Actuators & Mechanisms

Actuator sizing

Prof. Dr. Magdy M. Abdelhameed
Contents

- Modelling of Mechanical System

- Mechanisms and Drives
The study of Mechatronics systems can be divided into five areas of specialty:

1. Physical systems modeling
2. Sensors and actuators
3. Signals and systems
4. Computers and logic system
5. Software and data acquisition
6. Controller design
   - Logic controller
   - Microprocessor
   - Microcontroller
   - Programmable controller
   - PC based controller

Key Elements of Mechatronics
FIGURE 1.1 The key elements of mechatronics.
Hardware, Software and Firmware

**Hardware** is the name given to the physical devices and circuitry of the computer.

**Software** refers to the programs written for the computer.

**Firmware** is the term given to programs stored in ROMs or in Programmable devices which permanently keep their stored information. In other words, firmware is the combination of persistent memory and program code and data stored in it. Typical examples of devices containing firmware are embedded systems.
Robot Platforms (1)

Indoor Robots

DLR Gripper

NASA Mars Rover

Asimo Humanoid

Outdoor Robots

Robot Base Station

KUKA Manipulator
Robot Platforms (2)

- RoboCup
- Aibo 4 legged Robot
- Qurio Humanoid
- Robocup Team
Robot Platforms (3)

Robot educational kits

Robot sensors
Stepper, AC and DC motors
PLC and Microcontrollers

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PC-based Measurement and Control

- GPIB
- Serial/parallel
- PC-based Measurement and Control
- CAN BUS

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Engineering Software

Matlab  Labview  HP-VEE  IDL

C++  Java  Linux  LaTeX  Qt
Modeling of Mechanical System

Displacement

\[ F = k \times d \]

Velocity

\[ F = c \times v \]

Acceleration

\[ F = m \times a \]
Acceleration, Velocity and Displacement

Time (Simple vibration)

Velocity

Frequency (real machine)

Acceleration

Displacement
Mechatronics (2)
4th Year- Mechatronics Major

FFT Transformation

d = D \sin \omega_n t

Displacement

Time

Displacement

Frequency

Period, $T_n$ in [sec]

Frequency, $f_n \frac{1}{T_n}$ in [Hz = 1/sec]

$\omega_n = 2 \pi f_n = \sqrt{\frac{k}{m}}$

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FFT Transformation

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Actuator sizing
Types of Motion and Motion Conversion

Linear and Angular Motion
The linear motion induced in a rigid object is governed by Newton’s second law of motion

\[ F = ma \]

\( F \) is the resultant of all forces acting on the object, \( m \) is the mass of the object and \( a \) is the resulting linear acceleration.

The constant force \( F \) produces a constant acceleration \( a \) and moves the object of mass \( m \) a certain distance \( s \) according to

\[ s = \frac{1}{2} at^2 \]

\( s \) is the displacement and \( t \) is the time

Thus, the time required to move mass \( m \) through distance \( s \) by means of a constant force \( F \) is given by

\[ t = \sqrt{\frac{2ms}{F}} \]
Types of Motion and Motion Conversion

For angular motion, Newton’s law reads:

\[ T = J \ddot{\theta} \]

\( T \) is the resultant of all torques acting on a mass rotating about a fixed axis, \( J \) is the moment of inertia of the mass about its axis of rotation and \( \dot{\theta} \) is the angular acceleration.

and the angular displacement equation analogous to that of linear motion is \( \theta = \frac{1}{2}at^2 \)

\( \theta \) is the angular displacement. Solving for \( t \) yields:

\[ t = \sqrt{\frac{2J\theta}{T}} \]
Example:
Consider a rotary motion axis driven by an electric servo motor. The rotary load is directly connected to the motor shaft without any gear reducer (Fig. 1). The rotary load is a solid cylindrical shape made of steel material, $d=75\text{mm}$, $l=50\text{mm}$, $\rho=7800\text{kg/m}^3$. The desired motion of the load is a periodic motion (Fig. 2).
Example (Cont.):

*The total distance* to be traveled is 1/4 of a revolution. *The period of motion* is \( t_{\text{cyc}} = 250 \text{ msec.} \), and dwell portion of it is \( t_{\text{dw}} = 100 \text{ msec.} \), and the remaining part of the cycle time is equally divided between acceleration, constant speed and deceleration periods, \( t_a = t_r = t_d = 50 \text{ msec.} \).

