Design and Production Engineering Department

Chapter 5
Acceleration Analysis

MCT 251

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5.1. Definitions

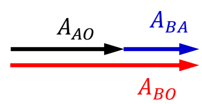
- Acceleration

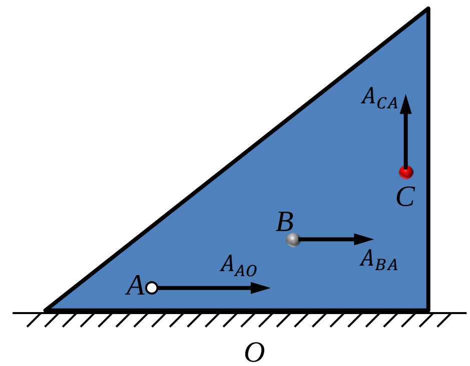
The vector represents the rate of change of the velocity with respect to time.

$$\vec{A} = \frac{d\vec{V}}{dt}$$

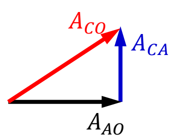
5.1. Definitions (cont.)

- Absolute and relative accelerations
  - $\overrightarrow{A_A} = \overrightarrow{A_{AO}}$
  - $\overrightarrow{A_{OA}} = -\overrightarrow{A_{AO}}$
  - $\overrightarrow{A_{BO}} = \overrightarrow{A_{BA}} + \overrightarrow{A_{AO}}$





  - $\overrightarrow{A_{CO}} = \overrightarrow{A_{CA}} + \overrightarrow{A_{AO}}$



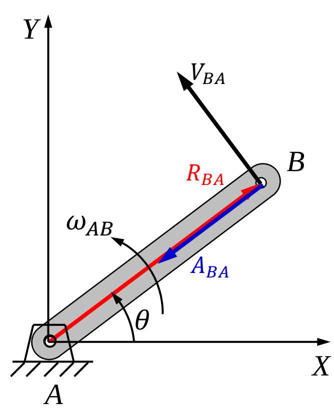
5.2. Analytical Acceleration Analysis

- Relative acceleration between two points on a rotating link
  - The link rotates with constant angular velocity
  - $\overrightarrow{V_{BA}} = R_{BA}\,\omega_{AB}\,i\,e^{i\theta}$
  - $\overrightarrow{A_{BA}} = \frac{d}{dt}\overrightarrow{V_{BA}} = R_{BA}\,\omega_{AB}\,i\,\frac{d}{dt}\left(e^{i\theta}\right)$

$\overrightarrow{A_{BA}} = R_{AB}\,\omega_{AB} i \times \omega_{AB}\, i\, e^{i\theta}$

$\overrightarrow{A_{BA}} = -R_{BA}\,\omega_{AB}^2\; e^{i\theta}$

$\overrightarrow{A_{BA}} = R_{AB}\,\omega_{AB}^2 @\angle(\theta + 180°)$



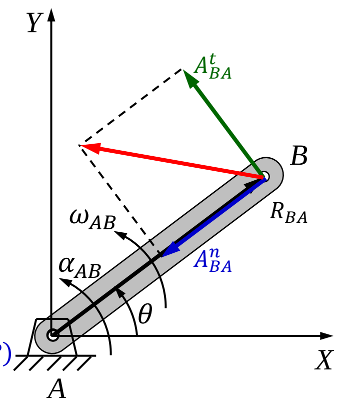
5.2. Analytical Acceleration Analysis (cont.)

- Relative acceleration between two points on a rotating link
  - The link rotates with variant angular velocity
  - $\overrightarrow{V_{BA}} = R_{BA}\, \omega_{AB}\, i\, e^{i\theta}$
  - $\overrightarrow{A_{BA}} = \frac{d}{dt}\overrightarrow{V_{BA}} = R_{BA}\, i \left[\frac{d}{dt}(\omega_{AB})\, e^{i\theta} + \omega_{AB}\frac{d}{dt}(e^{i\theta})\right]$

$$\overrightarrow{A_{BA}} = R_{BA}\, i\left[\alpha_{AB} e^{i\theta} + \omega_{AB} i \omega_{AB} e^{i\theta}\right]$$

$$\overrightarrow{A_{BA}} = \alpha_{AB} R_{BA} i\, e^{i\theta} - R_{BA}\, \omega_{AB}^2 e^{i\theta}$$

$$\overrightarrow{A_{BA}} = \alpha_{AB} R_{BA} @\angle(\theta + 90^\circ) + R_{BA}\omega_{AB}^2 @\angle(\theta + 180^\circ)$$



5.2. Analytical Acceleration Analysis (cont.)

- Relative acceleration between two points, which have relative sliding along a rotating link $\overrightarrow{V_{DA}} = R_{DA}\, \omega_{AB} i\, e^{i\theta} + \dot{R}_{DA} e^{i\theta}$
  - $\overrightarrow{A_{DA}} = \dot{R}_{DA}\omega_{AB} i\, e^{i\theta} + R_{DA}\alpha_{AB} i\, e^{i\theta} + R_{DA}\omega_{AB} i \omega_{AB} i\, e^{i\theta}$

