Design and Production Engineering Department

Chapter 3
Position Analysis

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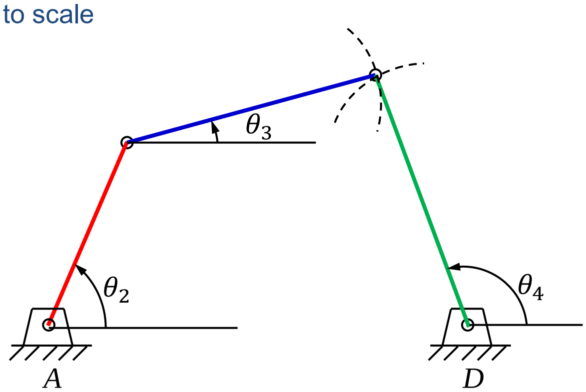
3.1. Graphical and Analytical Position Analysis

- Graphical
  - By drawing the mechanism to scale



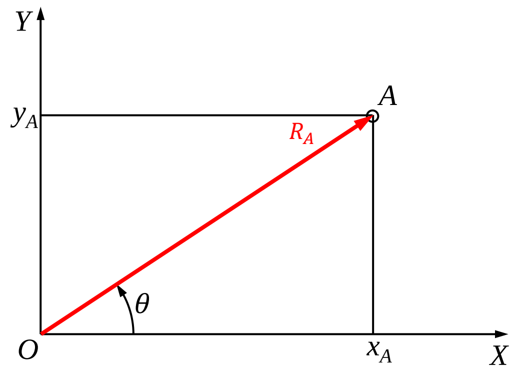
- Analytical
  - Loop equation

3.2. Definitions

- Position

$\overrightarrow{R_{OA}} = \overrightarrow{R_A}$

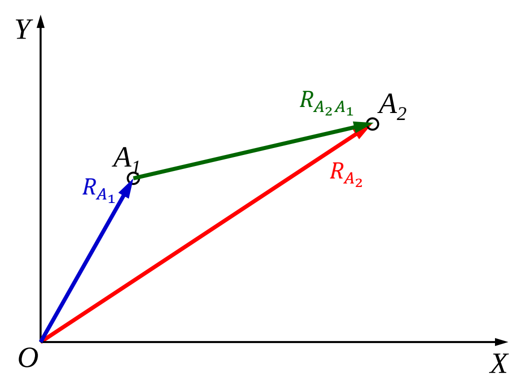
- Polar: $|\overrightarrow{R_A}|@\angle\theta$
- Cartesian: x_A, y_A
- $|\overrightarrow{R_A}| = \sqrt{x_A^2 + y_A^2}$
- $\theta = \tan^{-1}\frac{y_A}{x_A}$



3.2. Definitions (cont.)

- Displacement

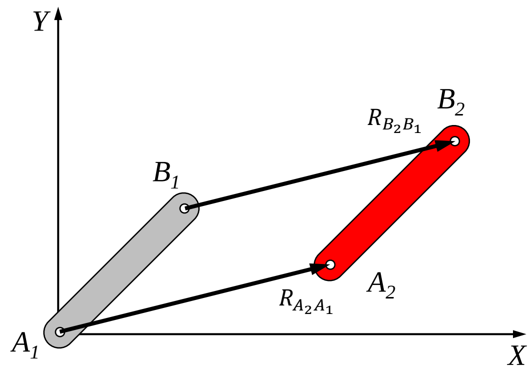
$\overrightarrow{R_{A_2A_1}} = \overrightarrow{R_{A_2}} - \overrightarrow{R_{A_1}}$