*Determine the required motor size for this application.*
Mechatronics (2)
4th Year - Mechatronics Major

Displacement

$\theta_a$

$\theta_r$

$\theta_d$

$\theta_{total}$

$\frac{1}{4}$ revolution

0 $t_a$ $t_a+tr$ $t_a+tr+td$ $t_{cyc}$

time (sec.)

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**Actuator Sizing Algorithm:**

1. Define the geometric relationship between the actuator and load. In other words, select the type of motion transmission mechanism between the motor and load ($N=$reduction ratio).

2. Define the inertia and torque/force characteristics of the load and transmission mechanisms, i.e. define the inertia of the tool as well as the inertia of the gear reducer mechanisms ($J_l$, $T_l$).

3. Define the desired cyclic motion profile in the load speed versus time ($\theta'_l(t)$).

4. Using the reflection equations developed above, calculate the reflected load inertia and torque/force ($J_{eff}$, $T_{eff}$) that will effectively act on the actuator shaft as well as the desired motion at the actuator shaft ($\theta'_m(t)$).

5. Guess a actuator/motor inertia from an available list (catalog) (or make the first calculation with zero motor inertia assumption), and calculate the torque history, $T_m(t)$, for the desired motion cycle. Then calculate the peak torque and RMS torque from $T_m(t)$. 

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Actuator Sizing Algorithm Cont.: 

6. Check if the actuator size meets the required performance in terms of peak and RMS torque, and maximum speed capacity ($T_p$, $T_{rms}$, $\theta'_\text{max}$). If the above selected actuator/motor from the available list does not meet the requirements (i.e. too small or too large), repeat the previous step by selecting a different motor. It should be noted that if a stepper motor is used, the torque capacity of the stepper motor is rated only in terms of the continuous rating, not peak. Therefore, the required peak and RMS torque must be smaller than the continuous torque capacity of the step motor.

7. Most servo motor continuous torque capacity rating is given for 25°C ambient temperature is different than 25°C, the continuous (RMS) torque capacity of the motor should be derated using the following equation for a temperature, 

$$T_{rms} = T_{rms}(25^\circ C) \sqrt{\frac{(155 - Temp^\circ C)}{130}} $$
Solution:

1) Determine the Net inertia:

where:

- $J_{\text{total}}$ = the total inertia reflected on the motor axis.
- $J_{\text{eff}}$ = the load inertias reflected on the motor shaft.
- $J_m$ = the motor rotor inertia.

$$J_{\text{eff}} = J_{L}$$

$$J_{\text{eff}} = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\left(\frac{\pi d^2}{4}\right)l)r^2$$

$$= \frac{1}{2} \times 7800 \times \left(\frac{\pi}{4} \times (75 \times 10^{-3})^2 \times 50 \times 10^{-3}\right) \times \left(\frac{75 \times 10^{-3}}{2}\right)^2 = 0.0012\text{kg.m}^2$$

The ratio of motor inertia to reflected load inertia should be between one-to-one and up to one-to-ten.

$$\frac{J_m}{J_{\text{eff}}} = \frac{1}{1} \sim \frac{1}{10}$$
The one-to-one is considered the optimal match (an ideal case), where the motor drives a purely inertia load and this inertia ratio results in minimum heating of the motor. Let us assume that we will pick a motor which has a rotor inertia same as the load so that there is an ideal load and motor inertia match.

\[ J_m = J_{\text{eff}} = 0.0012 \text{kg.m}^2 \]

\[ \therefore J_{\text{total}} = J_m + J_{\text{eff}} = 0.0024 \text{kg.m}^2 \]

