MCT321: Introduction to Nan-Mechatronics

Lecture #2: MEMS Scaling Laws

Mostafa Soliman, *Ph.D.*

Outline

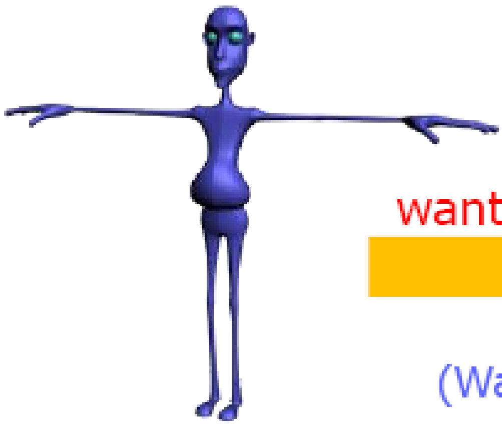
- Scaling Systems
- Scaling of Mechanical Systems
- Scaling of Electrical Components
- Scaling of Electrostatic Force
- Scaling of Electromagnetic Force

Scaling of Systems

- As the size of a system changes, its geometrical and physical parameters change.

- If you drop an ant from ten times its height, it will survive. But That is not the case with big beings.

Scaling of Systems, Example



175cm length
50 kg weight

Scaling $S=10^{-3}$

1.75mm length

wants to walk on water

γ_{water} = 72 mN/m
(Water Surface Tension)

- Weight: 50 x 9.8 = 490 N
- Weight = force by surface tension

→ Leg Perimeter = 490/72e-3 = 6005 m !!

- Weight: 490 N x $(10^{-3})^3$ = 4.9 x 10^{-7}

→ Leg Perimeter = 4.9 x 10^{-7} / 72e-3 = 6.8um

Scaling of Systems

- When designing a MEMS system, first thing we think about is what kind of force we use for actuation.
- The actuating force depends on the actuation phenomenon, for example,
  - Electrostatic force ➔ electric field
  - Magnetic force ➔ magnetic field
- It is important to know how such systems will behave as the physical dimensions (size) reduces.
- Scaling theory is a valuable guide to what may work and what may not when we go to the micro scale.

Scaling of Mechanical Systems

- We define the scale variable "*s*" which represents the linear scale of the system.
- Then, $0 < s < 1$
- Nominally *s* = 1, then if ***s*** changes to 0.1, all the dimensions of the system are decreased by a factor of 10.
- Now if a system is scaled down 10 times its original size, then,

$$x_s = s x_o$$

$$A_s = x_s y_s = s x_o s y_o = s^2 A_o$$

$$V_s = x_s y_s z_s = s x_o s y_o s z_o = s^3 V_o$$

- Which means that the area scales as s^2 and the volume scales as s^3.
- Area to volume ratio (1/*s*) increases with scaling down the system.

Scaling of Mechanical Systems

- Mass scales as follows:

$$M_s = \rho V_s = (s^0 \rho) \times (s^3 V_0) = s^3 M_0$$

  - Where ρ, the density, is constant and does not change with scaling the system.

- Assuming *four force laws* are described as follows:

$$F = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix}$$

- This matrix form is used to show a number of different force cases and scale sizes in a simple way.

Scaling of Acceleration

- The acceleration is defined as:

$$a = \frac{Force}{Mass} = \frac{F}{M}$$

- The acceleration can be scaled as:

$$a = \left[s^F\right]\left[s^{-3}\right] = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix} \begin{bmatrix} s^3 \\ s^3 \\ s^3 \\ s^3 \end{bmatrix}^{-1} = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix} \begin{bmatrix} s^{-3} \\ s^{-3} \\ s^{-3} \\ s^{-3} \end{bmatrix} = \begin{bmatrix} s^{-2} \\ s^{-1} \\ s^0 \\ s^1 \end{bmatrix}$$

- As the force scales down (s^1 to s^4), the acceleration decreases.
- This is an interesting result. When the force scales as [s^1], the acceleration scales as [s^{-2}]. If the size of the system decreases by a factor of 100, the acceleration increases by $(1/100)^{-2} = 10000$.
- **Small structures can be accelerated faster than big ones.**

