Faculty of Engineering

MCT242: Electronic Instrumentation

Lecture 7:
Filters

Overview

- Introduction to Passive and Active Filters
- Filter Categories
- Butterworth/Chebyshev/Bessel Filters
- Poles and Multiple Stages
- Transfer Function
- Bode Plot

Definition

- A *filter* is a system that processes a signal in some desired fashion.

Types of Filters

- There are two broad categories of filters:
  - An *analog filter* processes continuous-time signals
  - A *digital filter* processes discrete-time signals.
- They can also be categorized as
  - Passive filters
  - Active filters

Passive Filters

- Made up of passive components - resistors, capacitors and inductors
- No amplifying elements (- transistors, op-amps, etc)
- No signal gain
- 1st order - design is simple (just use standard equations to find resonant frequency of the circuit)
- 2nd order - complex equations
- Require no power supplies
- Not restricted by the bandwidth limitations of the op-amps
- Can be used at very high frequencies
- Can handle larger current or voltage levels than active devices
- Buffer amplifiers might be required

Passive elements : Inductor

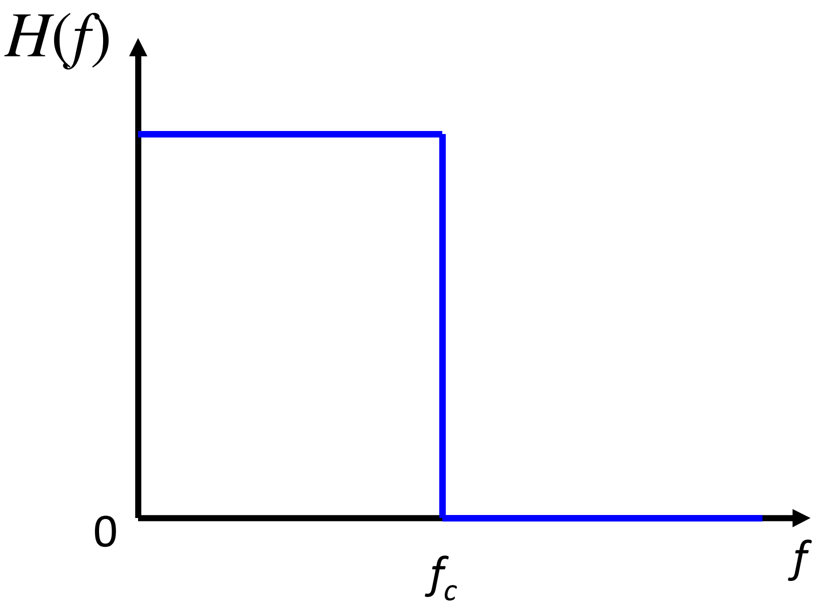
BIG PROBLEM!

- High accuracy (1% or 2%), small physical size, or large inductance values are required
- Standard values of inductors are not very closely spaced
- Difficult to find an off-the-shelf inductor within 10% of any arbitrary value
- Adjustable inductors are used
- Tuning such inductors to the required values is time-consuming and expensive for larger quantities of filters
- Inductors are often prohibitively expensive

