



Chapter 1

Mechanisms

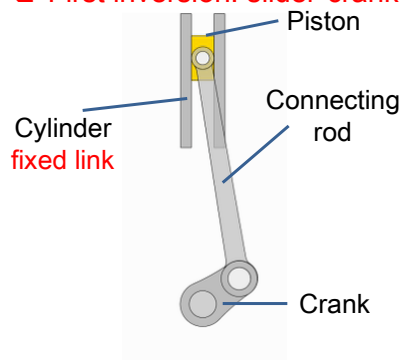
MCT 251

Asst. Prof. Mohammed M. Hedaya

1.6. Slider-crank mechanism

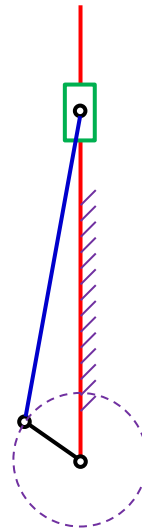


□ First inversion: slider-crank



□ Applications

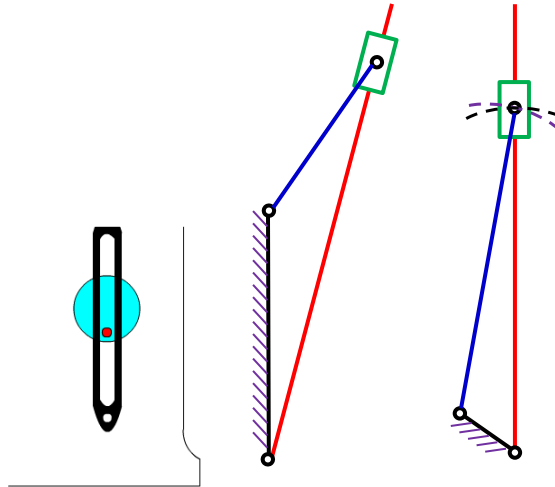
- Reciprocating engine
- Reciprocating compressor



1.6. Slider-crank mechanism (cont.)



□ Second inversion: quick-return mechanism



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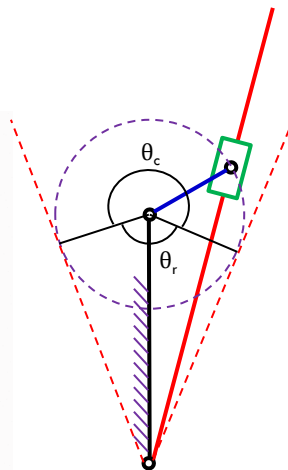
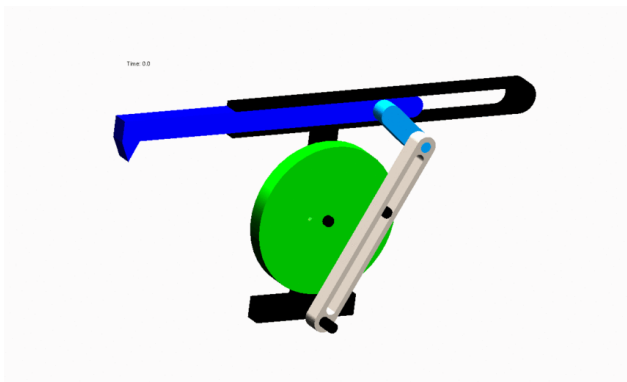
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1.6. Slider-crank mechanism (cont.)