Scaling of Transient Time

- The transient time t_{tr} to move from point A to B can be calculated as:

$$x = \frac{1}{2} a t_{tr}^{2} \qquad t_{tr} = \sqrt{\frac{2x}{a}} = \sqrt{2} \cdot (x)^{0.5} \cdot (a)^{-0.5}$$

- In matrix form:

$$t_{tr} = \begin{bmatrix} s^0 \\ s^0 \\ s^0 \\ s^0 \end{bmatrix} \begin{bmatrix} s^1 \\ s^1 \\ s^1 \\ s^1 \end{bmatrix}^{0.5} \begin{bmatrix} s^{-2} \\ s^{-1} \\ s^0 \\ s^1 \end{bmatrix}^{-0.5} = \begin{bmatrix} s^0 \\ s^0 \\ s^0 \\ s^0 \end{bmatrix} \begin{bmatrix} s^{0.5} \\ s^{0.5} \\ s^{0.5} \\ s^{0.5} \end{bmatrix} \begin{bmatrix} s^1 \\ s^{0.5} \\ s^0 \\ s^{-0.5} \end{bmatrix} = \begin{bmatrix} s^{1.5} \\ s^1 \\ s^{0.5} \\ s^0 \end{bmatrix}$$

- As the force scales down (s^1 to s^4), the transient time increases.
- This is an interesting result. Even in the worst case, where $F=[s^4]$, the time required to perform a task is constant when the scale goes down.
- when $F=[s^2]$, the time required to perform a task decreases as the scale "*s*" goes down.
- **Small things are quick.**

Scaling of Power Density

- The power, *P*, or work done on the object per unit time is:

$$P = \frac{Fx}{t_{tr}}$$

- In matrix form:

$$P = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix} \begin{bmatrix} s^1 \\ s^1 \\ s^1 \\ s^1 \end{bmatrix} \begin{bmatrix} s^{-1.5} \\ s^{-1} \\ s^{-0.5} \\ s^0 \end{bmatrix} = \begin{bmatrix} s^{0.5} \\ s^2 \\ s^{3.5} \\ s^5 \end{bmatrix}$$

- The power that can be produced per unit volume is:

$$\frac{P}{\text{volume}} = \begin{bmatrix} s^{0.5} \\ s^2 \\ s^{3.5} \\ s^5 \end{bmatrix} \begin{bmatrix} s^3 \\ s^3 \\ s^3 \\ s^3 \end{bmatrix}^{-1} = \begin{bmatrix} s^{-2.5} \\ s^{-1} \\ s^{0.5} \\ s^2 \end{bmatrix}$$

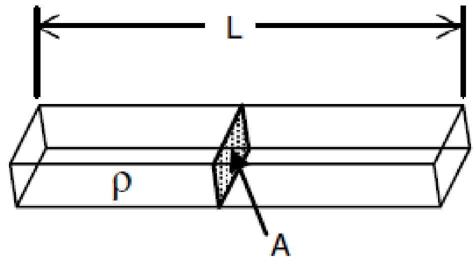
Scaling of Power Density

$$\frac{P}{\text{volume}} = \begin{bmatrix} s^{0.5} \\ s^{2} \\ s^{3.5} \\ s^{5} \end{bmatrix} \begin{bmatrix} s^{3} \\ s^{3} \\ s^{3} \\ s^{3} \end{bmatrix}^{-1} = \begin{bmatrix} s^{-2.5} \\ s^{-1} \\ s^{0.5} \\ s^{2} \end{bmatrix}$$

- When the force scales as [s^2], then the power per unit volume scales as [s^{-1}]. For example, when the scale decreases by a factor of 10, the force reduces by a factor of 100 but the power that can be generated per unit volume increases by a factor of 10. Thus, for force laws with a power higher than [s^2], the power generated per unit volume degrades as the scale decreases.