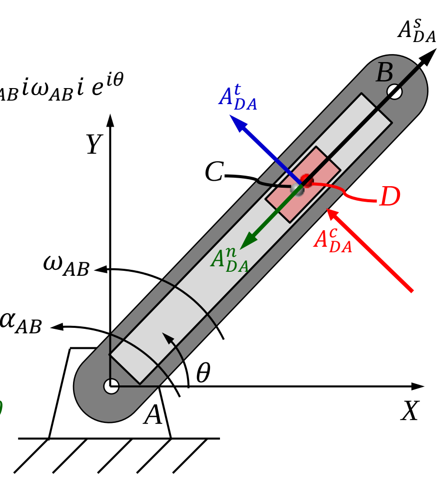
$$+\ddot{R}_{DA} e^{i\theta} + \dot{R}_{DA} i \omega_{AB} e^{i\theta}$$

  - $\overrightarrow{A_{DA}} = R_{DA}\alpha_{AB} i\, e^{i\theta} - R_{DA}\omega_{AB}^2 e^{i\theta}$

$$+\ddot{R}_{DA} e^{i\theta} + 2\dot{R}_{DA}\omega_{AB} i\, e^{i\theta}$$

  - $\overrightarrow{A_{DA}} = R_{DA}\alpha_{AB} @\angle(\theta + 90^\circ) - R_{DA}\omega_{AB}^2 @\angle\theta$

$$+\ddot{R}_{DA} @\angle\theta + 2\dot{R}_{DA}\omega_{AB} @\angle(\theta + 90^\circ)$$



5.2. Analytical Acceleration Analysis (cont.)

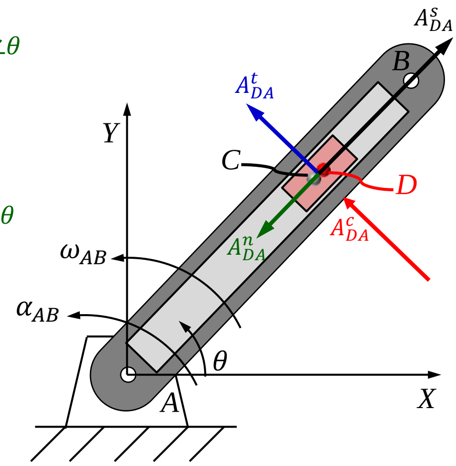
- $\overrightarrow{A_{DA}} = R_{DA}\alpha_{AB} i\, e^{i\theta} - R_{DA}\omega_{AB}^2 e^{i\theta} + \ddot{R}_{DA} e^{i\theta} + 2\dot{R}_{DA}\omega_{AB} i\, e^{i\theta}$

- $\overrightarrow{A_{DA}} = R_{DA}\alpha_{AB}@\angle(\theta + 90°) - R_{DA}\omega_{AB}^2@\angle\theta$
  $+\ddot{R}_{DA}@\angle\theta + 2\dot{R}_{DA}\omega_{AB}@\angle(\theta + 90°)$

C is a point on a rotating link

- $\overrightarrow{A_{CA}} = R_{CA}\alpha_{AB} i\, e^{i\theta} - R_{CA}\omega_{AB}^2 e^{i\theta}$
  $\overrightarrow{A_{CA}} = R_{CA}\alpha_{AB}@\angle(\theta + 90°) - R_{CA}\omega_{AB}^2@\angle\theta$

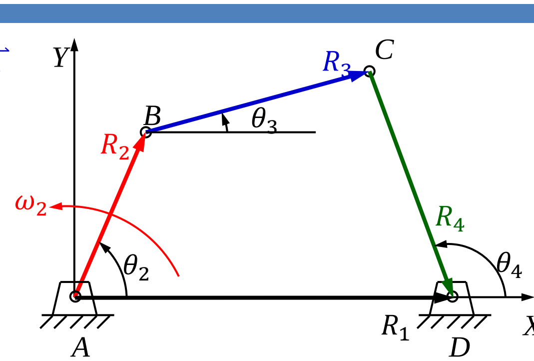
- $\overrightarrow{A_{DC}} = \ddot{R}_{DC} e^{i\theta} + 2\dot{R}_{DC}\omega_{AB} i\, e^{i\theta}$
  $\overrightarrow{A_{DC}} = \ddot{R}_{DC}@\angle\theta + 2\dot{R}_{DC}\omega_{AB}@\angle(\theta + 90°)$



- $\overrightarrow{A_{DA}} = \overrightarrow{A_{CA}} + \overrightarrow{A_{DC}}$

4.2. Analytical Acceleration Analysis (cont.)

- Velocity Equation: $\overrightarrow{V_C} = \overrightarrow{V_B} + \overrightarrow{V_{CB}}$
- $\overrightarrow{A_C} = \overrightarrow{A_B} + \overrightarrow{A_{CB}}$