□ Second inversion: quick-return mechanism

$$\text{Speed ratio} = \frac{\text{Cutting time}}{\text{return time}} = \frac{\theta_c}{\theta_r}$$



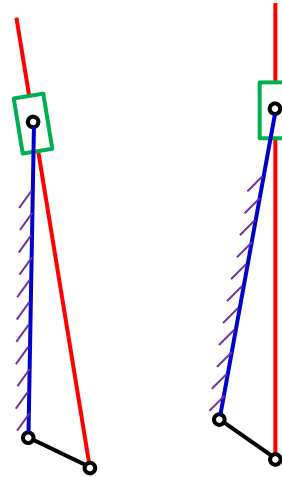
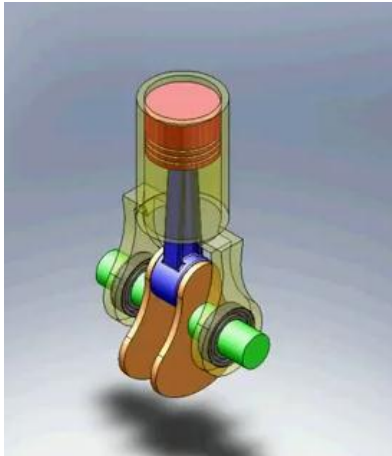
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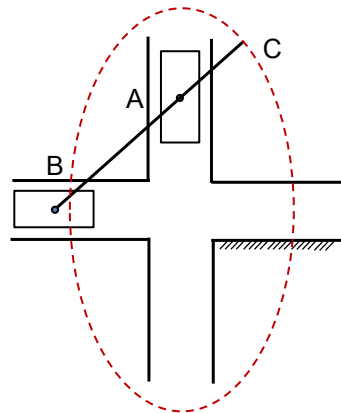
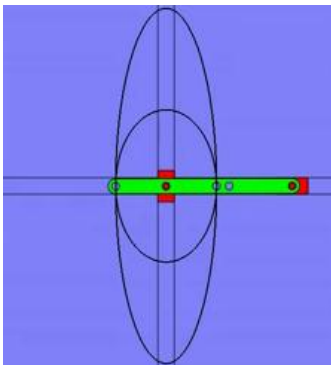
1.6. Slider-crank mechanism

□ Third inversion: oscillating cylinder



1.7. Double slider mechanism

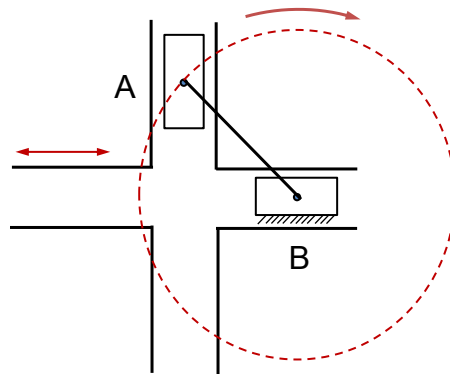
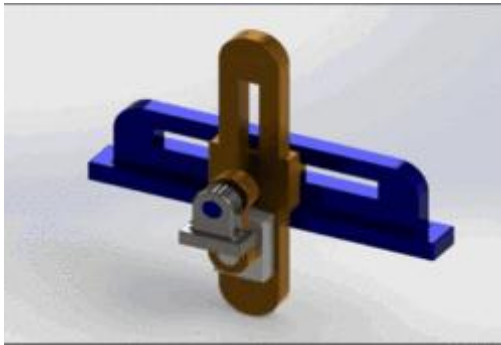
□ First inversion: Ellipse trammels



1.7. Double slider mechanism (cont.)



□ Second inversion: Scotch yoke mechanism



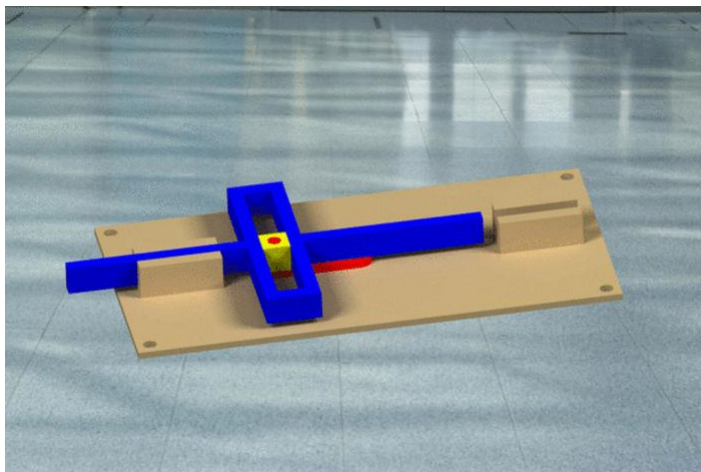
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1.7. Double slider mechanism (cont.)



