Faculty of Engineering

MEP 382: Design of Applied Measurement Systems

Lecture 2:
Instrumentation Definitions

Overview

- Measurement Error
- Accuracy
- Precision and Mean
- Resolution
- Mean
- Variance and Standard deviation
- Finesse
- Sensitivity
- Range
- Offset (bias) and scale factor shift
- Linearity and Linear Regression
- Hysteresis
- Response Time
- Real Time
- Gain

Measurement Error Definition

- Error defined as

$$\varepsilon_n = X_n - Y_n$$

- Where error in the n^{th} measurement is defined as the difference between Y , the actual, true, defined or calculated value of the Quantity under Measurement and X is the measurement

Accuracy

- The accuracy A of the n^{th} measurement is defined as

$$A_n \equiv 1 - \left| \frac{Y_n - X_n}{Y_n} \right|$$

- Percent Accuracy = 100 x A
- A measurement is more accurate if it is closer to what is defined as the “truth” as compared to a reference standard

Precision & Mean

- Precision of the nth measurement is defined as

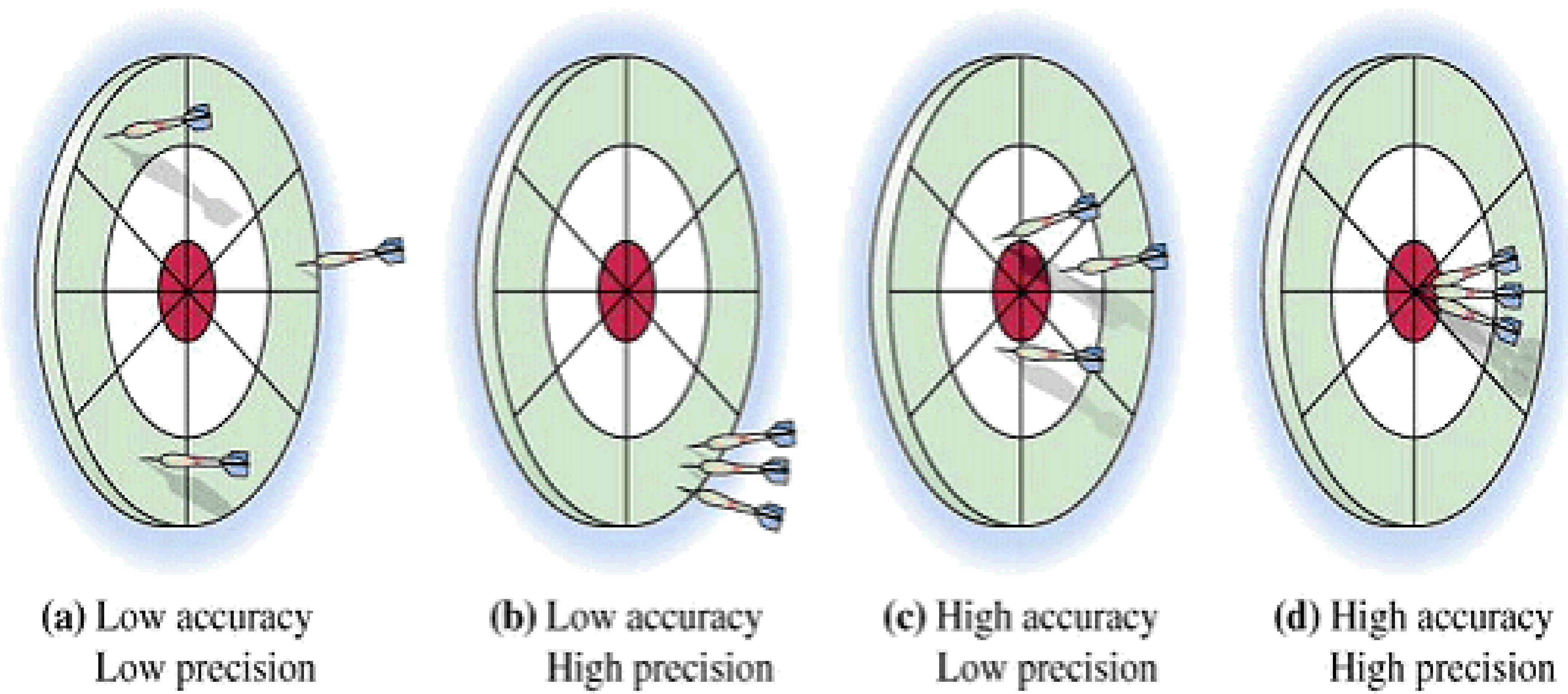
$$P_n \equiv 1 - \left| \frac{X_n - \overline{X}}{\overline{X}} \right|$$

$$where$$

$$\overline{X} \equiv \frac{1}{N} \sum_{n=1}^{N} X_n$$

N is the total number of samples and Xbar is the MEAN
Precision is a measure of the reproducibility of the measurement

Accuracy & Precision



From www.ni.com Measurement Fundamentals, Sampling Quality

Resolution

- Resolution is defined as the smallest unit of the quantity under measurement that can be detected
- Example : if a measurement is taken of a quantity under measurement that can vary between 0 and 5 volts and it is converted into one of 4095 binary values (steps) then the resolution is
- = 5 volts/4095 steps = .00122 volts per step
- A converter of n bits has $2^n - 1$ **steps**

