# Assignment 3

## Sensors Characteristics

**Problem 1:**

(a) An instrument is calibrated in an environment at a temperature of 20°C and the following output readings y are obtained for various input values x:

| **x** | 13.1 | 26.2 | 39.3 | 52.4 | 65.5 | 78.6 |
|---|---|---|---|---|---|---|
| **y** | 5 | 10 | 15 | 20 | 25 | 30 |

Determine the measurement sensitivity, expressed as the ratio y/x.

(b) When the instrument is subsequently used in an environment at a temperature of 50°C, the input/output characteristic changes to the following:

| **x** | 14.7 | 29.4 | 44.1 | 58.8 | 73.5 | 88.2 |
|---|---|---|---|---|---|---|
| **y** | 5 | 10 | 15 | 20 | 25 | 30 |

Determine the new measurement sensitivity. Hence determine the sensitivity drift due to the change in ambient temperature of 30°C.

**Problem 2:**

A load cell is calibrated in an environment at a temperature of 21°C and has the following deflection/load characteristic:

| **Load (Kg)** | 0 | 50 | 100 | 150 | 200 |
|---|---|---|---|---|---|
| **Deflection (mm)** | 0 | 1 | 2 | 3 | 4 |

When used in an environment at 35°C, its characteristic changes to the following:

| **Load (Kg)** | 0 | 50 | 100 | 150 | 200 |
|---|---|---|---|---|---|
| **Deflection (mm)** | 0.2 | 1.3 | 2.4 | 3.5 | 4.6 |

(a) Determine the sensitivity at 21°C and 35°C.
(b) Calculate the total zero drift and sensitivity drift at 35°C.
(c) Hence determine the zero drift and sensitivity drift coefficients (in units of µm/°C and(µm per kg)/(°C)).

**<u>Problem :3</u>**

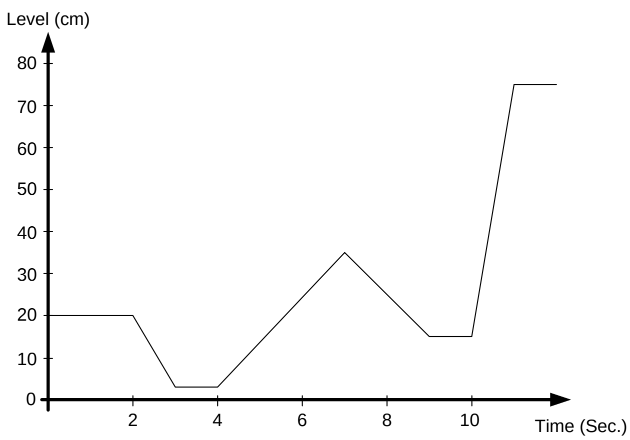
An unmanned submarine is equipped with temperature and depth measuring instruments and has radio equipment that can transmit the output readings of these instruments back to the surface. The submarine is initially floating on the surface of the sea with the instrument output readings in steady state. The depth-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 50 seconds. The water temperature on the sea surface, $T_0$, is20°C and the temperature $T_x$ at a depth of x metres is given by the relation: $T_x = T_0 - 0.01x$

(a) If the submarine starts diving at time zero, and thereafter goes down at a velocity of 0.5 metres/second, draw a table showing the temperature and depth measurements reported at intervals of 100 seconds over the first 500 seconds of travel. Show also in the table the error in each temperature reading.
(b) What temperature does the submarine report at a depth of 1000 metres?

**<u>Problem 4:</u>**

A capacitive level sensor is used to detect the level of water in a reservoir with 1m length. The sensor is made of a co-axial type capacitor with inner and out radii equal to 2mm and 4mm, respectively. The length of the sensor is 40cm and put at a 10cm from the bottom of the reservoir, assume the relative permittivity of water to be 2.5.

(a) Determine the sensitivity of the sensor
(b) If the water level in the reservoir is changing with time with the pattern shown below. Sketch the output of the sensor due to such changes indicating the reading of the sensor at each crossing point.



**Problem 5:**

You have a capacitive sensor that changes capacitance in response to changes in humidity according to the following equation:

C = 0.90μF + 0.002μF*(% humidity)

You integrate this capacitor into an LC circuit and measure the resonant frequency to determine the sensor's change in capacitance due to change in humidity. The inductor has an inductance of 10 mH. The resonant frequency, $f_{res}$, (in Hz) of an LC oscillator is

$f_{res} = 1/(2\pi\sqrt{LC})$

You are an expert in circuit design and quickly whip up a circuit that measures the resonant frequency and outputs a voltage with the following transfer function:

$V_{out} = f_{res}\,\alpha$ where $\alpha = 1$ *mV/Hz*

Assume all system elements of the system are ideal and precisely the value specified.

(a) What is the transfer function of the overall sensor system (humidity → voltage)?
(b) What is the range of possible outputs? What is the offset?
(c) What is the nominal sensitivity of the sensor system using this 10 mH inductor? What is the sensitivity at 75% humidity? At 100% humidity?
(d) Derive the Taylor Series expansion (through the 2nd derivative) of the output signal about the nominal capacitance value (Co = 1 μF when RH = 50%). (Hint: Taylor series of a function *f(x)* around point (*a*) is:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

(e) At 75% humidity, what is the ratio of the quadratic term to the linear term? What would we like this ratio to be? Give your answer both symbolically and in terms of the provided component values. Does the ratio get larger or smaller as the humidity gets closer to 50%?

**Problem 6:**

A resistor has a temperature dependence of resistance given by:

$$R = R_{T0} e^{\beta\frac{(T-T_0)}{TT_0}}$$

In this expression $T_0$ = 300 K, β = 3000 K, and $R_{T0}$ is 1000 Ω. This is a typical temperature dependence for a Negative Temperature Coefficient (NTC) thermistor, commonly used as a temperature sensor in low-accuracy applications.

(a) What is the sensitivity of this sensor at 300K?
(b) We are interested in using this sensor for temperature measurements near 300K (23C). Carry out a Taylor Series Expansion to the second derivative terms, and evaluate the terms of the expansion.
(c) How large is the second derivative term relative to the other two terms at 350K? at 400K?

**Problem 7:**

A cantilever beam has a pair of strain gauges mounted on the top and bottom of the beam, so that bending of the beam causes one to increase in resistance and the other to decrease in resistance. In equation form, $R_1 = R_o\left(1+\frac{F}{\beta}\right)$ and $R_2 = R_o\left(1-\frac{F}{\beta}\right)$, where $R_0$=1000Ω,

β = 1000N, F is the applied force in Newtons, and $R_1$ and $R_2$ are the resistance values of the sensors. These are wired together as a voltage divider as shown below.

(a) If the bias voltage is 5V, what is the output voltage as a function of load force, F?
(b) Using the Taylor series expansion, estimate the size of the quadratic error term relative to the linear term.