**2) Determine the Net Torque:**

\[ \sum T(t) = T_{\text{total}}(t) = T_m(t) - T_R(t) \]

where:
- \( T_{\text{total}}(t) \) = the total torque.
- \( T_m(t) \) = the torque generated by the motor.
- \( T_R(t) \) = the resistive load torques on the system, where \( T_R(t) \) represent the sum of all external torques. If the load torque is in the direction of assisting the motion, it will be negative, and net result will be the addition of two torques. The \( T_R(t) \) may include friction (\( T_f \)), gravity (\( T_g \)), and process related torque and forces (i.e. an assembly application may required the mechanism to provide a desired force pressure (\( T_R \)).
\[ \therefore T_R(t) = 0 \]

\[ \therefore T_{total}(t) = T_m(t) \]

**3) Fundamental Equations for torque calculation:**

The torque and motion relationship is:

\[ J_{total} \ddot{\theta} = \sum T \]

\[ (J_m + J_{eff}) \ddot{\theta} = T_m \]

The required torque to move the load through the desired cyclic motion given in the figure can be calculated if the value of \( \ddot{\theta} \) calculated.
4) Define the desired cyclic motion profile in the form of load (motor) speed versus time:

From the desired motion profile specification, we can determine the velocity and acceleration of the actuator can be deliver using the kinematic relations.

\[ t_a = t_r = t_d = \frac{250 - 100}{3} = 50 \text{ m/sec}. \]

where:

- \( t_a \) = acceleration mode time.
- \( t_r \) = constant speed mode time.
- \( t_d \) = deceleration speed mode time.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_a$</td>
<td>Angle (\frac{1}{4}) revolution</td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>Angle reversal</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$\theta_{total}$</td>
<td>Total displacement</td>
</tr>
</tbody>
</table>

**Velocity Graph:**
- $\theta'_{max}$

**Displacement Graph:**
- $0$ to $\frac{1}{4}$ revolution
- $\theta_a$
- $\theta_r$
- $\theta_d$
- $\theta_{total}$

**Time Graph:**
- $0$ to $t_a$
- $t_a$ to $t_{a+tr}$
- $t_{a+tr}$ to $t_{a+tr+td}$
- $t_{a+tr+td}$ to $t_{cyc}$
Acceleration diagram

- \( \theta_{\text{max}} \)

418 rad/sec

-418 rad/sec

Velocit

0 = t_{\text{cy}} - t_a + t_{r} + t_{d}

\( t_a \)

\( t_{a+r} \)

\( t_{a+r+d} \)

t_{\text{cy}}
5) Calculate the maximum speed $\dot{\theta}_{\text{max}}$ required to define this profile from the motor:

From displacement motion profile:

$$\theta_{\text{total}} = \theta_a + \theta_r + \theta_d = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$$

note that:

$$\theta(t) = \int \dot{\theta}(t) dt$$

From velocity motion profile:

$$\theta_{\text{total}} = \frac{1}{2} \dot{\theta}_{\text{max}} t_a + \frac{1}{2} \dot{\theta}_{\text{max}} t_r + \frac{1}{2} \dot{\theta}_{\text{max}} t_d$$

$$\dot{\theta}_{\text{max}} = \frac{2\theta_{\text{total}}}{t_a + t_r + t_d}$$

$$\dot{\theta}_{\text{max}} = \frac{\pi}{150 \times 10^{-3}} = 20.9 \text{ rad/sec}$$

$$n = \frac{60 \dot{\theta}_{\text{max}}}{2\pi} = 199.6 \text{ rpm}$$
6) Calculate the angular acceleration:

Note that:

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt}$$

From velocity motion profile:

$$\ddot{\theta}_a = \frac{d\dot{\theta}_a}{dt} = \frac{\dot{\theta}_{\text{max}} - 0}{t_a - 0} = \frac{\dot{\theta}_{\text{max}}}{t_a} = \frac{20.9}{50 \times 10^{-3}} = 418 \text{ rad/sec}^2$$

$$\ddot{\theta}_r = \frac{d\dot{\theta}_r}{dt} = \frac{\dot{\theta}_{\text{max}} - \dot{\theta}_{\text{max}}}{(t_a + t_r) - t_a} = 0 = 0$$

$$\ddot{\theta}_d = \frac{d\dot{\theta}_d}{dt} = \frac{0 - \dot{\theta}_{\text{max}}}{(t_d + t_r + t_a) - (t_r + t_a)} = \frac{-\dot{\theta}_{\text{max}}}{t_d}$$

$$= \frac{-20.9}{50 \times 10^{-3}} = -418 \text{ rad/sec}^2$$
7) Use Fundamental equation for torque calculation at each mode:

\[(J_m + J_{eff})\ddot{\theta} = T_m\]

\[\therefore T_{ma} = (J_m + J_{eff})\ddot{\theta}_a = 0.0024 \times 418 = 1.0032 N.m\]

\[\therefore T_{mr} = (J_m + J_{eff})\ddot{\theta}_r = 0\]

\[\therefore T_{md} = (J_m + J_{eff})\ddot{\theta}_d = 0.0024 \times -418 = -1.0032 N.m\]

The torque diagram profile is shown in Fig 8.

8) The Peak torque (maximum torque):

Hence, the peak torque requirement is

\[T_{max} = 1.0032 N.m\]
9) \( T_{\text{rms}} = \text{root mean square torque over entire cycle:} \)

\[
T_{\text{rms}} = \sqrt{\frac{\int_0^{t_{\text{cycle}}} T_m(t)^2 \, dt}{t_{\text{cycle}}}}
\]

From torque diagram:

\[
T_{\text{rms}} = \sqrt{\frac{T_{m_a}^2 t_a + T_{m_r}^2 t_r + T_{m_d}^2 t_d + T_H^2 t_{d\text{w}}}{t_a + t_r + t_d + t_{d\text{w}}}}
\]

where: \( T_H = \text{holding torque required in dwell mode}=0 \)

\[
T_{\text{rms}} = \sqrt{\left(1.0032\right)^2 \times 50 \times 10^{-3} + 0 + \left(-1.0032\right)^2 \times 50 \times 10^{-3} + 0} = 0.6344 \, \text{N.m}
\]

Therefore, a motor which has rotor inertia of about \( 0.0012 \, \text{kg.m}^2 \), maximum speed capability of \( 20.9 \, \text{rad/sec}(199.6 \, \text{rpm}) \) or better, peak and RMS torque rating in the range of \( 1.0032 \, \text{N.m} \) and \( 0.6344 \, \text{N.m} \) range would be sufficient for the task.
Mechanisms and Drives

Direct Drive

where

\[ S_m = \text{motor speed (rpm)} \]
\[ S_1 = \text{load speed (rpm)} \]
\[ T_m = \text{motor torque (lb-in)} \]
\[ T_1 = \text{load torque (lb-in)} \]
\[ J_t = \text{total inertia (lb-in-s}^2) \]
\[ J_1 = \text{load inertia (lb-in-s}^2) \]
\[ J_m = \text{motor inertia (lb-in-s}^2) \]

speed (motor) = speed (load)
\[ S_m = S_1 \]

torque at motor = torque at load
\[ T_m = T_1 \]

total inertia = inertia (load) + inertia (motor)
\[ J_t = J_1 + J_m \]
Mechatronics (2)
4th Year- Mechatronics Major

Mechanisms and Drives

\[ J_{\text{total}} = J_{\text{motor}} + J_{\text{coupling}} + J_{\text{load}} \]

\[ T_{\text{acc}} = J_{\text{total}} \frac{dn}{dt} \]

\[ T_{\text{motor}} = T_{\text{acc}} + T_{\text{friction}} \]

\[ T_{\text{r.m.s.}} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + T_4^2 t_4}{t_1 + t_2 + t_3 + t_4}} \]

\[ n_{\text{average}} = \frac{\frac{1}{2} V t_1 + V t_2 + \frac{1}{2} V t_3}{t_1 + t_2 + t_3 + t_4} \]