3.2. Definitions (cont.)

- Motion
  - Translation

$$\overrightarrow{R_{B_2B_1}} = \overrightarrow{R_{A_2A_1}}$$

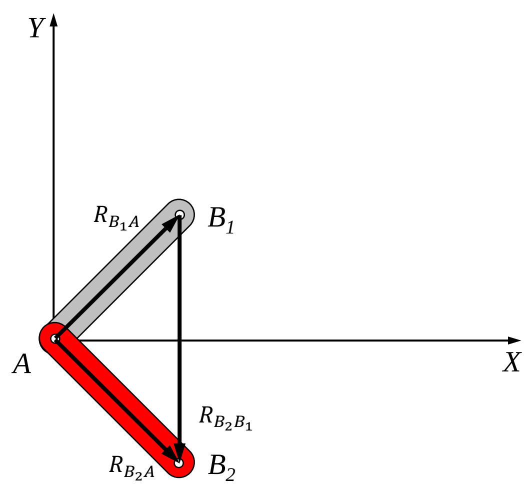


3.2. Definitions (cont.)

- Motion
  - Rotation

$$\overrightarrow{R_{A_2A_1}} = 0$$

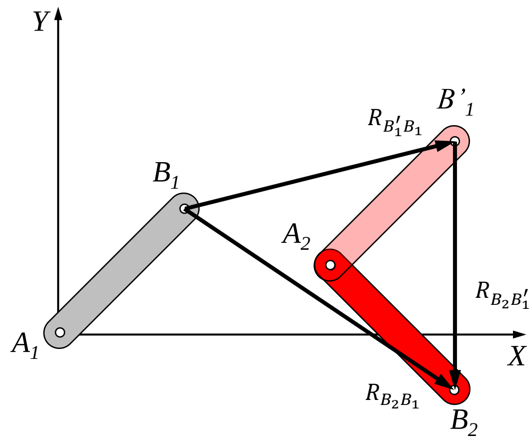
$$\overrightarrow{R_{B_2A}} = \overrightarrow{R_{B_1A}} + \overrightarrow{R_{B_2B_1}}$$



3.2. Definitions (cont.)

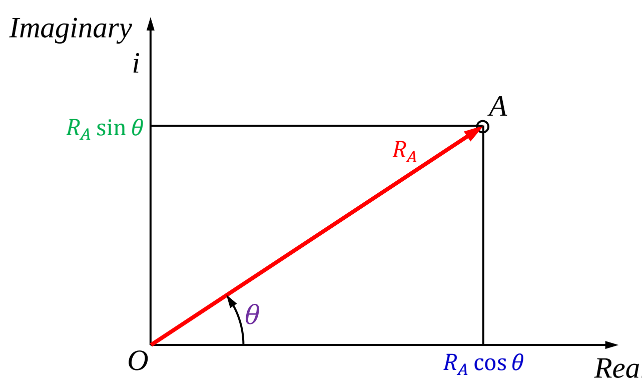
- Motion
  - Complex motion

$$\overrightarrow{R_{B_2B_1}} = \overrightarrow{R_{B_1'B_1}} + \overrightarrow{R_{B_2B_1'}}$$



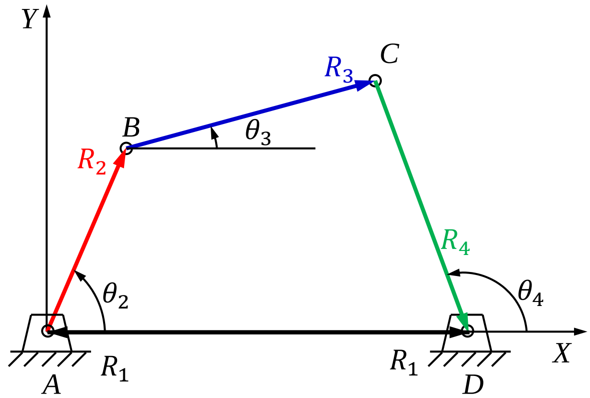
3.3. Complex numbers versus vectors

- $|\overrightarrow{R_A}| = R_A$
- $\overrightarrow{R_A} = R_A \cos\theta + R_A \sin\theta \, i$
- $\overrightarrow{R_A} = R_A e^{i\theta}$



3.4. Vector Loop Equation

- The closed loop of vector = zero
- $\overrightarrow{R_1} + \overrightarrow{R_2} + \overrightarrow{R_3} + \overrightarrow{R_4} = 0$
- $\overrightarrow{R_2} + \overrightarrow{R_3} + \overrightarrow{R_4} = \overrightarrow{R_1}$



3.5. Solving 3-Vector Loop Equation

- Slider-crank mechanism

$\overrightarrow{R_2} + \overrightarrow{R_3} = \overrightarrow{R_1}$

- Known: $R_2, R_3, \theta_1, \theta_2$ Unknown: R_1, θ_3

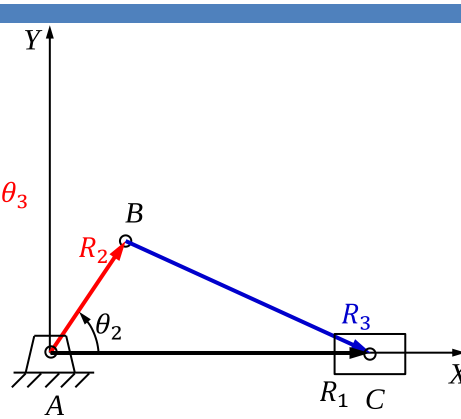
$R_2 \cos\theta_2 + R_3 \cos\theta_3 = R_1 \cos\theta_1$

