Design and Production Engineering Department

# Chapter 4
# Velocity Analysis

**MCT 251**

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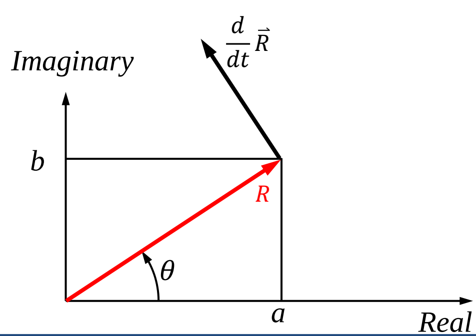
## Revision on vectors & complex number differentiation

- $\vec{R} = a + b\,i = Re^{i\theta}$
- $\frac{d}{dt}\vec{R} = \frac{d}{dt}\left(Re^{i\theta}\right) = R\,i\,e^{i\theta}\frac{d\theta}{dt} = R\omega\,i\,e^{i\theta}$

$$i = 0 + i = \cos 90° + \sin 90°\, i = e^{i\times 90°}$$

$$\frac{d}{dt}\vec{R} = R\omega\, e^{i\times 90°}\, e^{i\theta} = R\omega\, e^{(\theta+90°)}$$

$$\boxed{\frac{d}{dt}\vec{R} = R\omega @\angle(\theta + 90°)}$$



## 4.1. Definitions

- Velocity

The vector represents the rate of change of position with respect to time.

$$\vec{V} = \frac{d\vec{R}}{dt}$$

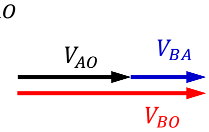
- Absolute Velocity

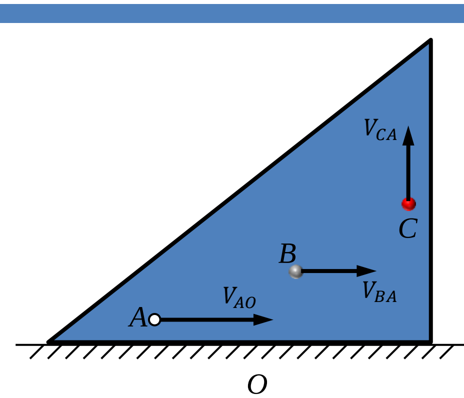
Absolute velocity of a point P is the velocity of point P relative to a fixed point, O.



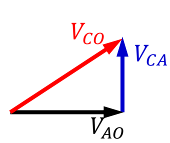
## 4.1. Definitions (cont.)

- Absolute and relative velocities
- $\overrightarrow{V_A} = \overrightarrow{V_{AO}}$
- $\overrightarrow{V_{OA}} = -\overrightarrow{V_{AO}}$
- $\overrightarrow{V_{BO}} = \overrightarrow{V_{BA}} + \overrightarrow{V_{AO}}$




- $\overrightarrow{V_{CO}} = \overrightarrow{V_{CA}} + \overrightarrow{V_{AO}}$

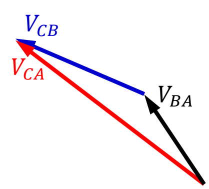


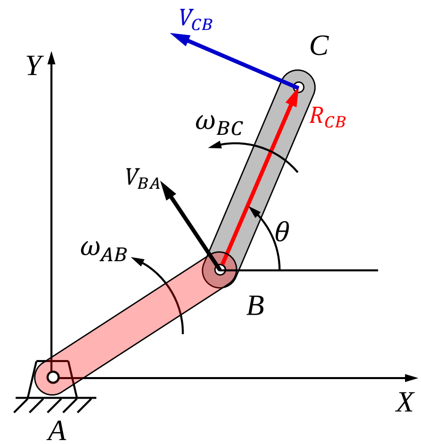
## 4.2. Analytical Velocity Analysis

- Relative velocity of two points of a moving link
- $\overrightarrow{V_{CB}} = \frac{d}{dt}\overrightarrow{R_{CB}} = \frac{d}{dt}\left(R_{CB}e^{i\theta}\right)$

  $\overrightarrow{V_{CB}} = R_{CB}\,\omega_{BC}\,i\,e^{i\theta}$

  $= R_{CB}\,\omega_{BC}\,@\angle(\theta + 90°)$
- $\overrightarrow{V_{CA}} = \overrightarrow{V_{CB}} + \overrightarrow{V_{BA}}$