Variance of a Population

$$\sigma_x^2 \equiv \frac{1}{N}\sum_{n=1}^{N}(X_n - \overline{X})^2$$

The **<u>standard deviation</u>** is the square root of the variance

Standard Deviation

- The standard deviation of a population is defined as

$$S_N \equiv \sqrt{\frac{1}{N}\sum_{n=1}^{N}(X_n - \overline{X})^2}$$

$$\sigma_x = S_N$$

This is the equation to use if you have 100 percent sampled the entire population

- The standard deviation of a sample is defined as
- $$S_N \equiv \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}(X_n - \overline{X})^2}$$

$$\sigma_x = S_N$$

This is the equation to use if you are dealing with a sample of the population and trying to estimate the entire population's characteristics

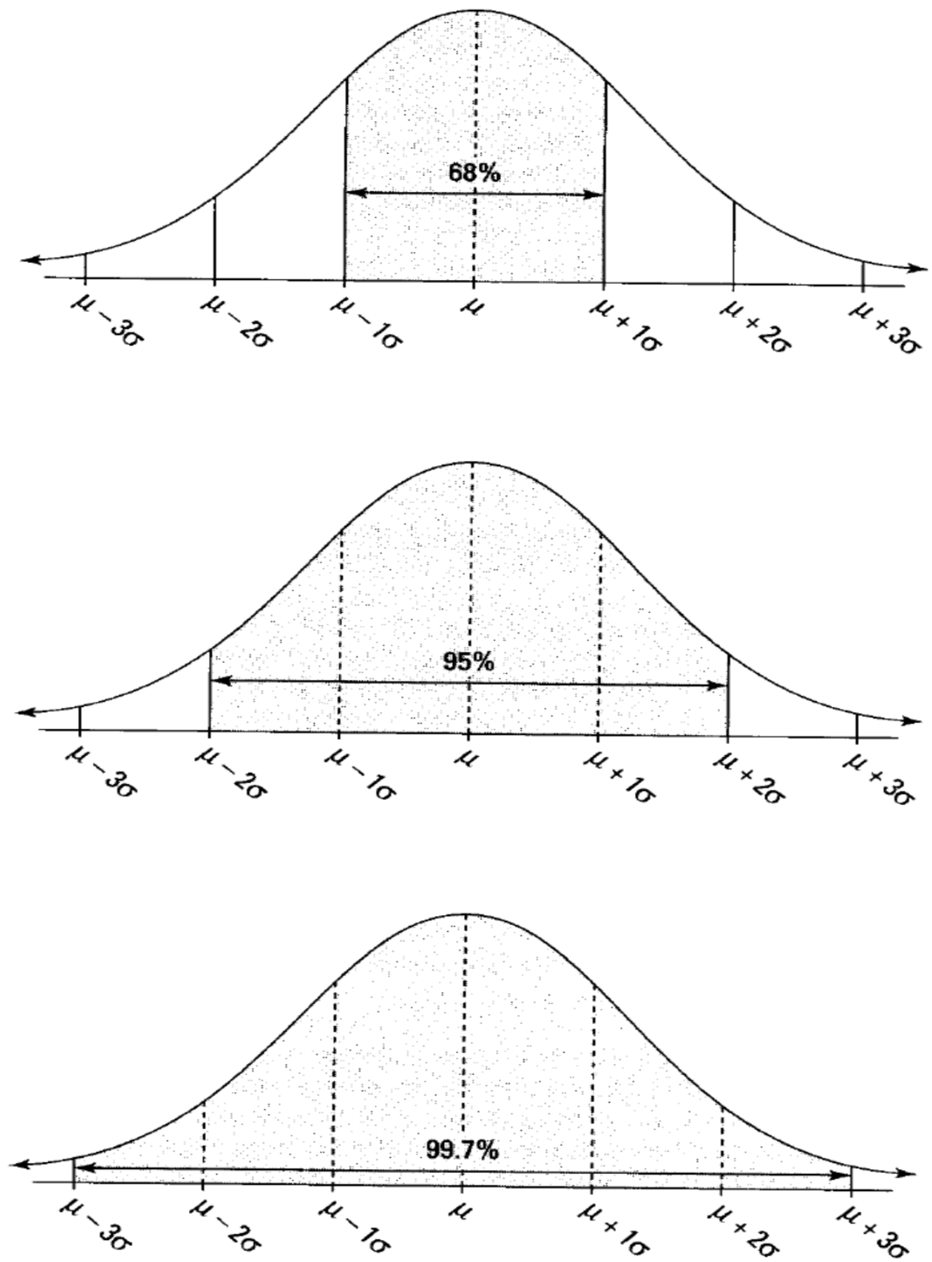


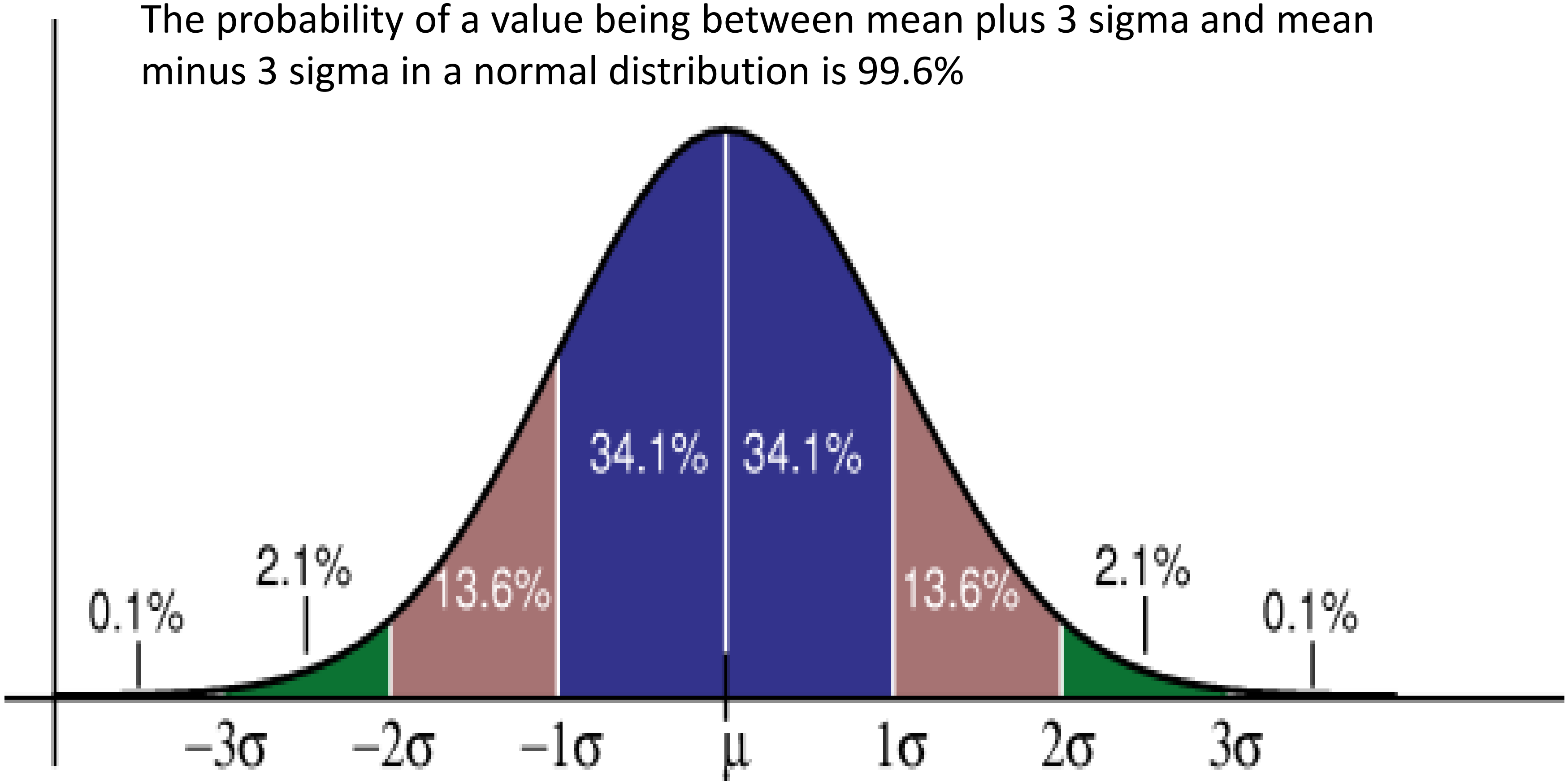
Figure 8-5: The empirical rule (68%, 95%, and 99.7%).

From "Statistics For Dummies"by Deborah Rumsey"

Caution – these probabilities are for normal distributions <u>only</u>. Not all data fits a normal distribution. It may be log-normal, exponential, etc…which have different interpretations.

Standard Deviations In A Normal Distribution

The probability of a value being between mean plus 3 sigma and mean minus 3 sigma in a normal distribution is 99.6%



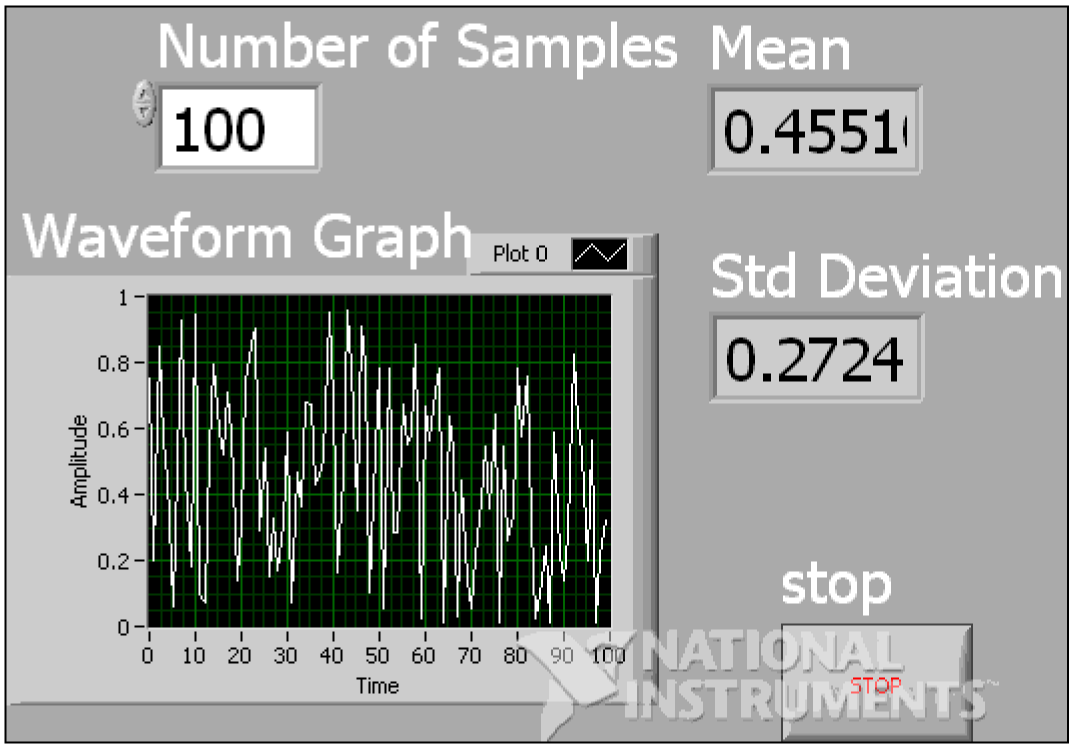
Note – This is two sided – can vary the same on either side of the mean – not all populations of data are two sided