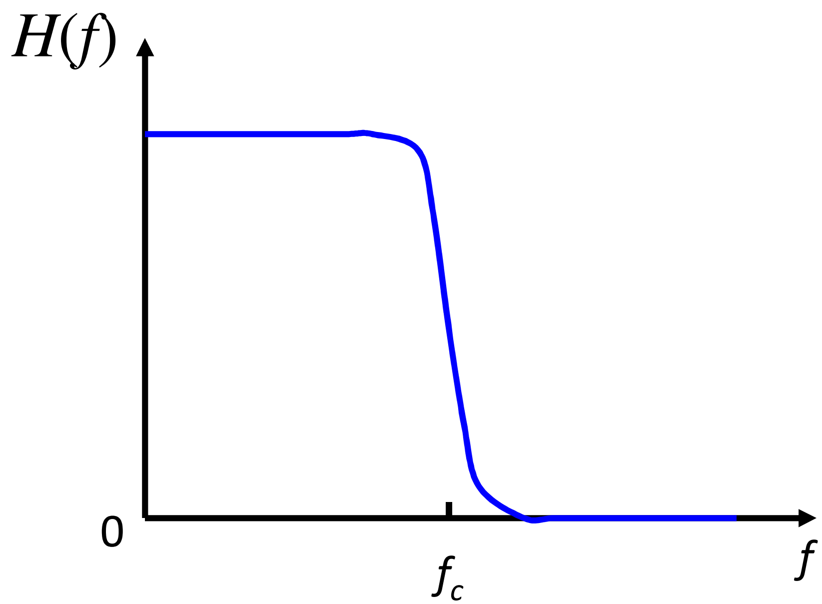
Active Filters

- No inductors
- Made up of op-amps, resistors and capacitors
- Provides virtually any arbitrary gain
- Generally easier to design
- High input impedance prevents excessive loading of the driving source
- Low output impedance prevents the filter from being affected by the load
- At high frequencies is limited by the gain-bandwidth of the op-amps
- Easy to adjust over a wide frequency range without altering the desired response

Analog Filter Response
(Bode Plot)



Ideal “brick wall” filter

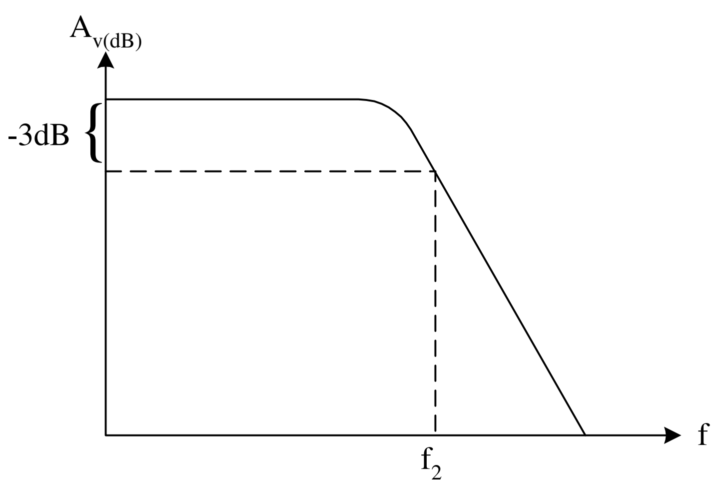


Practical filter

Categories of Filters

Low Pass Filters (LPF):

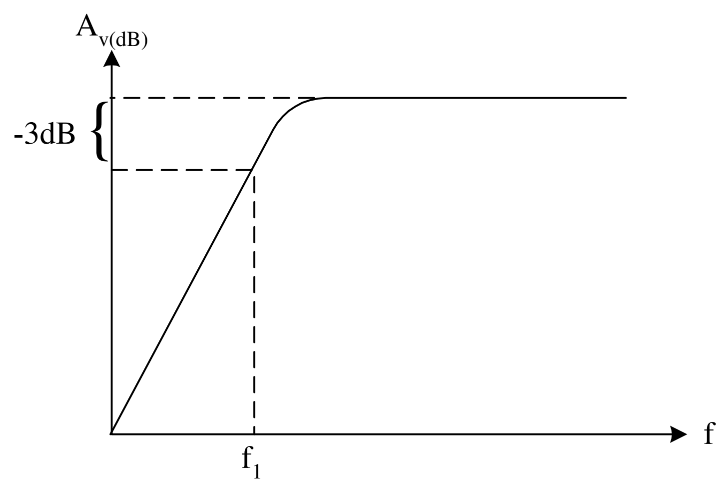
pass all frequencies from dc up to the upper cutoff frequency.



Low-pass response

High Pass Filters (HPF):

pass all frequencies that are above its lower cutoff frequency

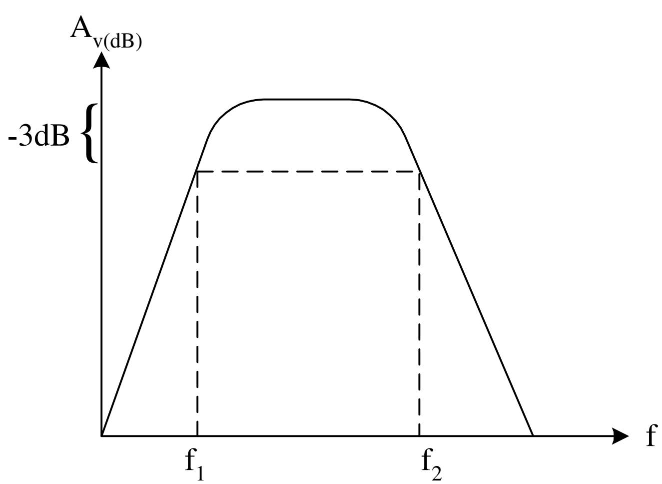


High-pass response

Categories of Filters

Band Pass Filters:

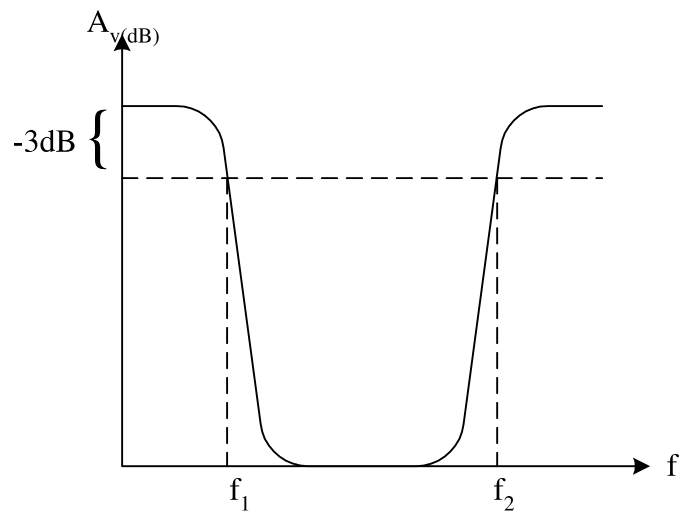
pass only the frequencies that fall between its values of the lower and upper cutoff frequencies.



Band Pass Response

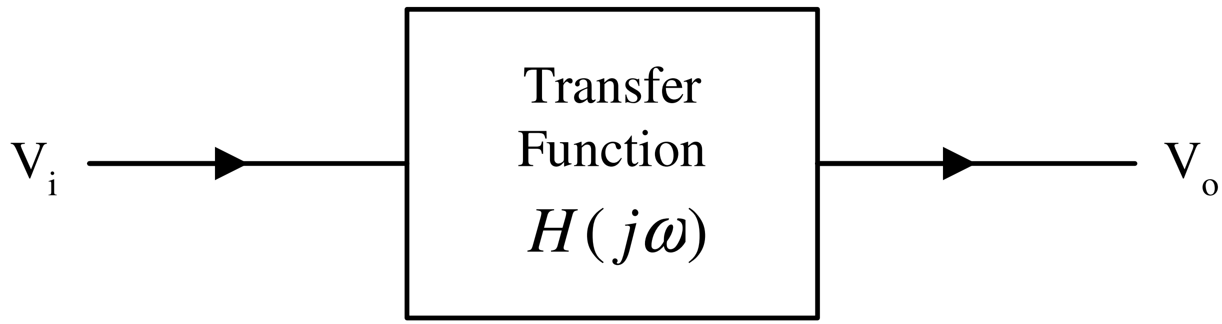
Band Stop (Notch) Filters:

eliminate all signals within the stop band while passing all frequencies outside this band.



Band Stop Response

Transfer function H(jω)



$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

$$H = \mathrm{Re}(H) + j\,\mathrm{Im}(H)$$

$$|H| = \sqrt{\mathrm{Re}(H)^2 + \mathrm{Im}(H)^2}$$

$$\angle H = \tan^{-1}\left(\frac{\mathrm{Im}(H)}{\mathrm{Re}(H)}\right) \qquad \mathrm{Re}(H) \succ 0$$

$$\angle H = 180^o + \tan^{-1}\left(\frac{\mathrm{Im}(H)}{\mathrm{Re}(H)}\right) \qquad \mathrm{Re}(H) \prec 0$$

Frequency transfer function of filter H(jω)