Desired Load Velocity

Required Motor Torque

Motor
Coupling
Load

Motion Requirements

Friction

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Rotating mass driven through a gear reducer

\[ T_0 = J_{eq} \alpha_1. \]

**gear ratio**  The speed ratio (or, the gear ratio) \( n \) is expressed by

\[ n = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1} = \frac{R_1}{R_2} = \frac{n_1}{n_2} \]

\[ F = \frac{T_1}{R_1} = \frac{T_2}{R_2} \quad T_1 = R_1 \frac{T_2}{R_2} = nT_2. \]

Taking gear 1 as a free body gives

\[ T_0 - T_1 = \alpha_1 J_1 \]

\[ T_2 = \alpha_2 J_2. \]

The equivalent moment of inertia as

\[ J_{eq} = \frac{T_0}{\alpha_1} = \frac{\alpha_1 J_1 + nT_2}{\alpha_1} = J_1 + \frac{nT_2}{\alpha_1} = J_1 + n^2 \left( \frac{T_2}{\alpha_2} \right) = J_1 + n^2 J_2. \]
How to calculate equivalent inertia?

The efficiency of a motion transmission mechanism is defined as the ratio between the output power and input power,

\[ \eta = \frac{P_{out}}{P_{in}} \]  

\[ P_{out} = \eta \cdot P_{in} \]

\[ T_{out} \cdot \dot{\theta}_{out} = \eta \cdot T_{in} \cdot \dot{\theta}_{in} \]

\[ KE_l = \eta \cdot KE_{in} \]

\[ KE_l = \frac{1}{2} \cdot J_l \cdot \dot{\theta}^2 \]

\[ = \eta \cdot \frac{1}{2} \cdot J_{in, eff} \cdot \dot{\theta}^2_{in} \]

\[ = \frac{1}{2} \cdot J_l \cdot (\dot{\theta}_{in}/N)^2 \]

\[ J_{in, eff} = \frac{1}{\eta \cdot N^2} \cdot J_l \]
Gear Drive:

where

\[ S_m = \text{motor speed (rpm)} \]
\[ S_1 = \text{load speed (rpm)} \]
\[ N = \text{gear ratio} \]
\[ N_1 = \text{number of load gear teeth} \]
\[ N_m = \text{number of motor gear teeth} \]
\[ T_m = \text{motor torque (lb-in)} \]
\[ T_1 = \text{load torque (lb-in)} \]
\[ e = \text{efficiency} \]
\[ J_t = \text{total inertia (lb-in-s}^2) \]
\[ J_1 = \text{load inertia (lb-in-s}^2) \]
\[ J_m = \text{motor inertia (lb-in-s}^2) \]

\[
\text{speed (motor)} = \text{speed (load)} \times \text{gear ratio}
\]
\[
S_m = S_1 \times N
\]
\[ \text{or } S_m = S_1 \times N_1 \div N_m
\]

\[
\text{torque at motor} = \text{torque at load} \div \text{gear ratio}
\]
\[
T_m = \frac{T_1}{N_e}
\]

\[
\text{total inertia} = \text{inertia (load)} \div \text{(gear ratio}^2) + \text{inertia (motor)}
\]
\[
J_t = J_1 \div N^2 + J_m
\]
Motor
\[ \theta_m, \omega_m, J_m, T_m \]

Timing Belt

\begin{align*}
\text{Ratio} &= N = \frac{\text{motor velocity}}{\text{load velocity}} = \frac{D_l}{D_m} \\
\theta_m &= N \theta_L, \quad \omega_m = N \omega_L \\
\text{Total Inertia} &= J_{\text{total}} = J_m + \frac{1}{N^2} J_L \\
\text{Load Torque Reflected to Motor} &= \frac{1}{N} T_L
\end{align*}

\[ 1 \text{ rev} = 2\pi \text{ (rad)} \]

\[ \theta = \text{angular distance (rad)} \]
\[ \omega = \text{angular velocity (rad/s)} \]
\[ J = \text{moment of inertia (lb-in-s}^2\text{)} \]
\[ T = \text{torque (lb-in)} \]
Conversion of Rotary to linear Motion