Source: Wikipedia

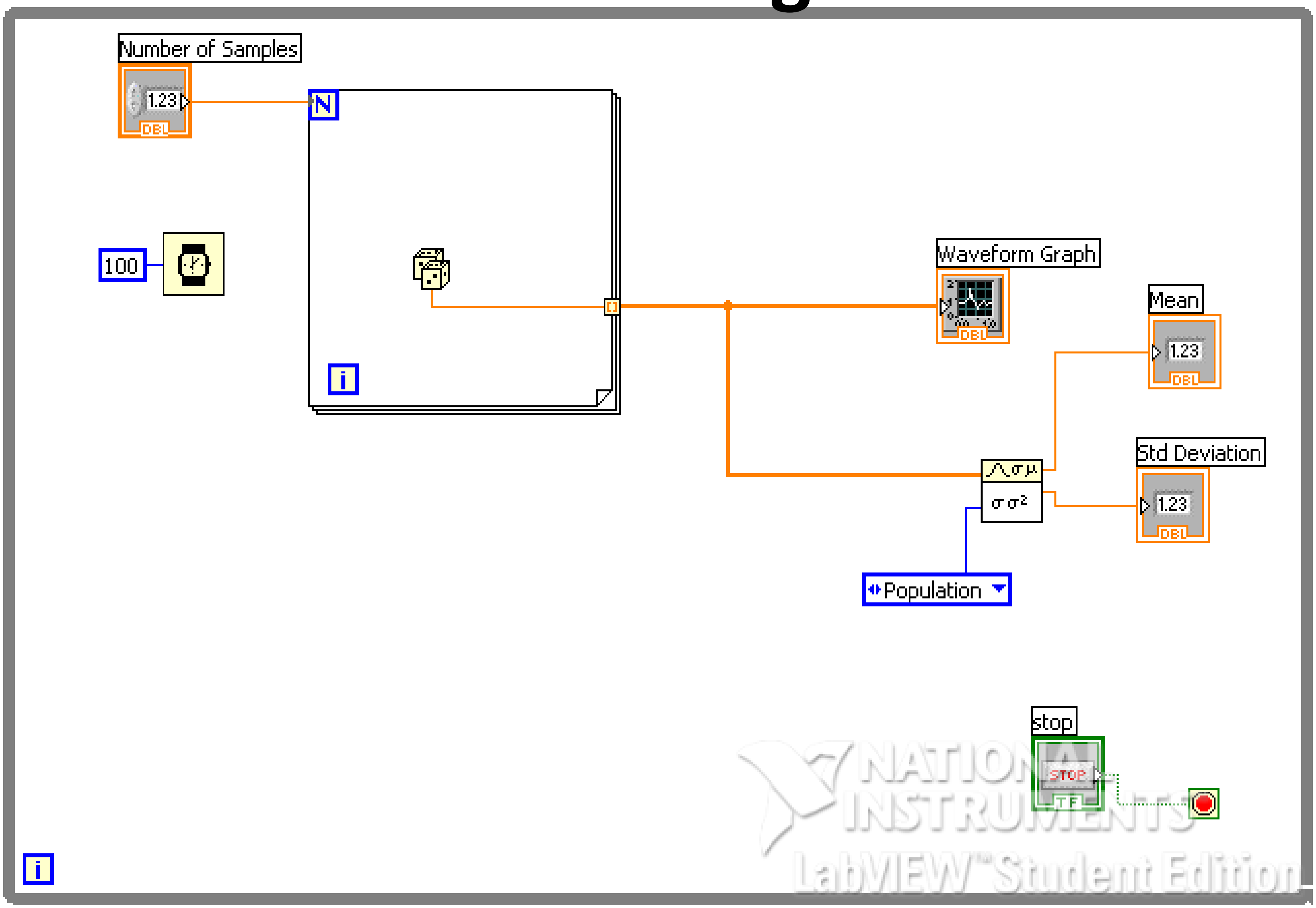
Finesse

- The degree to which a quantity under measurement is influenced by the measurement process
- For example a thermistor or RTD can heat to something being measured at the same time it is measuring it