(I) Low - Pass Filter

$|H(j\omega)| = 1 \qquad f < f_o$

$|H(j\omega)| = 0 \qquad f > f_o$

(II) High - Pass Filter

$|H(j\omega)| = 0 \qquad f < f_o$

$|H(j\omega)| = 1 \qquad f > f_o$

(III) Band - Pass Filter

$|H(j\omega)| = 1 \qquad f_L < f < f_H$

$|H(j\omega)| = 0 \qquad f < f_L$ and $f > f_H$

(IV) Band - Stop (Notch) Filter

$|H(j\omega)| = 0 \qquad f_L < f < f_H$

$|H(j\omega)| = 1 \qquad f < f_L$ and $f > f_H$

(V) All - Pass (or phase - shift) Filter

$|H(j\omega)| = 1 \qquad$ for all f

has a specific phase response

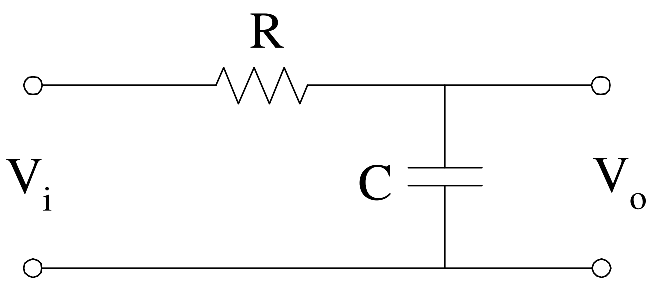
Bode Plots

- Bode plots are important when considering the frequency response characteristics of amplifiers. They plot the magnitude or phase of a transfer function in dB versus frequency.

Poles & Zeros of the transfer function

- pole—value of *s* where the denominator goes to zero.
- zero—value of *s* where the numerator goes to zero.

Passive Single-Pole LPF



$$V_o = \frac{X_C}{X_C + R} V_i$$

$$V_o = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i = \frac{1}{1 + j\omega CR} V_i$$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$ where $$\omega_o = \frac{1}{RC}$$

or

$$H(s) = \frac{\omega_0}{s + \omega_0}$$

where

$$s = j\omega$$

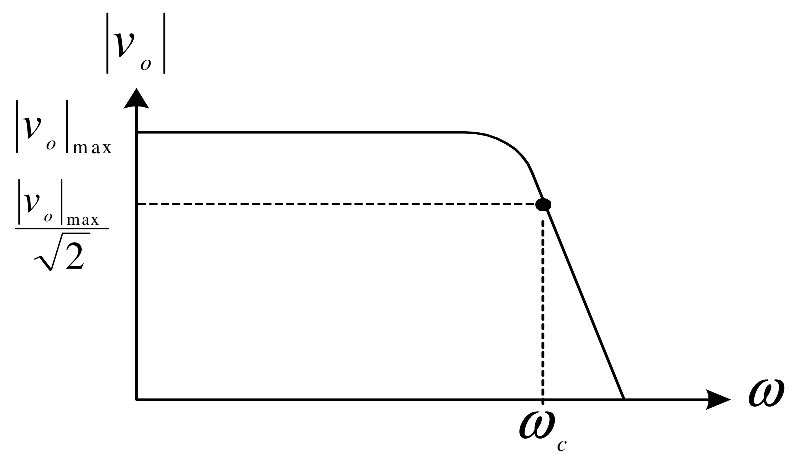
$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$\omega \to 0 \Rightarrow |V_o| = |V_i| \leftarrow$ **max. value**

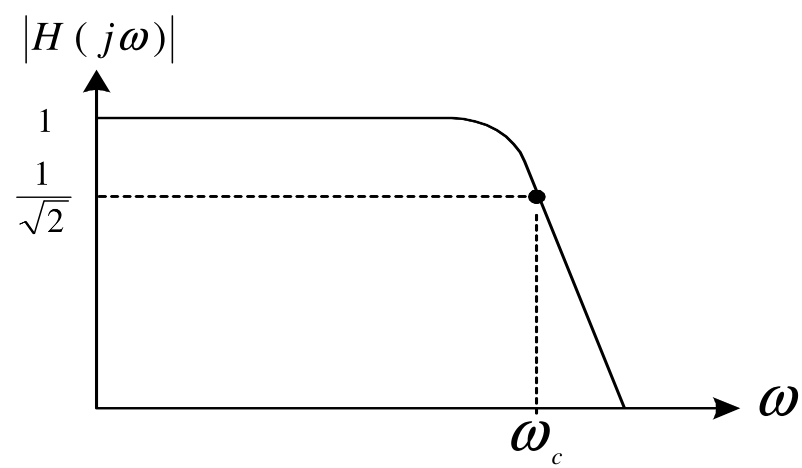
$\omega \to \infty \Rightarrow |V_o| = 0 \leftarrow$ **min. value**

