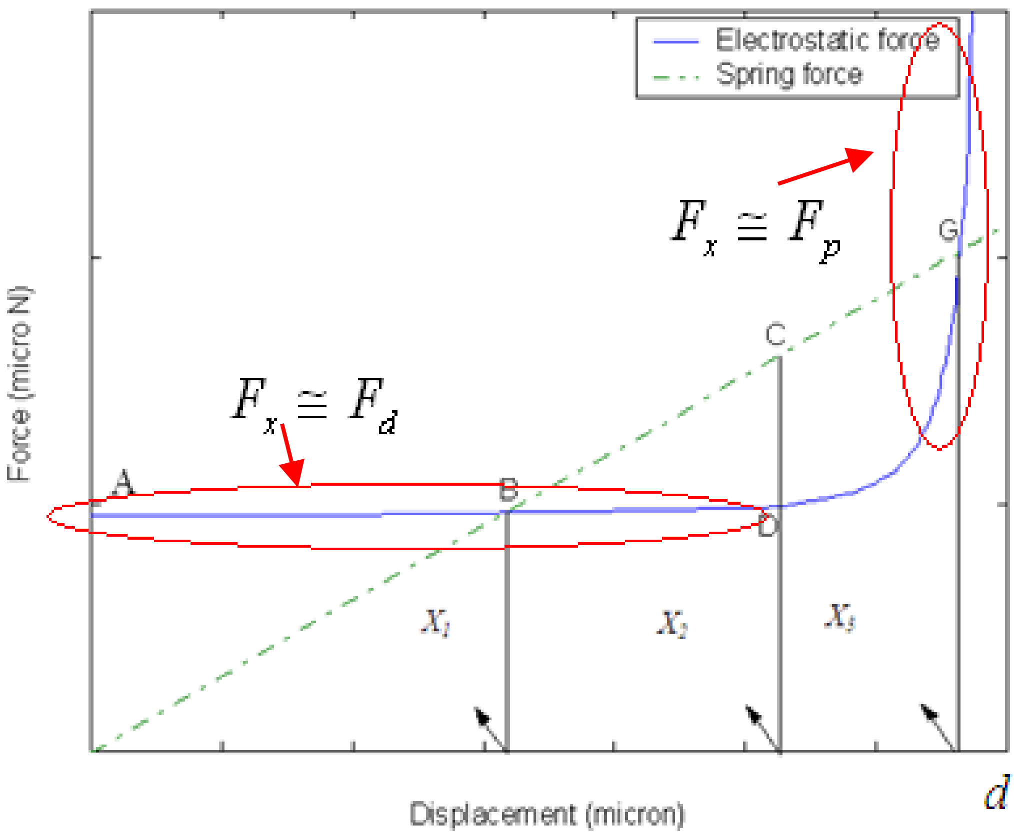
Comb Drive Actuators

- **<u>Electrostatic Force Analysis (F_x)</u>**
- If $x \ll d$, the F_d dominates

$$F_x \cong F_d = n\frac{\varepsilon_o \varepsilon_r t V^2}{g}$$

- If $x \cong d$, the F_p dominates

$$F_x \cong F_p = n\frac{\varepsilon_o \varepsilon_r t V^2 b}{(d-x)^2}$$



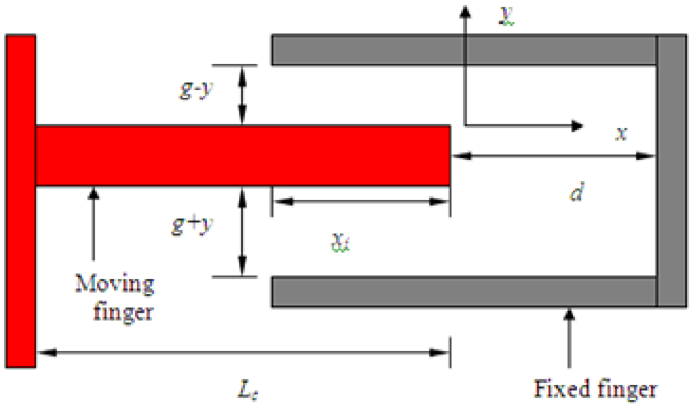
Force-Displacement diagram of a comb drive electrostatic actuator.