$R_2 \sin\theta_2 + R_3 \sin\theta_3 = R_1 \sin\theta_1$

- $\theta_1 = 0$

$R_2 \cos\theta_2 + R_3 \cos\theta_3 = R_1$

$R_2 \sin\theta_2 + R_3 \sin\theta_3 = 0$



3.5. Solving 3-Vector Loop Equation (cont.)

$R_3 \cos\theta_3 = R_1 - R_2 \cos\theta_2$ (1)

$R_3 \sin\theta_3 = -R_2 \sin\theta_2$ (2)

- (2) →

$\sin\theta_3 = \frac{-R_2 \sin\theta_2}{R_3}$ → θ_3

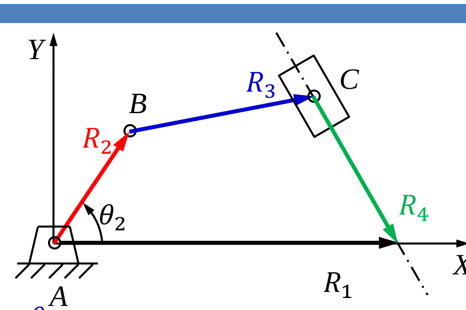
- (1) →

$R_1 = R_2 \cos\theta_2 - R_3 \cos\theta_3$

3.6. Solving 4-Vector Loop Equation

- Inclined slider-crank mechanism

$\overrightarrow{R_2} + \overrightarrow{R_3} + \overrightarrow{R_4} = \overrightarrow{R_1}$



- Known: $R_1, R_2, R_3, \theta_1, \theta_2, \theta_4$

  Unknown: R_4, θ_3

- $R_2 \cos\theta_2 + R_3 \cos\theta_3 + R_4 \cos\theta_4 = R_1 \cos\theta_1$

  $R_2 \sin\theta_2 + R_3 \sin\theta_3 + R_4 \sin\theta_4 = R_1 \sin\theta_1$

- $\theta_1 = 0$

  $R_2 \cos\theta_2 + R_3 \cos\theta_3 + R_4 \cos\theta_4 = R_1$

  $R_2 \sin\theta_2 + R_3 \sin\theta_3 + R_4 \sin\theta_4 = 0$

3.6. Solving 4-Vector Loop Equation (cont.)

$R_3 \cos\theta_3 = R_1 - R_2 \cos\theta_2 - R_4 \cos\theta_4$ (1)

$R_3 \sin\theta_3 = -R_2 \sin\theta_2 - R_4 \sin\theta_4$ (2)

- $(1)^2 + (2)^2 \rightarrow$
- $R_3^2 = (R_1 - R_2 \cos\theta_2 - R_4 \cos\theta_4)^2 + (-R_2 \sin\theta_2 - R_4 \sin\theta_4)^2$

$\cdots\cdots R_4^2 + \cdots\cdots R_4 + \cdots\cdots = 0$

- Solving quadratic equation $\rightarrow$ R_4
- By substitution in (2)

$\sin\theta_3 = \frac{-R_2 \sin\theta_2 - R_4 \sin\theta_4}{R_3}$ $\rightarrow$ θ_3

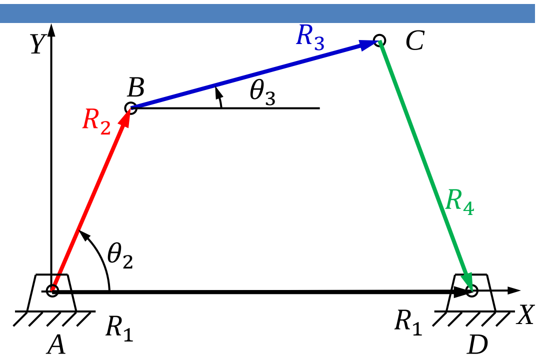
3.6. Solving 4-Vector Loop Equation (cont.)