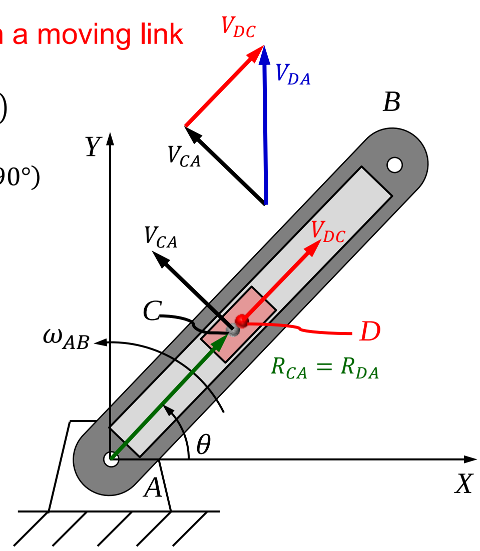

## 4.2. Analytical Velocity Analysis (cont.)

- Relative velocity of a point sliding on a moving link
- $\overrightarrow{V_{CA}} = \frac{d}{dt}\overrightarrow{R_{CA}} = \frac{d}{dt}\left(R_{CA}e^{i\theta}\right) = R_{CA}\frac{d}{dt}\left(e^{i\theta}\right)$

  $\overrightarrow{V_{CA}} = R_{CA}\,\omega_{AB}\,i\,e^{i\theta} = R_{CA}\,\omega_{AB}\,@\angle(\theta + 90°)$
- $\overrightarrow{V_{DA}} = \frac{d}{dt}\overrightarrow{R_{DA}} = \frac{d}{dt}\left(R_{DA}e^{i\theta}\right)$

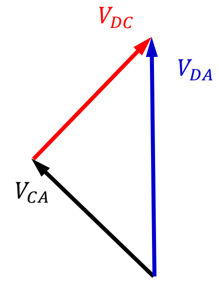
  $\overrightarrow{V_{DA}} = R_{DA}\frac{d}{dt}\left(e^{i\theta}\right) + e^{i\theta}\frac{d}{dt}\left(R_{DA}\right)$

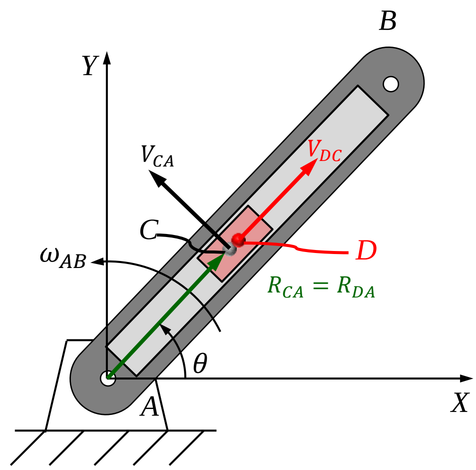
  $\overrightarrow{V_{DA}} = R_{DA}\,\omega_{AB}\,@\angle(\theta + 90°) + \dot{R}_{DA}\angle\theta$
- $\overrightarrow{V_{DA}} = \overrightarrow{V_{CA}} + \overrightarrow{V_{DC}}$



## 4.2. Analytical Velocity Analysis (cont.)

- Relative velocity of a point sliding on a moving link
- $\overrightarrow{V_{DA}} = \overrightarrow{V_{CA}} + \overrightarrow{V_{DC}}$



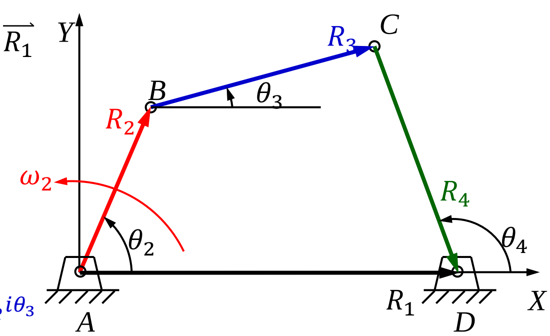



## 4.2. Analytical Velocity Analysis (cont.)

- Loop Equation: $\overrightarrow{R_2} + \overrightarrow{R_3} + \overrightarrow{R_4} = \overrightarrow{R_1}$
- $\overrightarrow{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{DC}} = 0$

  $\overrightarrow{V_B} + \overrightarrow{V_{CB}} - \overrightarrow{V_C} = 0$

  $\overrightarrow{V_C} = \overrightarrow{V_B} + \overrightarrow{V_{CB}}$




- $\omega_4 R_4 i e^{i\theta_4} = \omega_2 R_2 i e^{i\theta_2} + \omega_3 R_3 i e^{i\theta_3}$
  - $-\omega_4 R_4 \sin\theta_4 = -\omega_2 R_2 \sin\theta_2 - \omega_3 R_3 \sin\theta_3$