Comb Drive Actuators

- **<u>Electrostatic Force Analysis (F_y)</u>**
- If there is a small displacement y, then F_y will be

$$F_y = \frac{1}{2}V^2\frac{\partial C_t}{\partial y} = \frac{1}{2}n\varepsilon_o\varepsilon_r t(x_i + x)\left[\frac{1}{(g-y)^2} - \frac{1}{(g+y)^2}\right]V^2$$

- As the moving finger of the comb drive moves between the two fixed fingers (x increases), K_y increases (K_y is the electrostatic force spring constant in y-direction) $K_y' = \left.\frac{\partial F_y}{\partial y}\right|_{y=0}$

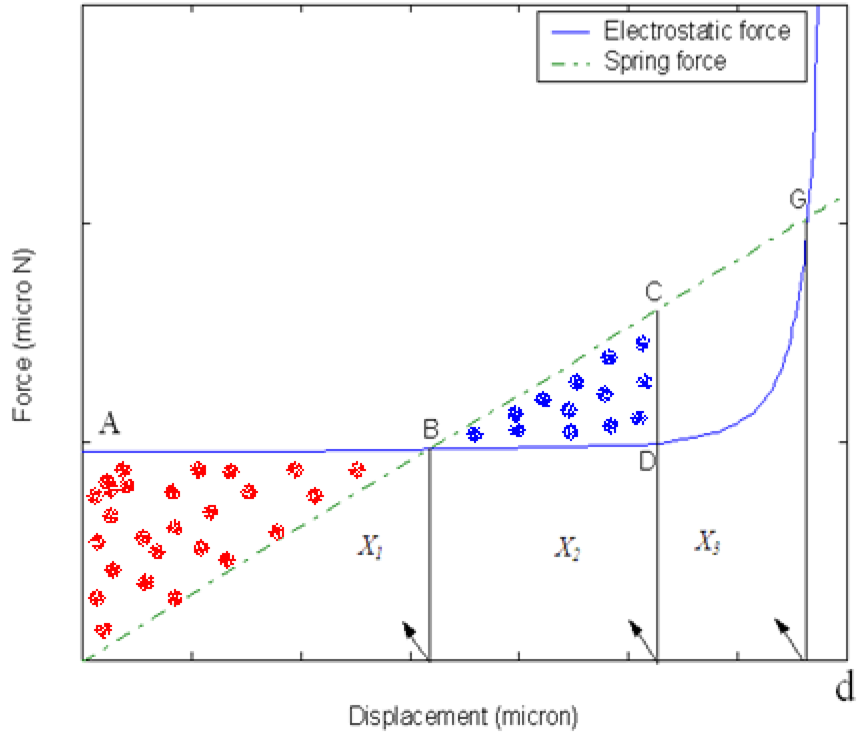


General position of moving finger between the two fixed fingers.

- F_y can cause the side instability (side sticking)
- If the moving finger is perfectly centered between the two fixed fingers, then $y = 0$ and $F_y = 0$.

Comb Drive Actuators

- **<u>Static Analysis:</u>**
- Balanced displacement occurs @ $F_e = F_s$, ➔ $n\varepsilon_o\varepsilon_r t\left(\frac{1}{g}+\frac{b}{(d-x)^2}\right)V^2 = K_x x$
- The intersection occurs at two points, G (unstable balanced position) and B (stable balanced position).
- B and G correspond to displacements of X_1 and X_3
- Point C (X_2) represents the overshoot.
- Maximum balanced displacement occurs when points B and G overlap.



Comb Drive Actuators

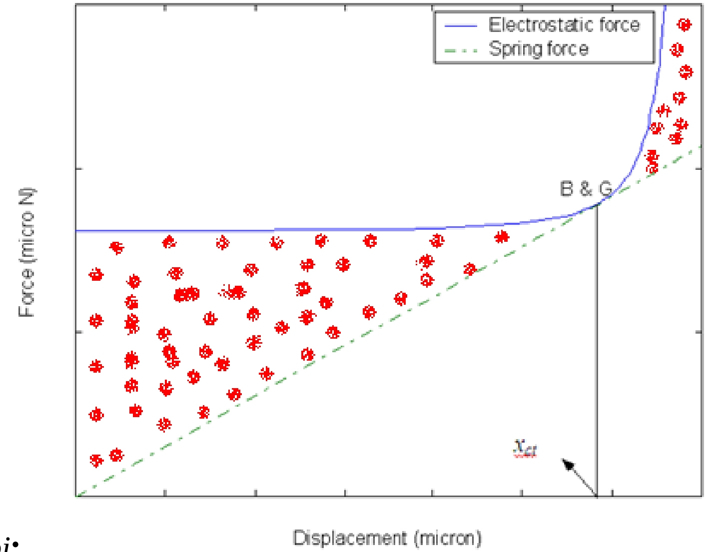
- **<u>Static Analysis (Pull-in): (front instability) (maximum balanced poistion under steady state)</u>**

- At point B & G, 2 conditions must satisfy:

1. $$F_e = F_s, \;\Rightarrow\; K_x x_{ct} = n\varepsilon_o\varepsilon_r t\left(\frac{1}{g}+\frac{b}{(d-x_{ct})^2}\right)V_{pi}^{\;2}$$

2. $$\frac{\partial F_S}{\partial x} = \frac{\partial F_e}{\partial x} \;\Rightarrow\; K_x = 2n\varepsilon_o\varepsilon_r t\left(\frac{b}{(d-x_{ct})^3}\right)V_{pi}^{\;2}$$

- Solving the above 2 equations we can find both x_{ct} and V_{pi}.