□ Second inversion: Scotch yoke mechanism (cont.)



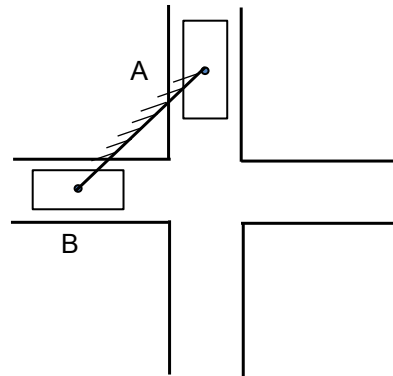
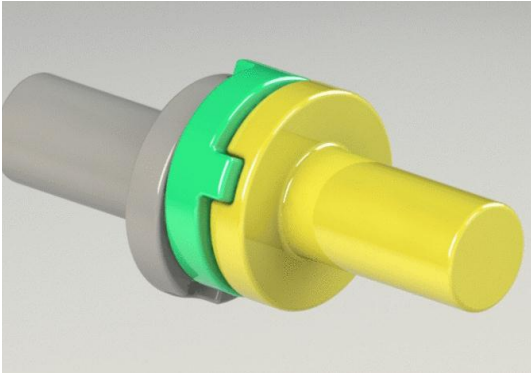
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1.7. Double slider mechanism (cont.)



□ Third inversion: Oldham's coupling



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1.7. Double slider mechanism (cont.)



□ Third inversion: Oldham's coupling



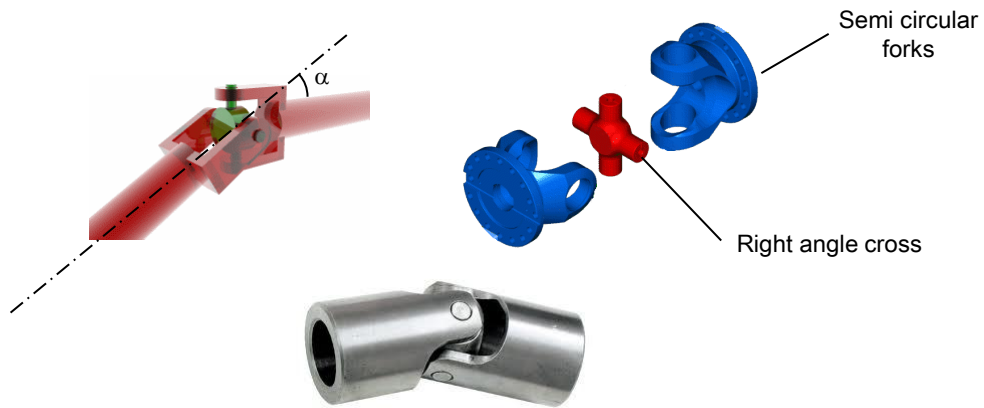
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1.8. Hooke's joint (Universal Joint)