1. Rack and pinion drives,
2. Power (lead) screws,
3. Linkages.

\[ V = R\omega \]

If the load attached to the rack has mass \( m \), then, total equivalent moment of inertia equals

\[ J_{eq} = J_1 + mR^2 = J_1 + m\left(\frac{V}{\omega}\right)^2 \]

Conversely, if the rack is the driver, then the moment of inertia \( J_1 \) attached to the pinion shaft must be reflected back to the rack, and the equivalent linear inertia as felt by the pinion driving the rack is

\[ m_{eq} = m + \frac{J_1}{R^2} \]
g = gravity constant = 386 in/s²

\[ W_{total} = \text{weight (lbs)} = W_{load} + \text{weight of moving parts} \]

\[ v = \text{linear velocity (in/min)} \]

\[ x = \text{linear distance (in)} \]

\[ \theta_L = x/r, \quad \omega_L = \frac{v}{60 \pi r}, \quad n \ (r.p.m.) = \frac{v}{2 \pi r} \]

\[ F_{gravity} = W_{total} \sin \theta, \quad F_{friction} = \mu W_{total} \cos \theta \]

\[ T_L = F_{total} r + T_{acc}, \quad T_{acc} = \left[ J_{pulleys/pinion} + \frac{W_{total} r^2}{g} \right] \frac{dn}{dt} \]

- Motor \( \theta_m, \omega_m, J_m, T_m \)
- Optional Timing Belt or Gear Reducer
- Load Side \( \theta_L, \omega_L, J_L, T_L \)
Mechatronics (2)
4th Year - Mechatronics Major

Motor
\( \theta_m, \omega_m, J_m, T_m \)

Optional Timing Belt or Gear Reducer

Load Side
\( \theta_L, \omega_L, J_L, T_L \)

Load (\( W_{load} \))

Table (\( W_{table} \))

\( \mu = \) coefficient of friction

Incline angle \( \theta \)

\( F_{external} \)

\( F_{friction} \)

\( F_{gravity} \)

\( F_{total} = F_{external} + F_{friction} + F_{gravity} \)

\( \theta_L = \frac{2 \pi P x}{60}, \omega_L = \frac{2 \pi P v}{60}, n \text{ (r.p.m.)} = \frac{P v}{60} \)

\( F_{gravity} = W_{total} \sin \theta, F_{friction} = \mu W_{total} \cos \theta \)

\( T_L = \frac{F_{total}}{2\pi P} + T_{acc}, T_{acc} = \frac{J_{screw} + \frac{W_{total}(1/2\pi P)^2}{g}}{\text{dn/dt}} \)

\( P = \) pitch = revs/inch of leadscrew

\( h = \) lead = inches/rev of leadscrew

\( g = \) gravity constant = 386 in/s²

\( W_{total} = \) weight (lbs) = \( W_{load} + W_{table} \)

\( v = \) linear velocity (in/min)

\( x = \) linear distance (in)
The Move Profile

Typical Move Profile (Figure 6)

Example Move Profile (Figure 7)
In addition to knowing the relationships for common mechanical transmissions, it is also very useful to know the relationships for some common shapes as follows:

**Hollow**

- Diameter: $D_o = 2r_o$
- Diameter: $D_i = 2r_i$
- Length: $L$

Moment of Inertia:

$$J = \frac{m (D_o^2 + D_i^2)}{8} = \frac{W (r_o^2 + r_i^2)}{2g} = \frac{\pi L \rho (r_o^4 - r_i^4)}{2g}$$