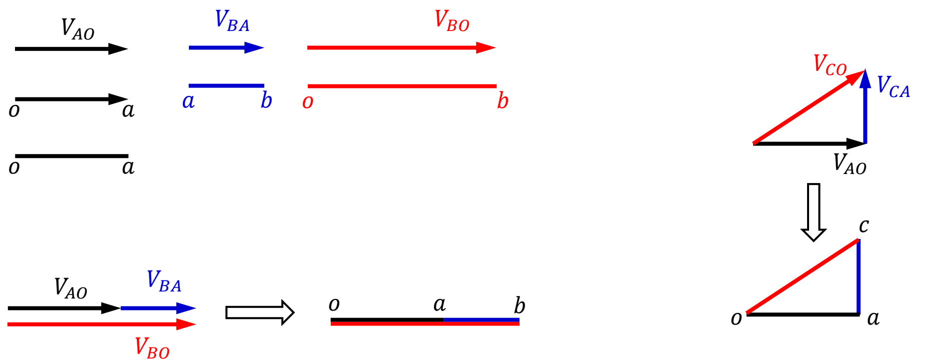
    $\dots\dots\,\omega_4 - \dots\dots\,\omega_3 = \ \dots\dots$ ———— (1)

  - $\omega_4 R_4 \cos\theta_4 = \omega_2 R_2 \cos\theta_2 + \omega_3 R_3 \cos\theta_3$

    $\dots\dots\,\omega_4 - \dots\dots\,\omega_3 = \ \dots\dots$ ———— (2)

## 4.3. Graphical Velocity Analysis

### ❑ Velocity Diagram




Velocity Diagram (V.D.)
Scale: $1\text{cm} \equiv \cdots m/s$

Velocity Diagram (V.D.)
Scale: $1\text{cm} \equiv \cdots m/s$



## 4.3. Graphical Velocity Analysis (cont.)

### ❑ Velocity Diagram

- A diagram represents the relative velocities of points to each other.
- The relative velocity between any two fixed points is equal to zero.

  All fixed points are represented by one point in velocity diagram.

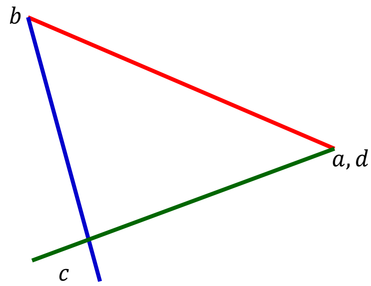
- The velocity of any point is equal to the vector from the fixed point to the considered point.

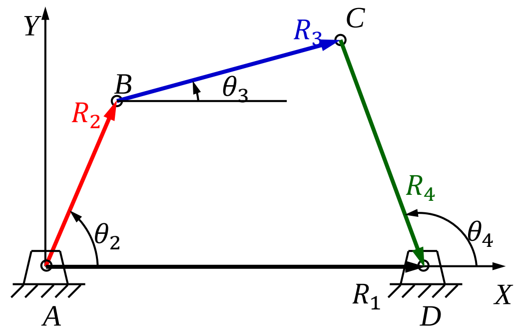
## 4.3. Graphical Velocity Analysis (cont.)

- $\overrightarrow{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{OC}} = 0$

  $\overrightarrow{V_B} + \overrightarrow{V_{CB}} - \overrightarrow{V_C} = 0$

  $\overrightarrow{V_C} = \overrightarrow{V_B} + \overrightarrow{V_{CB}}$

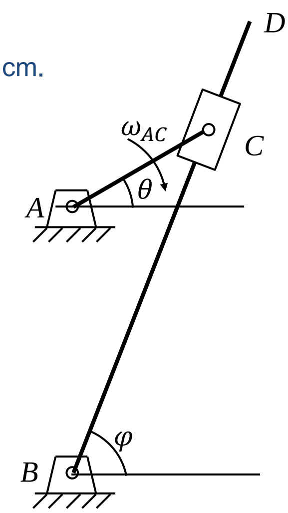





## 4.4. Solved examples

- Quick-return mechanism
  - AB =6.6 cm, AC = 5 cm, BC = 10.4 cm and BD = 13 cm.
  - $\theta = 36.86°$, $\varphi = 68.38°$ and $\omega_{AC} = 10\ rad/s$.
  - Find the angular velocity of link BD
  - Find the velocity of point D.



## 4.4. Solved examples (cont.)

### Four-bar mechanism

- AB = 6 cm, BC = CD =BE = 8 cm, AD = 13 cm
- $\omega_{AB} = 10\frac{rad}{s}$.
- Find the angular velocities of links BC and DC.
- Find the velocity of point E.