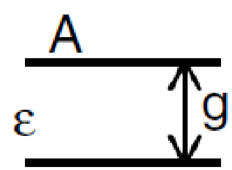
Scaling of electrical components

Resistance - R



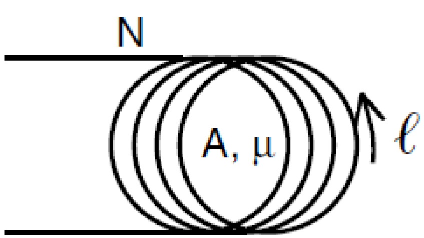
$$R = \frac{\rho L}{A} \propto \frac{1}{S}$$

Capacitance - C



$$C = \frac{\varepsilon A}{g} \propto S$$

Inductance - L

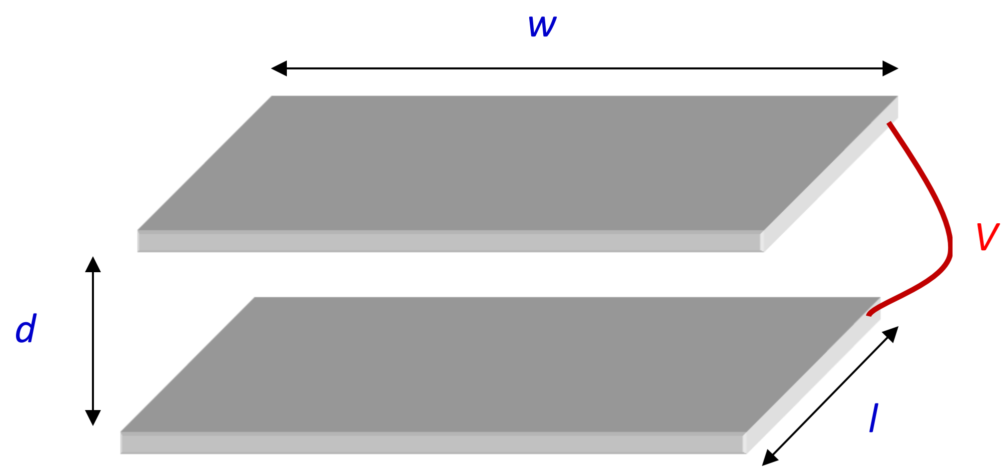


$$L = \frac{\mu N^2 A}{\ell} \propto S$$

Scaling of electrostatic force

- Stored potential energy is:

$$U = \frac{1}{2} C V^2$$



- The capacitance is:

$$C = \frac{\varepsilon_o A}{d} = \frac{\varepsilon_o wl}{d}$$

- The electrostatic force is expressed as:

$$F = \frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2}\varepsilon_o \frac{wl}{d} V^2\right]$$

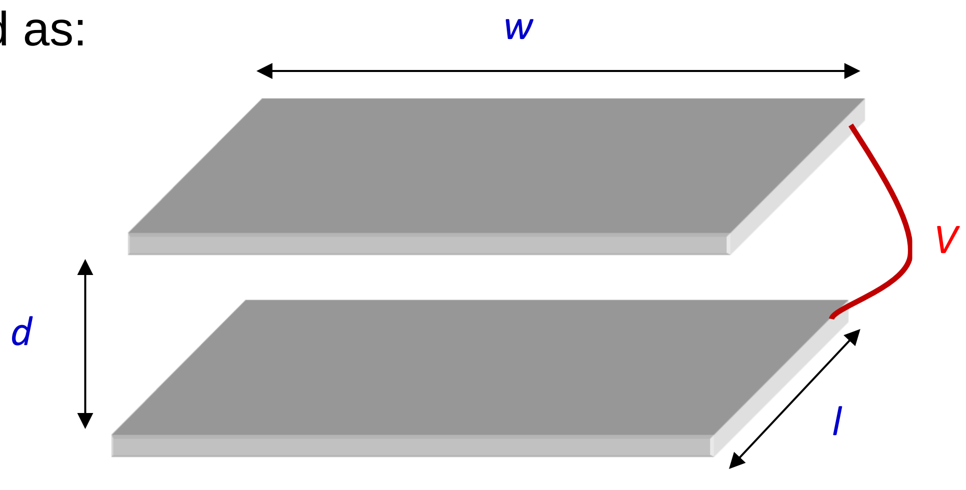
- The applied voltage can be expressed as $V = E\,d$, where E is the electric field strength (Volt/m).