Volume:

$$Volume = \frac{\pi}{4} (D_o^2 - D_i^2) L$$

**Solid**

- Diameter: $D = 2r$
- Length: $L$

Moment of Inertia:

$$J = \frac{m D^2}{8} = \frac{W r^2}{2g} = \frac{\pi L \rho r^4}{2g}$$

Volume:

$$Volume = \pi r^2 L$$

**Solid Cube**

- Length: $L$
- Width: $w$
- Height: $h$

Moment of Inertia:

$$J = \frac{W}{12 g} (h^2 + w^2)$$

Volume:

$$Volume = L w h$$
### Material Densities

<table>
<thead>
<tr>
<th>Material</th>
<th>oz/in³</th>
<th>lb/in³</th>
<th>gm/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>1.57</td>
<td>0.098</td>
<td>2.72</td>
</tr>
<tr>
<td>Brass</td>
<td>4.96</td>
<td>0.31</td>
<td>8.6</td>
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<tr>
<td>Bronze</td>
<td>4.72</td>
<td>0.295</td>
<td>8.17</td>
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<tr>
<td>Copper</td>
<td>5.15</td>
<td>0.322</td>
<td>8.91</td>
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<tr>
<td>Plastic</td>
<td>0.64</td>
<td>0.04</td>
<td>1.11</td>
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<tr>
<td>Steel</td>
<td>4.48</td>
<td>0.28</td>
<td>7.75</td>
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### Friction Coefficients (Sliding)

<table>
<thead>
<tr>
<th>Surface Combination</th>
<th>( \mu )</th>
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<tbody>
<tr>
<td>Steel on Steel</td>
<td>0.58</td>
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<tr>
<td>Steel on Steel (Greased)</td>
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<tr>
<td>Aluminum on Steel</td>
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<tr>
<td>Copper on Steel</td>
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<tr>
<td>Brass on Steel</td>
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<tr>
<td>Plastic on Steel</td>
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<tr>
<td>Linear Bearings</td>
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## Conversion Tables

(To convert A to B, multiply by value in table)

### Length

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Inch</th>
<th>Feet</th>
<th>Micro Inch</th>
<th>Micron</th>
<th>Millimeter</th>
<th>Centimeter</th>
<th>Meter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inch</td>
<td>1</td>
<td>8.33 x 10⁻²</td>
<td>1.0 x 10⁶</td>
<td>2.54 x 10⁴</td>
<td>25.4</td>
<td>2.54</td>
<td>2.54 x 10⁻²</td>
<td></td>
</tr>
<tr>
<td>Feet</td>
<td>12</td>
<td>1</td>
<td>1.2 x 10⁷</td>
<td>3.05 x 10⁵</td>
<td>305</td>
<td>30.5</td>
<td>0.305</td>
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</tr>
<tr>
<td>Micro-Inch</td>
<td>1.0 x 10⁻⁶</td>
<td>1.2 x 10⁴</td>
<td>1</td>
<td>2.54 x 10⁻²</td>
<td>2.54 x 10⁻⁵</td>
<td>2.54 x 10⁻⁶</td>
<td>2.54 x 10⁻⁸</td>
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</tr>
<tr>
<td>Micron</td>
<td>3.937 x 10⁻⁵</td>
<td>3.28 x 10⁻⁶</td>
<td>39.37</td>
<td>1</td>
<td>0.001</td>
<td>1.0 x 10⁻⁴</td>
<td>1.0 x 10⁻⁶</td>
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<tr>
<td>Millimeter</td>
<td>3.937 x 10⁻²</td>
<td>3.28 x 10⁻³</td>
<td>3.937 x 10⁴</td>
<td>1000</td>
<td>1</td>
<td>0.1</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>Centimeter</td>
<td>0.3937</td>
<td>3.28 x 10⁻²</td>
<td>3.937 x 10⁵</td>
<td>1 x 10⁴</td>
<td>10</td>
<td>1</td>
<td>0.01</td>
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</tr>
<tr>
<td>Meter</td>
<td>39.37</td>
<td>3.28</td>
<td>3.937 x 10⁷</td>
<td>1 x 10⁶</td>
<td>1000</td>
<td>100</td>
<td>1</td>
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### Mass