$$V_o = \frac{1}{1 + j\omega CR} V_i$$



$$\omega = \frac{1}{RC}$$ $\Rightarrow$ **$|V_o|$ = ??**

$$V_o = \frac{1}{1 + j} V_i$$

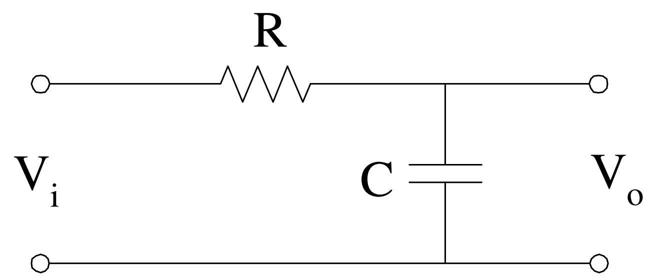


$$|V_o| = \frac{1}{\sqrt{1^2 + 1^2}} |V_i| = \frac{1}{\sqrt{2}} |V_i|$$

$$\omega_c = \omega_o = \frac{1}{RC} \quad \text{(cut-off frequency)}$$

Bode Plot (single pole)

$$H(j\omega) = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j\left(\frac{\omega}{\omega_o}\right)}$$



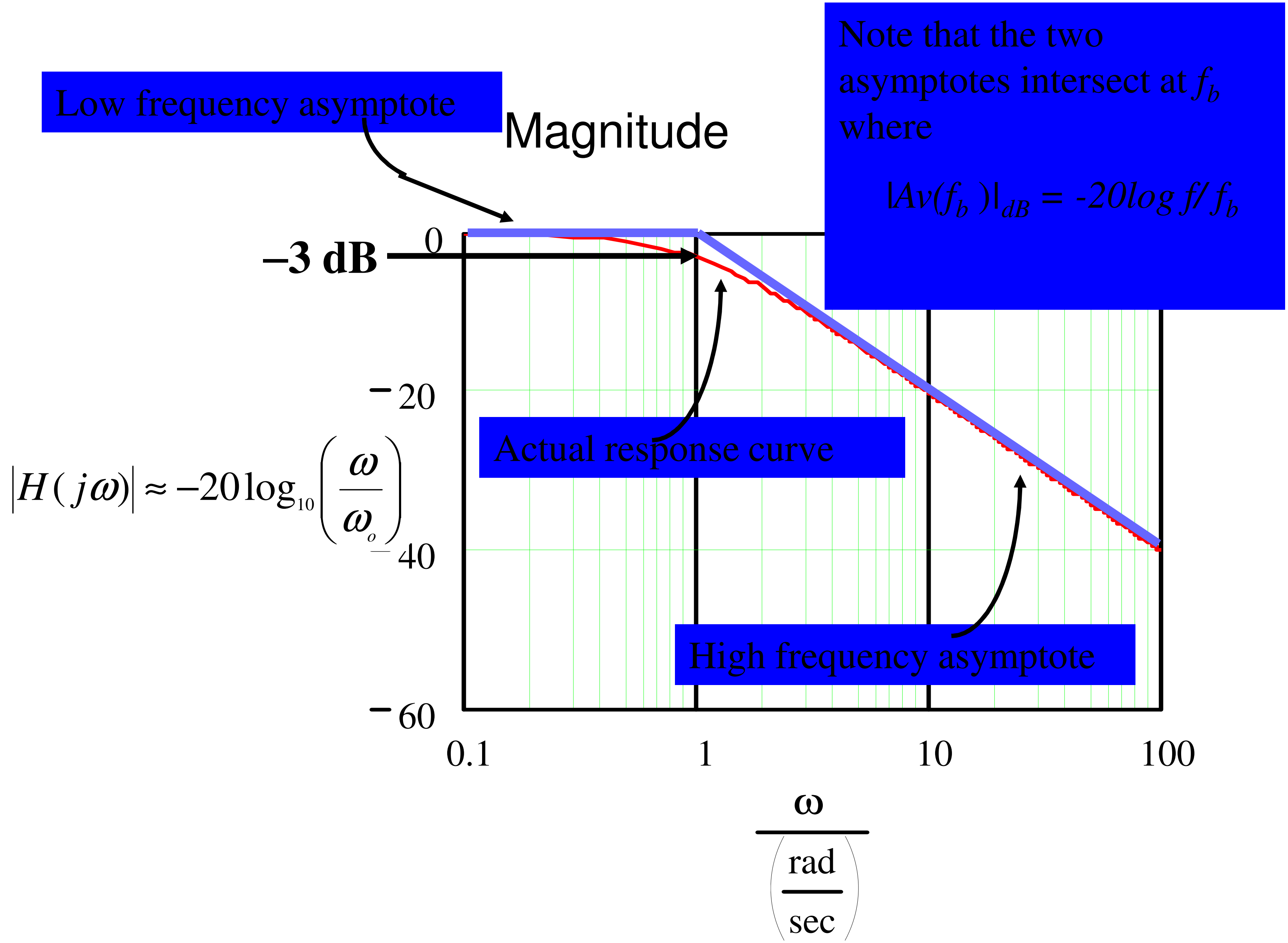
Single pole low-pass filter

$$\Rightarrow \quad |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}}$$