Scaling of electrostatic force

- The electrostatic force is expressed as:

$$F = \frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2}\varepsilon_o \frac{wl}{d} E^2 d^2\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{1}{2}\varepsilon_o wldE^2\right]$$



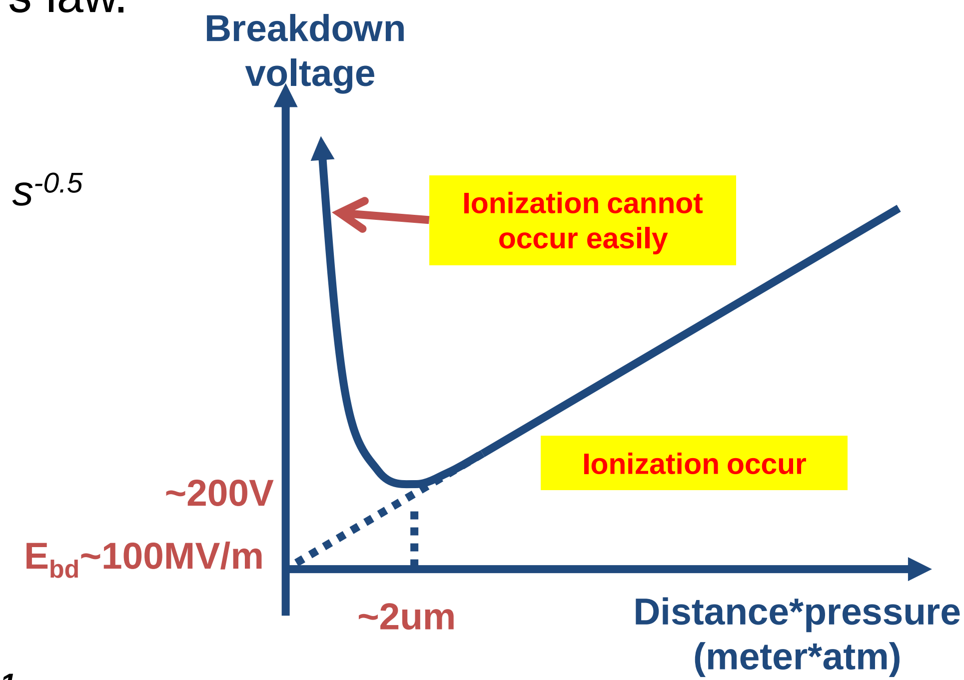
- The Electrostatic force will scale as

$$F = s^2 E^2 \text{ or, } F \alpha\, s^2$$

- In this case, scaling of the force depends on the scaling of the electric field *E*.
- If the field strength, *E*, does not change with the scale, then $E \alpha\, s^0$ and *F* will scale as s^2.

Scaling of electrostatic force

- For micro-scale, where the separation between electrodes and surfaces is reduced dramatically, the voltage can be increased without breaking down the air gap. This is explained by Paschen's law.



- For micro domain,

the breakdown electric field can be scaled as $s^{-0.5}$

- Then the force can be scaled even better as

$$F \ \alpha \ s^2 (s^{-0.5})^2 \ \alpha \ s^1$$

- **<u>The electrostatic force scales as s^2 or s^1 .</u>**
- **<u>The electrostatic force is surface force, so it scales well in the micro domain.</u>**

Scaling of electromagnetic force

- **Force between two wires:**

a- Constant current density: $F=[s^4]$.

b- Constant heat flow through the surface of the wire: $F=[s^3]$.

c- Constant temperature rise of the wire: $F=[s^2]$.

- **Force between a wire and a permanent magnet:**

a- Constant current density: $F=[s^3]$.

b- Constant heat flow through the surface of the wire: $F=[s^{2.5}]$.

c- Constant temperature rise of the wire: $F=[s^2]$.

- The electromagnetic force scales as s^2 for best case scenario.
- The electromagnetic force is body, or volume, force, so it does not scale well in the micro domain.
- **In micro domain, electrostatic forces are more favorable.**

Summary

- Geometrical and physical parameters of a system change according to the change in its scale.
- Small structures can be accelerated faster than big ones.
- Small things are quick.
- MEMS designer should be aware of which forces are favorable and which are not.
- As the force scales down, the power density reduces.
- Electrostatic force is a surface force.
- Electromagnetic force is a body force, or volume force.
- Electrostatic force scales better than electromagnetic force in micro domain.
- In <u>macro</u> world Electromagnetic force is favorable more than electrostatic force.