<table>
<thead>
<tr>
<th>A</th>
<th>ozm</th>
<th>lbm</th>
<th>slug</th>
<th>gm</th>
<th>kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>oz-m</td>
<td>1</td>
<td>6.25 x 10⁻²</td>
<td>1.94 x 10⁻³</td>
<td>28.35</td>
<td>2.835 x 10⁻²</td>
</tr>
<tr>
<td>Lb-m</td>
<td>16</td>
<td>1</td>
<td>3.11 x 10⁻²</td>
<td>453.6</td>
<td>0.453</td>
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<tr>
<td>slug</td>
<td>514.72</td>
<td>32.2</td>
<td>1</td>
<td>14590</td>
<td>14.59</td>
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<tr>
<td>gm</td>
<td>3.53 x 10⁻²</td>
<td>2.205 x 10⁻³</td>
<td>6.85 x 10⁻⁵</td>
<td>1</td>
<td>0.001</td>
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<tr>
<td>kg</td>
<td>35.274</td>
<td>2.205</td>
<td>6.85 x 10⁻²</td>
<td>1000</td>
<td>1</td>
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</tbody>
</table>
# Force

<table>
<thead>
<tr>
<th>A</th>
<th>ozf</th>
<th>lbf</th>
<th>Newtons</th>
<th>dyne</th>
<th>gmf</th>
<th>Kgf</th>
</tr>
</thead>
<tbody>
<tr>
<td>oz-f</td>
<td>1</td>
<td>6.25 x 10^{-2}</td>
<td>0.278</td>
<td>2.78 x 10^{4}</td>
<td>28.35</td>
<td>2.835 x 10^{-2}</td>
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<tr>
<td>lb-f</td>
<td>16</td>
<td>1</td>
<td>4.448</td>
<td>4.448 x 10^{5}</td>
<td>453.6</td>
<td>0.4535</td>
</tr>
<tr>
<td>Newtons</td>
<td>3.596</td>
<td>0.225</td>
<td>1</td>
<td>1 x 10^{6}</td>
<td>101.9</td>
<td>0.1019</td>
</tr>
<tr>
<td>dyne</td>
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<td>2.248 x 10^{-6}</td>
<td>1.0 x 10^{-5}</td>
<td>1</td>
<td>1.02 x 10^{-3}</td>
<td>1.02 x 10^{-6}</td>
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<tr>
<td>gm-f</td>
<td>3.53 x 10^{-2}</td>
<td>2.205 x 10^{-3}</td>
<td>9.81 x 10^{-3}</td>
<td>981</td>
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<td>0.001</td>
</tr>
<tr>
<td>kg-f</td>
<td>35.3</td>
<td>2.205</td>
<td>9.81</td>
<td>9.81 x 10^{5}</td>
<td>1000</td>
<td>1</td>
</tr>
</tbody>
</table>

# Power

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Watts</th>
<th>Kilowatts</th>
<th>ft.lb/sec</th>
<th>in-lb/sec</th>
<th>Hp (Imperial)</th>
<th>Hp (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watts</td>
<td>1</td>
<td>1 x 10^{-3}</td>
<td>0.74</td>
<td>8.85</td>
<td>1.34 x 10^{-3}</td>
<td>1.33 x 10^{-3}</td>
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</tr>
<tr>
<td>Kilowatts</td>
<td>1000</td>
<td>1</td>
<td>738</td>
<td>8850</td>
<td>1.34</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>ft-lb/sec</td>
<td>1.35</td>
<td>1.36 x 10^{-3}</td>
<td>1</td>
<td>12</td>
<td>1.82 x 10^{-3}</td>
<td>1.81 x 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>in-lb/sec</td>
<td>0.113</td>
<td>1.13 x 10^{-4}</td>
<td>8.3 x 10^{-2}</td>
<td>1</td>
<td>1.52 x 10^{-4}</td>
<td>1.53 x 10^{-4}</td>
<td></td>
</tr>
<tr>
<td>Hp(Imperial)</td>
<td>746</td>
<td>0.746</td>
<td>550</td>
<td>6600</td>
<td>1</td>
<td>0.995</td>
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</tr>
<tr>
<td>Hp (SI)</td>
<td>750</td>
<td>0.750</td>
<td>553</td>
<td>6636</td>
<td>1.005</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Thank You For Your Attention!

Questions?