Our First LabVIEW Virtual Instrument – Front Panel



Our First LabVIEW Virtual Instrument – Block Diagram



Sensitivity

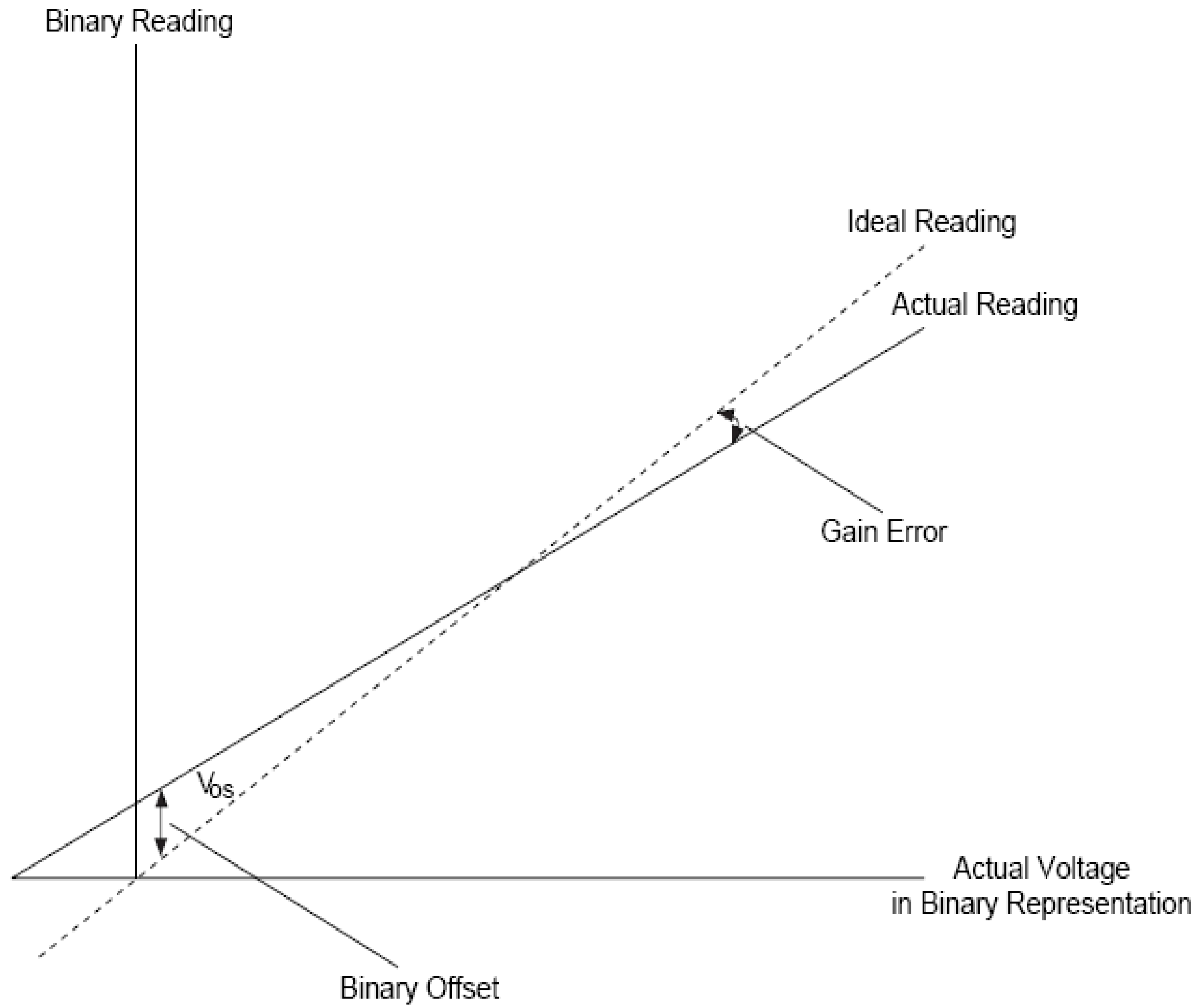
- The sensitivity of the sensor is defined as the slope of the output characteristic curve or, more generally, the *minimum input of physical parameter that will create a detectable output change.*
- In some sensors, the sensitivity is defined as the input parameter change required to produce a standardized output change.
- In others, it is defined as an output voltage change for a given change in input parameter

Range

- The range of the sensor is the maximum and minimum values of applied parameter that can be measured.