Maximum critical displacement.

$$\therefore 2x_{ct}bg = bg(d-x_{ct})+(d-x_{ct})^3$$

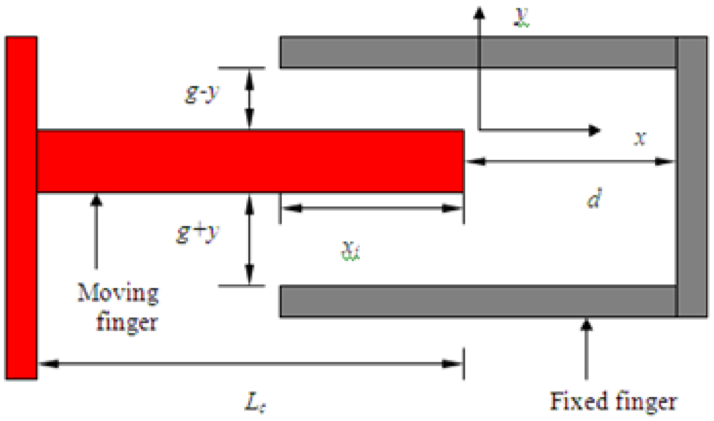
- If we assume that b is very small w.r.t. t, then the above equation reduced to $x_{ct} = d$.

This condition cannot happen as there is no decelerating energy to balance with the accelerating energy gained by the mass

Comb Drive Actuators

❑ Side instability:

❑ A movable finger placed between two fixed fingers is in an inherently unstable position if a voltage is applied.



General position of moving finger between the two fixed fingers.

❑ A small displacement y in the y – direction will cause an unbalance between the forces and then a net force F_y will be created.

❑ For a comb drive with n fingers, the force F_y is expressed as:

$$F_y = \frac{1}{2} V^2 \frac{\partial C_t}{\partial y} = \frac{1}{2} n \varepsilon_o \varepsilon_r t (x_i + x) \left[\frac{1}{(g-y)^2} - \frac{1}{(g+y)^2} \right] V^2$$

Comb Drive Actuators

- **<u>Side instability:</u>**

- To avoid the side sticking, the spring constant in the y – direction, i.e. K_y, must be higher than the y – direction electrostatic force stiffness (K'_y), i.e. $K_y > K'_y$.

$$\text{Where, } K_y = \frac{2Ebh}{L}, \; bh \text{ is the beam cross section area}$$

- The y – direction electrostatic force stiffness (K'_y) can be obtained by differentiating F_y *w.r.t.* the displacement y at y = 0:

$$K'_y = \left.\frac{\partial F_y}{\partial y}\right|_{y=0} = 2n\varepsilon_o\varepsilon_r t \frac{x + x_i}{g^3} V^2$$

- So to avoid side sticking,

$$\frac{2Ebh}{L} > 2n\varepsilon_o\varepsilon_r t \frac{x + x_i}{g^3} V^2$$

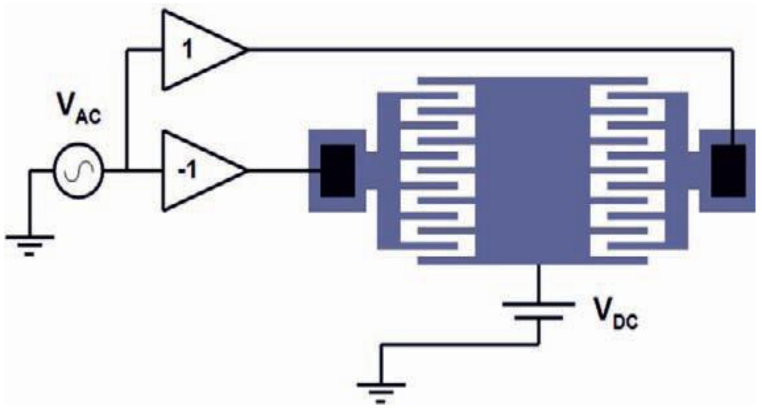
Comb Drive Actuators

Balanced Actuation:

- **If an AC voltage is applied to a comb drive, it will oscillate with double the AC signal frequency since the generated force is proportional to V^2 .**
- **Sometimes it is desired to generate a sinusoidal net actuation force with same frequency of the applied ac voltage signal ➔ ➔ ➔ Balanced Actuation is utilized.**
- **Balanced Actuation is used to**
  - **Generate force with same driving frequency.**
  - **Linearize the force with respect to the applied voltage ($Force \ \alpha \ V$).**

Comb Drive Actuators

- **Balanced Actuation:**



- The method is based on applying

$V_1 = V_{dc} + v_{ac} \sin \omega t$ to one set of electrodes and

$V_2 = V_{dc} - v_{ac} \sin \omega t$ to the opposing set.