$$|H(j\omega)|_{dB} = 20\log_{10}|H(j\omega)| = 20\log_{10}\left(1 \Big/ \sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}\right)$$

For $\omega >> \omega_o$

$$|H(j\omega)|_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_o}\right)$$

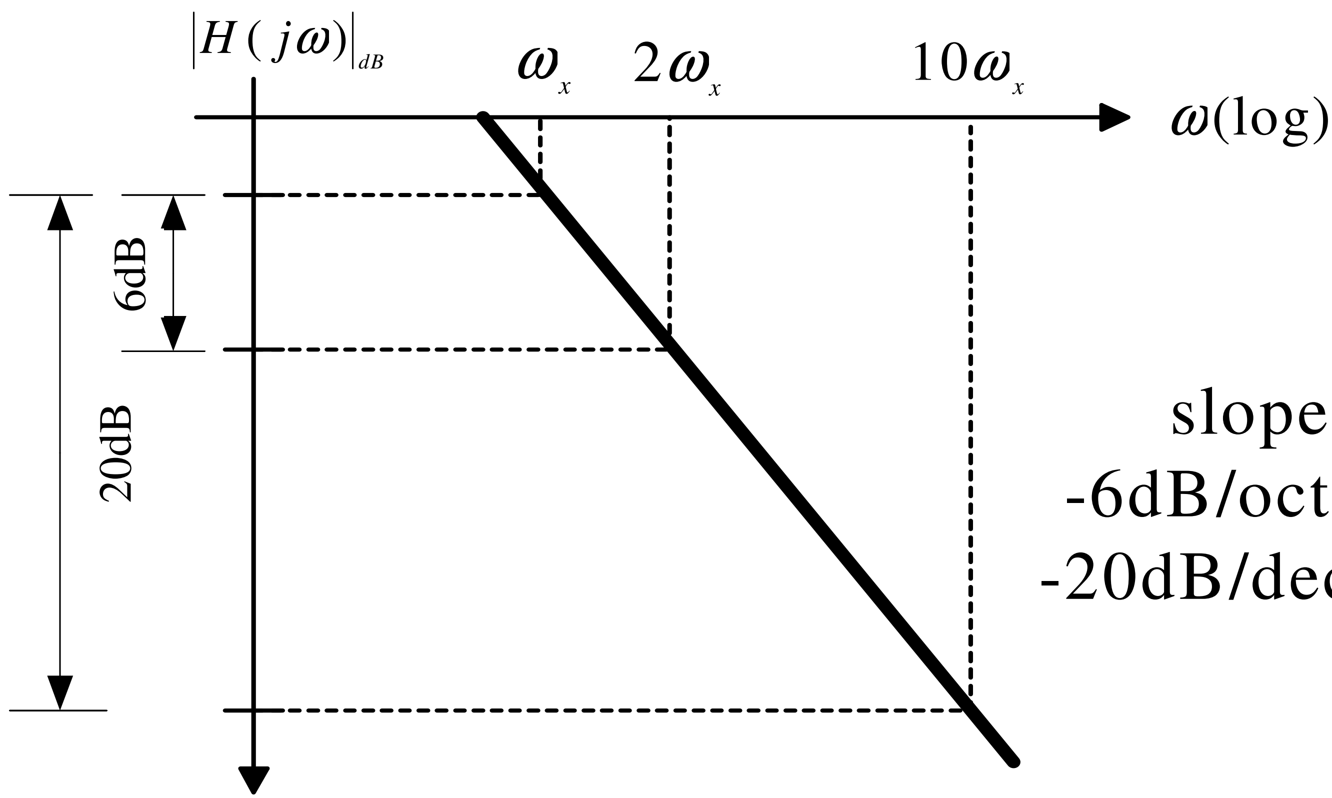


Low frequency asymptote
Magnitude
Note that the two asymptotes intersect at f_b where
$|Av(f_b)|_{dB} = -20log\ f/f_b$
–3 dB
0
–20
–40
–60
$|H(j\omega)| \approx -20\log_{10}\left(\frac{\omega}{\omega_o}\right)$
Actual response curve
High frequency asymptote
0.1
1
10
100
$\frac{\omega}{\left(\frac{rad}{sec}\right)}$

$$\left|H(j\omega)\right| \approx -20\log_{10}\left(\frac{\omega}{\omega_o}\right)$$