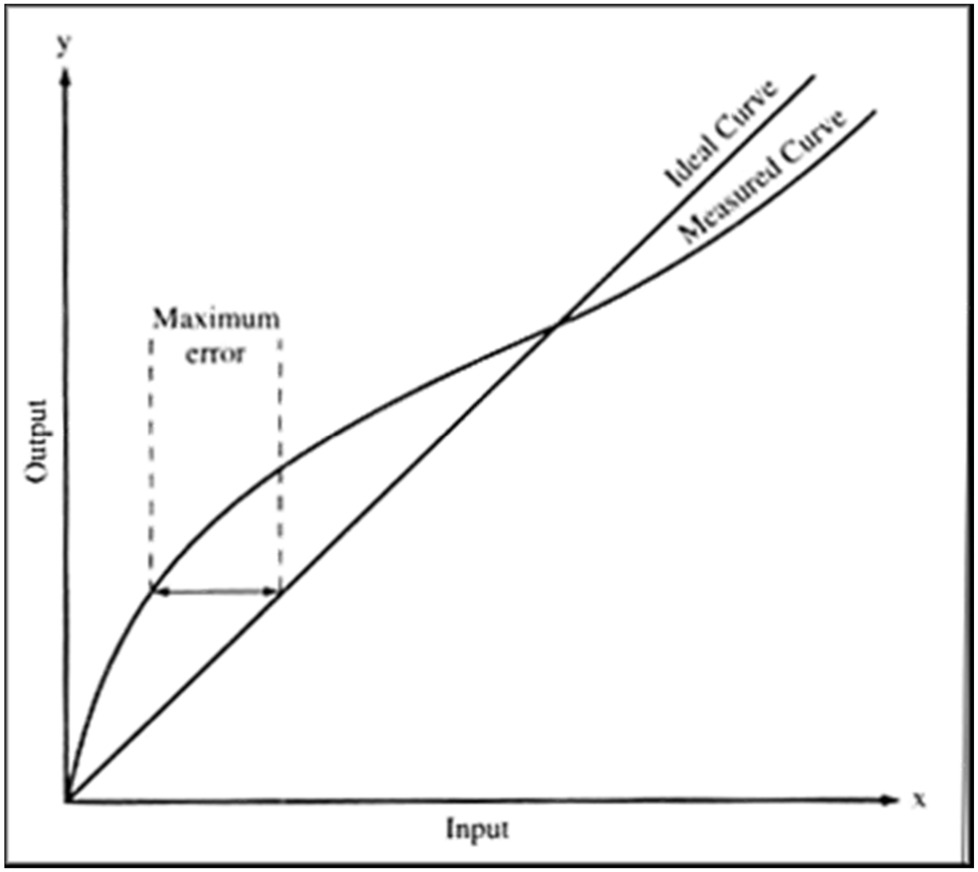
Offset (Bias) and Scale Factor

- The offset error of a transducer is defined as the output that will exist when it should be zero
- Alternatively, the difference between the actual output value and the specified output value under some particular set of conditions
- Offset is a linear error
- If a line is $y = mx + b$, an offset is an error in the b term
- A scale factor error is an error in the slope
  - An error in the m term
  - A change in scale factor can also be viewed as a change in sensitivity



From LabVIEW Data Acquisition Basics Manual

Linearity



- The linearity of the transducer is an expression of the extent to which the actual measured curve of a sensor departs from the ideal curve.
- Figure 3 shows a somewhat exaggerated relationship between the ideal, or least squares fit, line and the actual measured or *calibration* line
- *(Note* in most cases, the static curve is used to determine linearity, and this may deviate somewhat from a dynamic linearity)

Diagram from Sensor Terminology" from www.ni.com Measurement Fundamentals.

LINEAR REGRESSION

Least Squares

From
NCEES
Handbo

$$y = \hat{a} + \hat{b}x \text{, where}$$

Note - this is not standard y=mx+b terminology

$$y\text{ - intercept} : \hat{a} = \overline{y} - \hat{b}\overline{x},$$

$$\text{and slope} : \hat{b} = S_{xy}/S_{xx},$$

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - (1/n)\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right),$$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - (1/n)\left(\sum_{i=1}^{n} x_i\right)^2,$$

$$n = \text{sample size},$$

$$\overline{y} = (1/n)\left(\sum_{i=1}^{n} y_i\right) \text{, and}$$

$$\overline{x} = (1/n)\left(\sum_{i=1}^{n} x_i\right).$$

Least Squares Regression – Alternate Description

$$b = \frac{\overline{Y}\,\overline{X^2} - \overline{X}\,R_{xy}(0)}{\sigma_x^2}$$

For Examples – Regression Analysis.vi
Linear Regression.xls

$$m = \frac{R_{xy}(0) - \overline{X}\,\overline{Y}}{\sigma_x^2}$$

$$R_{xy}(0) = \frac{1}{N}\sum_{k=1}^{N} X_k Y_k$$

$$r \equiv \frac{1}{\sigma_x \sigma_y}\left| R_{xy}(0) - \overline{X}\,\overline{Y} \right|$$

$0 \le r \le 1$ A r of 1 indicates a perfect fit of the line to the data

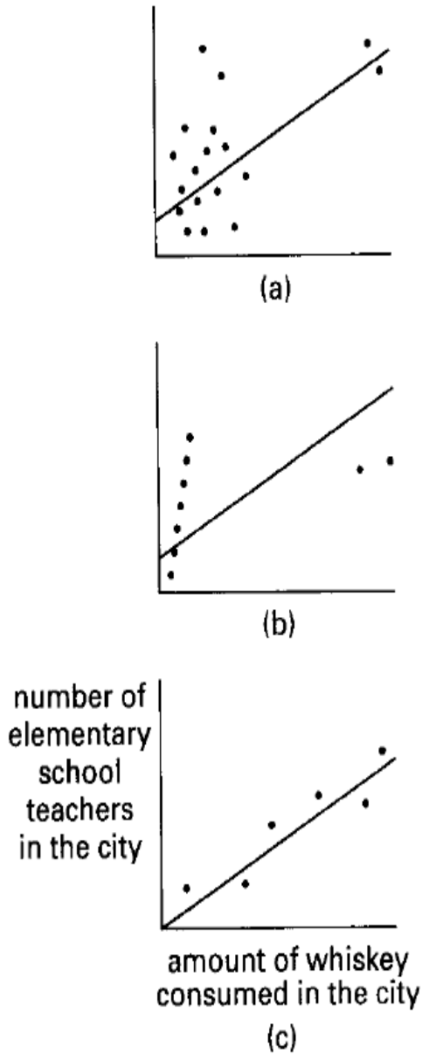


Figure 12.8 *Common Regression Difficulties*

- Common Errors in using linear regression analysis
  - (a) moderate correlation coefficient due to extreme points even though there is little actual correlation at the lower points
  - (b) good correlation in general but extreme points are missed and the overall correlation is moderate
  - (c) correlation does not mean that a cause and effect relationship exists

From “Electrical Engineering Reference for the Electrical and Computer PE Exam” by Camara

Hysteresis

- A transducer should be capable of following the changes of the input parameter regardless of which direction the change is made;
- Hysteresis is the measure of this property.
- Figure 4 shows a typical hysteresis curve.
- Note that it matters from which *direction* the change is made.
- Approaching a fixed input value (point B in Figure 4) from a higher value (point P) will result in a different indication than approaching the same value from a lesser value (point Q or zero).