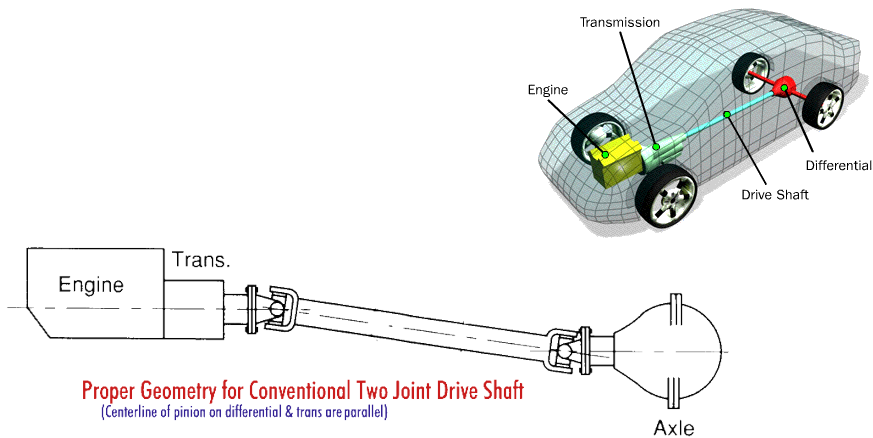
- Transmits the rotation between two intersecting shafts
 - The centre of the cross must lie on the axis of each shaft



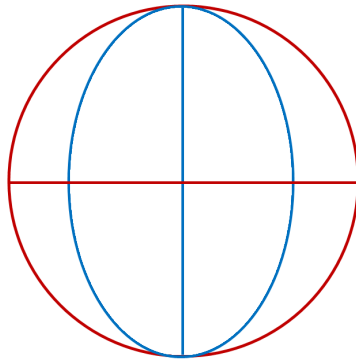
1.8. Hooke's joint (cont.)



- Gear box to back axel



1.8. Hooke's joint (cont.)



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1.8. Hooke's joint (cont.)



□ Relation between the angular velocities

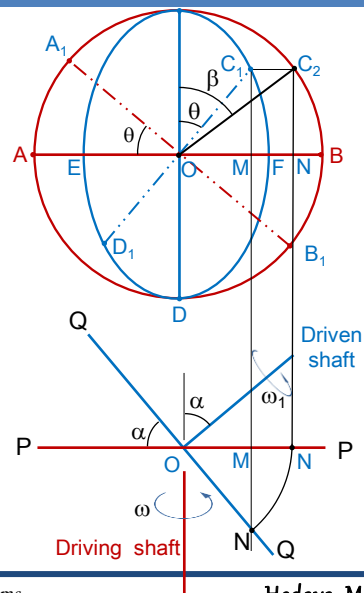
θ : Angular displacement of the driver $\omega = \frac{d\theta}{dt}$

β : Angular displacement of the driven $\omega_1 = \frac{d\beta}{dt}$

$$\tan \beta = \frac{ON}{NC_2} \quad \tan \theta = \frac{OM}{MC_1} = \frac{OM}{NC_2}$$

$$\frac{\tan \beta}{\tan \theta} = \frac{ON}{OM} = \frac{1}{\cos \alpha}$$

$$\tan \theta = \tan \beta \cdot \cos \alpha$$



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1.8. Hooke's joint (cont.)



$$\tan \theta = \tan \beta \cdot \cos \alpha$$

Differentiating this equation

$$\sec^2 \theta \frac{d\theta}{dt} = \cos \alpha \cdot \sec^2 \beta \cdot \frac{d\beta}{dt}$$

ω (under $\frac{d\theta}{dt}$) ω_1 (under $\frac{d\beta}{dt}$)

$$\frac{\omega}{\omega_1} = \cos \alpha \cdot \cos^2 \theta \sec^2 \beta$$

$$\begin{aligned} \sec^2 \beta &= 1 + \tan^2 \beta = 1 + \frac{\tan^2 \theta}{\cos^2 \alpha} = 1 + \frac{\sin^2 \theta}{\cos^2 \theta \cdot \cos^2 \alpha} \\ &= \frac{\cos^2 \theta \cdot \cos^2 \alpha + \sin^2 \theta}{\cos^2 \theta \cdot \cos^2 \alpha} = \frac{\cos^2 \theta \cdot \cos^2 \alpha + 1 - \cos^2 \theta}{\cos^2 \theta \cdot \cos^2 \alpha} = \frac{1 - \cos^2 \theta (1 - \cos^2 \alpha)}{\cos^2 \theta \cdot \cos^2 \alpha} \\ &= \frac{1 - \cos^2 \theta \cdot \sin^2 \alpha}{\cos^2 \theta \cdot \cos^2 \alpha} \end{aligned}$$