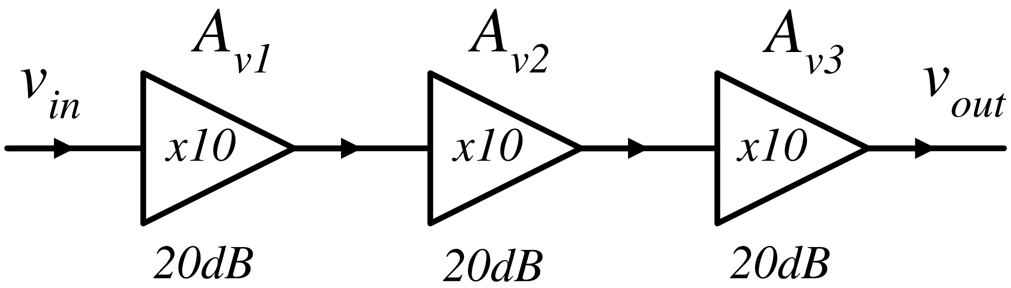
For octave apart, $\frac{\omega}{\omega_o} = \frac{2}{1}$ $\qquad \left|H(j\omega)\right| \approx -6dB$

For decade apart, $\frac{\omega}{\omega_o} = \frac{10}{1}$ $\qquad \left|H(j\omega)\right| \approx -20dB$



slope
-6dB/octave
-20dB/decade

Cascaded System



$$A_v = A_{v1} \times A_{v2} \times A_{v3}$$

$$A_v = 10 \times 10 \times 10 = 10^3$$

$$A_v(dB) = 20\log_{10}\left(A_{v1} \times A_{v2} \times A_{v3}\right)$$

$$A_v(dB) = 20\log_{10}\left(A_{v1}\right) + 20\log_{10}\left(A_{v2}\right) + 20\log_{10}\left(A_{v3}\right)$$

$$A_v(dB) = A_{v1}(dB) + A_{v2}(dB) + A_{v3}(dB)$$

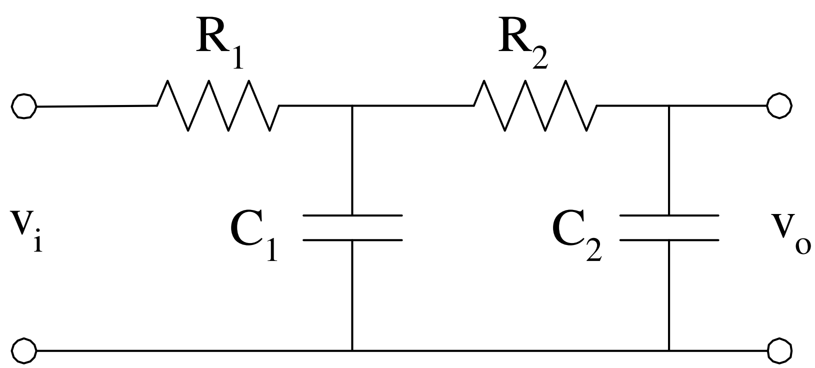
$$A_v(dB) = 20dB + 20dB + 20dB$$

$$A_v(dB) = 60dB$$

$$20\log_{10}(10) = 20dB$$

$$20\log_{10}(10^3) = 60dB$$

Bode plot (Two-pole)



$$\omega_{o1} \neq \omega_{o2}$$

$$\left|H(j\omega)\right| = 20\log_{10}\left\{1 \Big/ \left(\sqrt{1+\left(\frac{\omega}{\omega_{o1}}\right)^2}\sqrt{1+\left(\frac{\omega}{\omega_{o2}}\right)^2}\right)\right\}$$

- The technique for approximating a filter function based on Bode plots is useful for low order, simple filter designs
- More complex filter characteristics are more easily approximated by using some well-described rational functions, the roots of which have already been tabulated and are well-known.