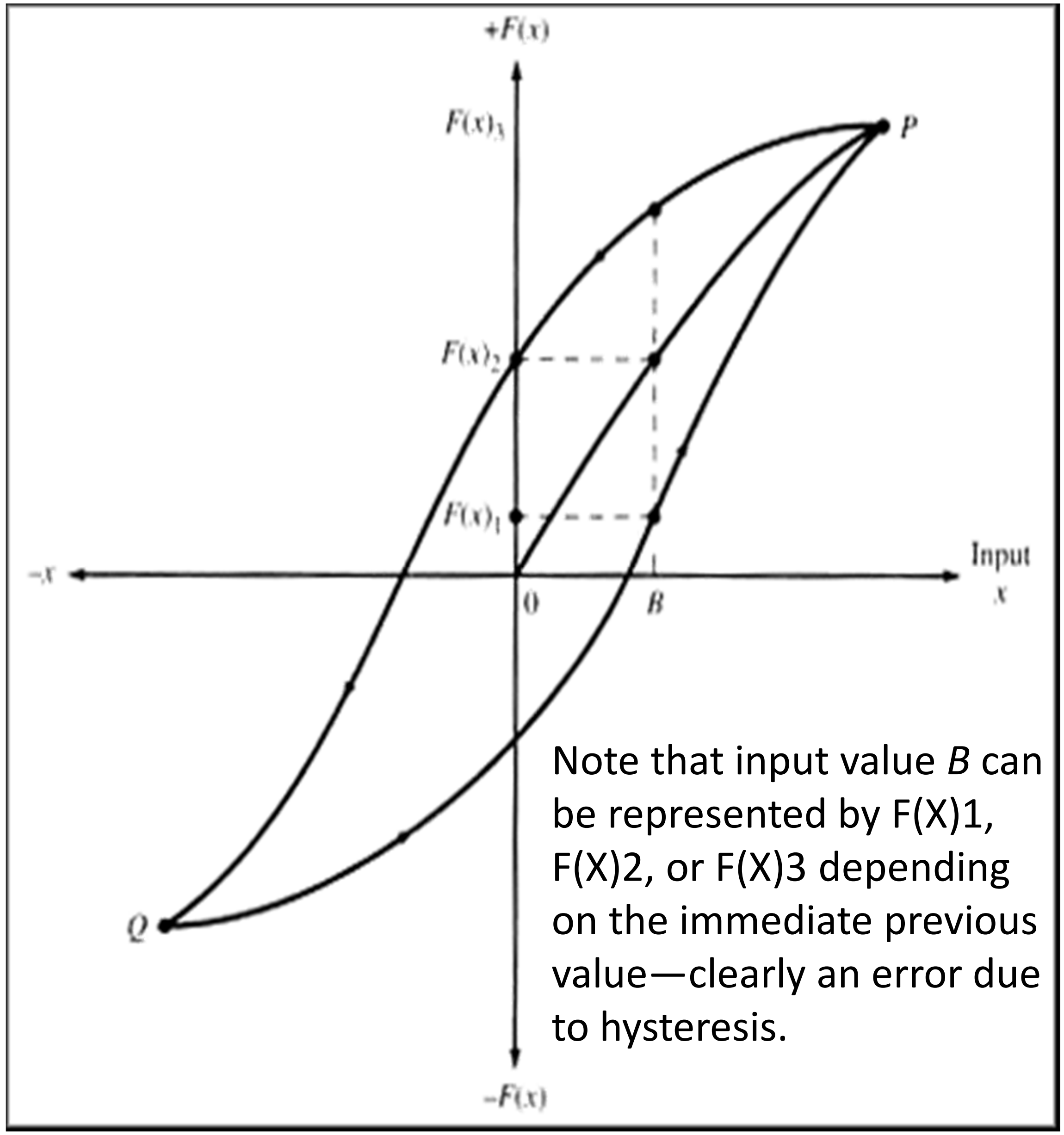


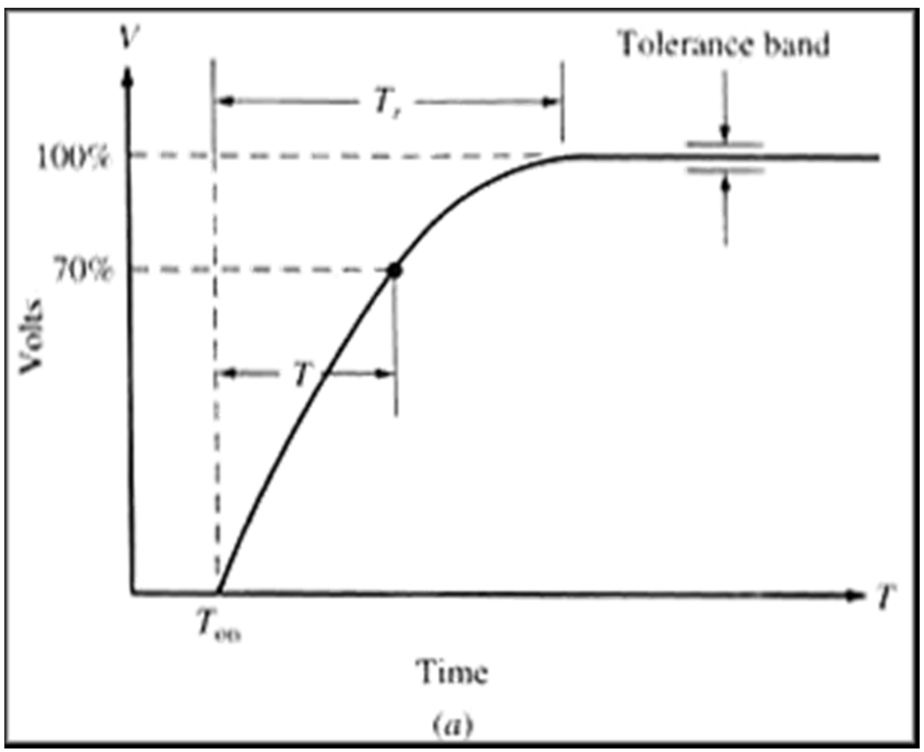
Diagram from Sensor Terminology" from www.ni.com Measurement Fundamentals.

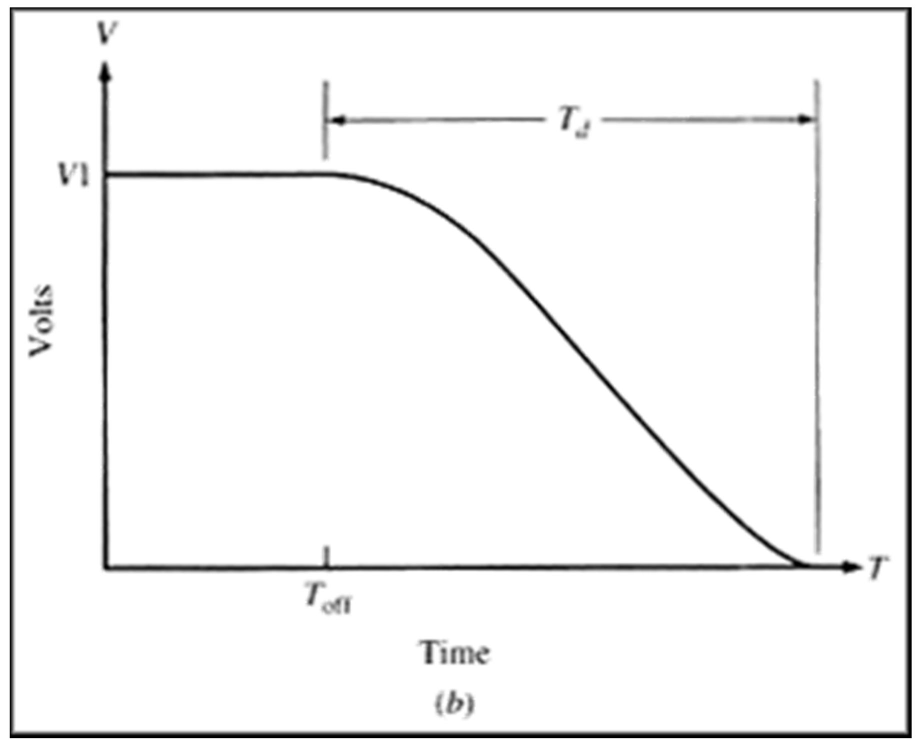
Response Time

- Sensors do not change output state immediately when an input parameter change occurs.
- Rather, it will change to the new state over a period of time, called the response time.
- The response time can be defined as the *time required for a sensor output to change from its previous state to a final settled value within a tolerance band of the correct new value.*
- Separating the response time of the sensor from the response time of the system is a critical issue
- In general sensors should be selected to have a much faster response than the system being measured

Response Time - Examples

- The curves below show two types of response time. In Figure a the curve represents the response time following an abrupt positive going step-function change of the input parameter. The form shown in Figure b is a decay time (Td to distinguish from Tr, for they are not always the same) in response to a negative going step-function change of the input parameter.





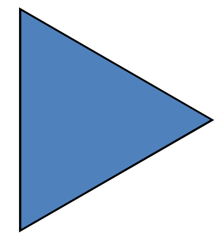
Real Time

- "Real Time Systems are defined as those systems in which the correctness of the system depends not only on the logical results of the computation, but also on the time at which the results are produced"
- Many data acquisition systems are real-time systems
- If you can afford to run it again and capture exactly the same conditions and results (not just similar), it is not real time

Dryden Flight Research Center ECN77-8608 Photographed 1977
Shuttle prototype Enterprise separating from 747 Shuttle Carrier
Aircraft for approach and landing test (ALT) research. NASA photo

Gain

- Gain is another term for multiplicative amplification
- Typically represented by a Triangle
- If an input is 1 unit and an output is 10 units, the Gain is a factor of 10



- Gain is also expressed in decibels (db)
- Decibels is calculated two ways
  - For Voltage it is calculated as $20\log_{10}(\frac{V}{V_{reference}})$
  - For Power Ratios it is calculated as $10\log_{10}(\frac{P}{P_{reference}})$