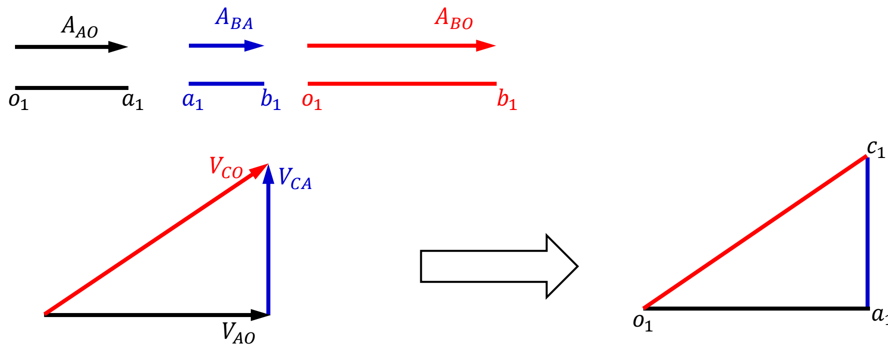
- $\alpha_4 R_4 i e^{i\theta_4} - \omega_4^2 R_4 e^{i\theta_4} = \alpha_2 R_2 i e^{i\theta_2} - \omega_2^2 R_2 e^{i\theta_2} + \alpha_3 R_3 i e^{i\theta_3} - \omega_3^2 R_3 e^{i\theta_3}$

$\dots\dots\alpha_4 - \dots\dots\alpha_3 = \dots\dots$ ______ (1)

$\dots\dots\alpha_4 - \dots\dots\alpha_3 = \dots\dots$ ______ (2)

5.3. Graphical Acceleration Analysis

❑ Acceleration Diagram



Acceleration Diagram (A.D.)
Scale: $1\text{cm} \equiv \cdots m/s^2$

Acceleration Diagram (A.D.)
Scale: $1\text{cm} \equiv \cdots m/s^2$

5.3. Graphical Acceleration Analysis (cont.)

❑ Acceleration Diagram

- A diagram represents the relative acceleration of points to each other.
- The relative acceleration between any two fixed points is equal to zero.

  All fixed points are represented by one point in acceleration diagram.

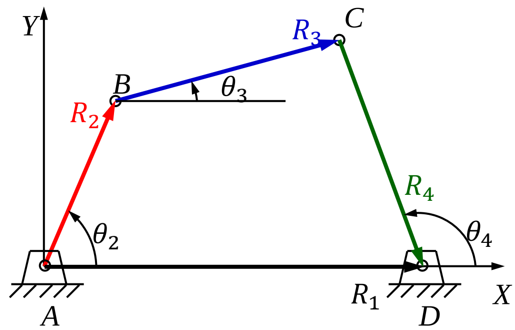
- The acceleration of any point is equal to the vector from the fixed point to the considered point.

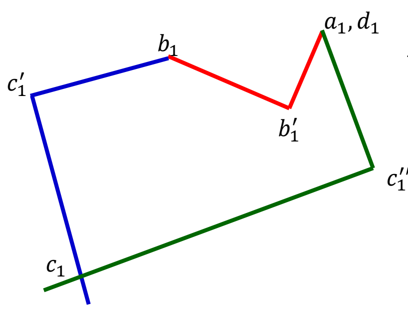
5.3. Graphical Acceleration Analysis (cont.)

- $\overrightarrow{A_B} + \overrightarrow{A_{CB}} + \overrightarrow{A_{OC}} = 0$

  $\overrightarrow{A_B} + \overrightarrow{A_{CB}} - \overrightarrow{A_C} = 0$

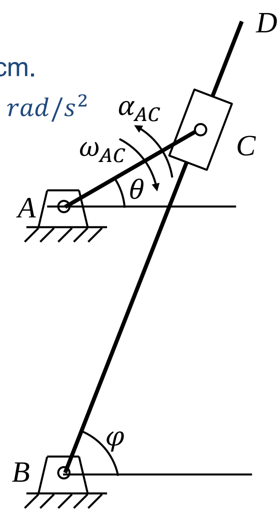
  $\overrightarrow{A_C} = \overrightarrow{A_B} + \overrightarrow{A_{CB}}$





5.4. Solved examples

- Quick-return mechanism
  - AB =6.6 cm, AC = 5 cm, BC = 10.4 cm and BD = 13 cm.
  - $\theta = 36.86°$, $\varphi = 68.38°$ and $\omega_{AC} = 10\ rad/s$, $\alpha_{AC} = 1\ rad/s^2$
  - Find the angular velocity of link BD
  - Find the acceleration of point D.



5.4. Solved examples

- Four-bar mechanism
  - AB = 6 cm, BC = 8cm, CD = 8 cm, AD = 13 cm
  - $\omega_{AB} = 10\, rad/s$, $\alpha_{AB} = 1\, rad/s^2$
  - Find the angular acceleration of links BC and DC.
  - Find the acceleration of point E.