- Assuming the two electrode sets are identical

($C_1 = C_2 = C$), the net electrostatic force is:

$$F_{balanced} = \frac{1}{2}\frac{\partial C_1}{\partial x} V_1^{\;2} - \frac{1}{2}\frac{\partial C_2}{\partial x} V_2^{\;2} = \frac{1}{2}\frac{\partial C}{\partial x}\left(V_1^{\;2} - V_2^{\;2}\right)$$

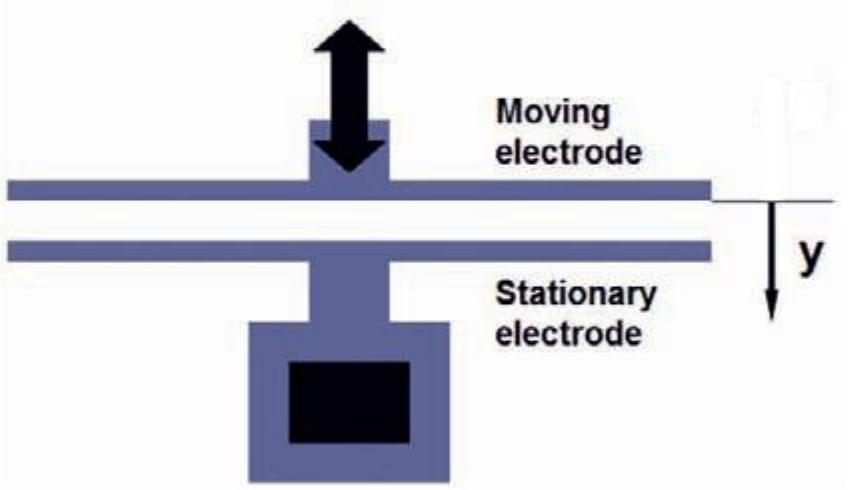
$$= \frac{1}{2}\frac{\partial C}{\partial x}\left((V_{dc} + v_{ac}\sin\omega t)^2 - (V_{dc} - v_{ac}\sin\omega t)^2\right)$$

$$\therefore F_{balanced} = 2\frac{\partial C}{\partial x} V_{dc} v_{ac} \sin\omega t$$

Capacitive Sensing

Variable (or Closing) Gap Capacitors:

- Most widely used method for detection of small displacements.
- The electrode gap, *d*, could be as small as sub-micron, determined by the technology used.
- The capacitance to displacement ratio (C2D) can be found as follows, where "*t*" is the thickness of the electrode inside the paper:
- Capacitance to displacement ratio is a measure of the sensitivity of the capacitance with any change in the gap, *y*.



$$C(y) = \frac{\varepsilon_o t L}{(d-y)}$$

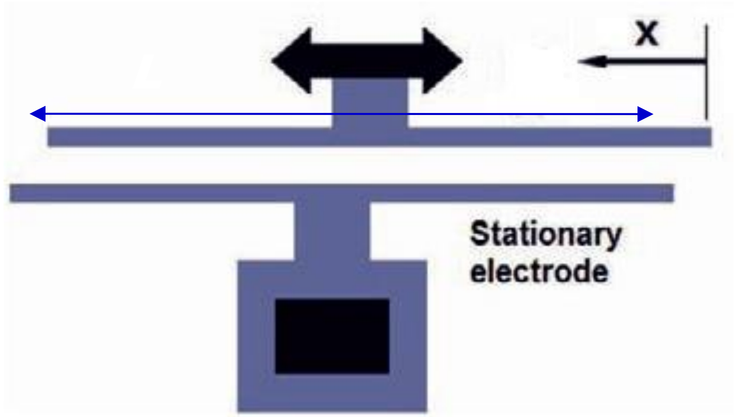
$$\therefore \frac{\partial C(y)}{\partial y} = \frac{\varepsilon_o t L}{(d-y)^2}$$

Capacitive Sensing

Variable Area Capacitors:

- Used for detection of large displacements.
- If the electrode gap and thickness are *d* and *t*,

  the capacitance is:



$$C(x) = \frac{\varepsilon_o t(L + x)}{d}$$

- The capacitance to displacement ratio can be found as follows:

$$\frac{\partial C(x)}{\partial x} = \frac{\varepsilon_o tL}{d} = \text{Constant}$$

- The C2D in variable gap is larger that the C2D in variable area sensing elements.

$$\frac{\left.\frac{\partial C(y)}{\partial y}\right|_{\text{closing gap}}}{\left.\frac{\partial C(x)}{\partial x}\right|_{\text{variable area}}} = \frac{d}{(d-y)^2} > 1 \, (\text{if } \ y > 0)$$