1.8. Hooke's joint (cont.)



Hence

$$\frac{\omega}{\omega_1} = \frac{1 - \cos^2 \theta \cdot \sin^2 \alpha}{\cos^2 \alpha}$$

$$\frac{\omega}{\omega_{1max}} \text{ at } \cos \theta = \pm 1 \quad \text{i.e. at } \theta = 0, \pi, 2\pi \dots \text{ etc.}$$

$$\frac{\omega}{\omega_{1max}} = \cos \alpha \quad , \quad \omega_{1max} = \frac{\omega}{\cos \alpha}$$

$$\frac{\omega}{\omega_{1min}} \text{ at } \cos \theta = 0 \quad \text{i.e. at } \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots \text{ etc.}$$

$$\frac{\omega}{\omega_{1min}} = \frac{1}{\cos \alpha} \quad , \quad \omega_{1min} = \omega \cos \alpha$$

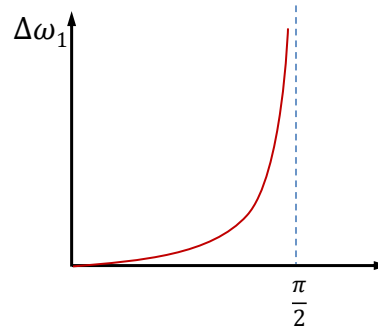
1.8. Hooke's joint (cont.)



$$\omega_{1max} = \frac{\omega}{\cos\alpha} \quad \omega_{1min} = \omega \cos\alpha$$

$$\begin{aligned} \frac{\Delta\omega_1}{\omega} &= \frac{1}{\cos\alpha} - \cos\alpha = \frac{1 - \cos^2\alpha}{\cos\alpha} \\ &= \frac{\sin^2\alpha}{\cos\alpha} = \frac{\sin\alpha \sin\alpha}{\cos\alpha} = \sin\alpha \tan\alpha \end{aligned}$$

$$\Delta\omega_1 \propto \alpha^2$$



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1.8. Hooke's joint (cont.)



□ Conditions of equal speeds

$$\frac{\omega}{\omega_1} = \frac{1 - \cos^2\theta \cdot \sin^2\alpha}{\cos\alpha} = 1$$

$$1 - \cos^2\theta \cdot \sin^2\alpha = \cos\alpha$$

$$\cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha}$$

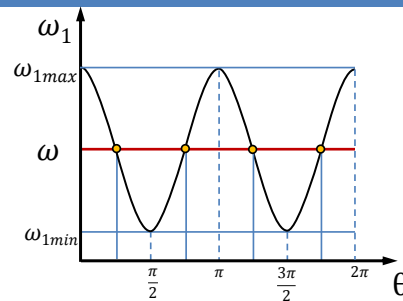
$$\cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha} = \frac{1 - \cos\alpha}{1 - \cos^2\alpha}$$

$$= \frac{1 - \cos\alpha}{(1 - \cos\alpha)(1 + \cos\alpha)} = \frac{1}{1 + \cos\alpha}$$

$$\frac{1}{1 + \tan^2\theta} = \frac{1}{1 + \cos\alpha}$$

$$\tan^2\theta = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha}$$



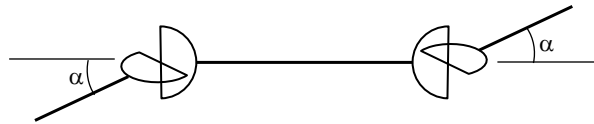
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1.8. Hooke's joint (cont.)