- Four-bar mechanism

$\overrightarrow{R_2} + \overrightarrow{R_3} + \overrightarrow{R_4} = \overrightarrow{R_1}$

- Known: $R_1, R_2, R_3, R_4, \theta_1, \theta_2$

Unknown: θ_3, θ_4



$R_3 e^{i\theta_3} + R_4 e^{i\theta_4} = R_1 e^{i\theta_1} - R_2 e^{i\theta_2}$

$R_3 e^{i\theta_3} + R_4 e^{i\theta_4} = Z,$ where Z: complex number

$Z = (R_1 \cos\theta_1 - R_2 cos\theta_2) + (R_1\, sin\,\theta_1 - R_2 sin\theta_2)i$

$R_3 s + R_4 t = Z,$ where $s = e^{i\theta_3},$ $t = e^{i\theta_4}$

3.6. Solving 4-Vector Loop Equation (cont.)

$$R_3 s + R_4 t = Z \qquad (1)$$

- Conjugate the equation

$$\frac{R_3}{s} + \frac{R_4}{t} = \bar{Z} \qquad (2)$$

$$\text{conj(s)} = \text{conj}\left(e^{i\theta_3}\right) = e^{-i\theta_3} = \frac{1}{s}$$

- $(1) \rightarrow s = \frac{Z - R_4 t}{R_3}$
- By substitution in (2)

$$\cdots\cdots t^2 + \cdots\cdots t + \cdots\cdots = 0, \qquad At^2 + Bt + C = 0$$

- A, B and C are complex numbers

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \qquad t = e^{i\theta_4} = \cos\theta_4 + i sin\theta_4 \longrightarrow \theta_4$$

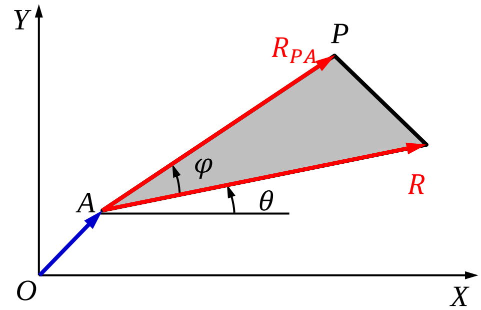
3.6. Solving 4-Vector Loop Equation (cont.)

- By substitution in (1)

$$s = \frac{Z - + R_4 t}{R_3} \qquad s = e^{i\theta_3} = \cos\theta_3 + i sin\theta_3 \longrightarrow \theta_3$$

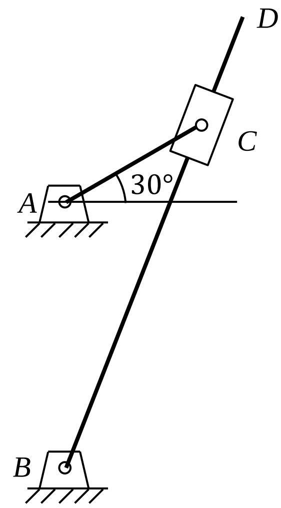
3.7. Position of Any point on the link

- $\overrightarrow{R_P} = \overrightarrow{R_A} + \overrightarrow{R_{PA}}$
- $\overrightarrow{R_{PA}} = \left|\overrightarrow{R_{PA}}\right|\left(cos + i\ sin(\theta + \varphi)\right)$



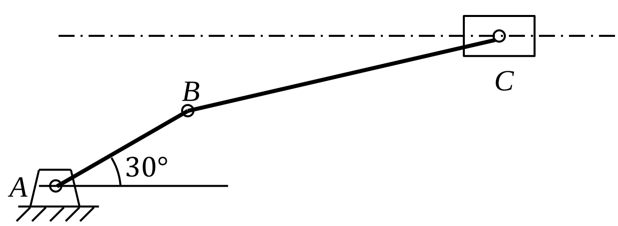
3.8. Solved Examples

- Quick-return mechanism
  - AB =7 cm, AC = 4 cm
  - Find the positions of block C and the link BD.



3.8. Solved Examples (cont.)

- Offset slider-crank mechanism
  - Offset = 5 cm, AB = 4 cm, BC 9 cm
  - Find the positions of block C and the link BC.



3.8. Solved Examples (cont.)

- Four-bar mechanism
  - AB = 6 cm, BC = 8cm, CD = 8 cm, AD = 13 cm
  - Find the positions of links BC and CD