- If the driving and the driven shafts are equally inclined to the intermediate shaft and the 2 forks on the intermediate shaft lie in the same plane, it is evident that speeds of driving and driven shafts are identical and the fluctuation of speed are confined to intermediate shaft, which may be made short and light



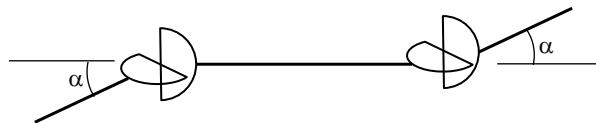
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1.8. Hooke's joint (cont.)



- If the forks of the intermediate shaft lie in planes perpendicular to each other, the fluctuation of the driven shaft shall vary between $\cos^2 \alpha$ and $\frac{1}{\cos^2 \alpha}$



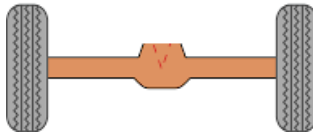
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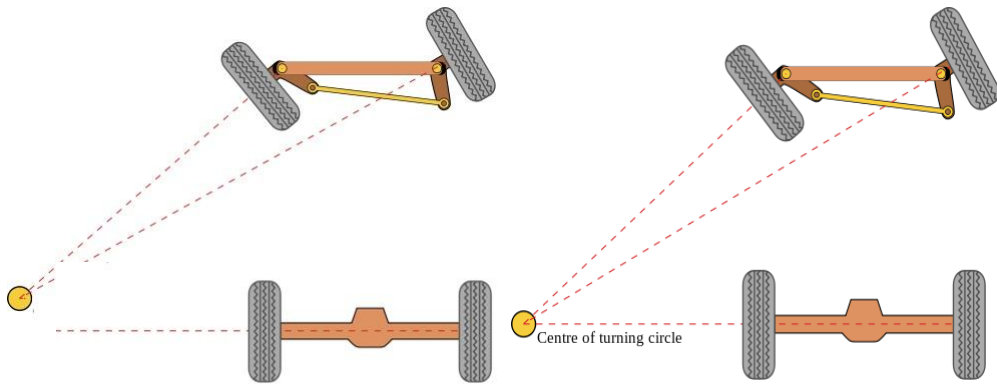
1.9 Steering mechanism

□ Ackermann steering



1.9 Steering mechanism (cont.)

□ Ackermann steering



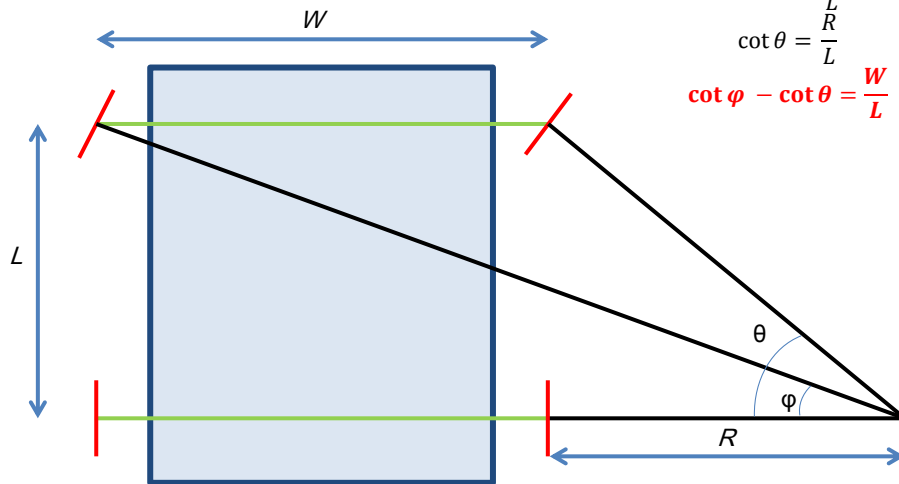
Ackermann Steering

Ideal Steering



1.9 Steering mechanism (cont.)

□ Condition of ideal steering



$$\cot \phi = \frac{W + R}{L}$$

$$\cot \theta = \frac{R}{L}$$

$$\cot \phi - \cot \theta = \frac{W}{L}$$