



ECE421: Electronics for Instrumentation

Lecture #2: MEMS Scaling Laws

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Outline

- Scaling Systems
- Scaling of Mechanical Systems
- Scaling of Electrical Components
- Scaling of Electrostatic Force
- Scaling of Electromagnetic Force



Scaling of Systems

- As the size of a system changes, its geometrical and physical parameters change.



- If you drop an ant from ten times its height, it will survive. But That is not the case with big beings.



Scaling of Systems, Example



wants to walk on water



$\gamma_{\text{water}} = 72 \text{ mN/m}$
(Water Surface Tension)

175cm length
50 kg weight

Scaling ↓ $S=10^{-3}$



1.75mm length



- Weight: $50 \times 9.8 = 490 \text{ N}$
- Weight = force by surface tension
- Leg Perimeter = $490/72e-3 = 6005 \text{ m} !!$



- Weight: $490 \text{ N} \times (10^{-3})^3 = 4.9 \times 10^{-7}$
- Leg Perimeter = $4.9 \times 10^{-7} / 72e-3 = 6.8\mu\text{m}$



Scaling of Systems

- When designing a MEMS system, first thing we think about is what kind of force we use for actuation.
- The actuating force depends on the actuation phenomenon, for example,
 - Electrostatic force → electric field
 - Magnetic force → magnetic field
- It is important to know how such systems will behave as the physical dimensions (size) reduces.
- Scaling theory is a valuable guide to what may work and what may not when we go to the micro scale.



Scaling of Mechanical Systems

- We define the scale variable “s” which represents the linear scale of the system.
- Then, $0 < s < 1$
- Nominally $s = 1$, then if s changes to 0.1, all the dimensions of the system are decreased by a factor of 10.
- Now if a system is scaled down 10 times its original size, then,

$$x_s = sx_o$$

$$A_s = x_s y_s = sx_o sy_o = s^2 A_o$$

$$V_s = x_s y_s z_s = sx_o sy_o sz_o = s^3 V_o$$

- Which means that the area scales as s^2 and the volume scales as s^3 .
- Area to volume ratio ($1/s$) increases with scaling down the system.



Scaling of Mechanical Systems

- Mass scales as follows:

$$M_s = \rho V_s = (s^0 \rho) \times (s^3 V_0) = s^3 M_0$$

- Where ρ , the density, is constant and does not change with scaling the system.

- Assuming four force laws are described as follows:

$$F = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix}$$

- This matrix form is used to show a number of different force cases and scale sizes in a simple way.



Scaling of Acceleration

- The acceleration is defined as:

$$a = \frac{\text{Force}}{\text{Mass}} = \frac{F}{M}$$

- The acceleration can be scaled as:

$$a = [s^F][s^{-3}] = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix} \begin{bmatrix} s^3 \\ s^3 \\ s^3 \\ s^3 \end{bmatrix}^{-1} = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix} \begin{bmatrix} s^{-3} \\ s^{-3} \\ s^{-3} \\ s^{-3} \end{bmatrix} = \begin{bmatrix} s^{-2} \\ s^{-1} \\ s^0 \\ s^1 \end{bmatrix}$$

- As the force scales down (s^1 to s^4), the acceleration decreases.
- This is an interesting result. When the force scales as $[s^1]$, the acceleration scales as $[s^{-2}]$. If the size of the system decreases by a factor of 100, the acceleration increases by $(1/100)^{-2} = 10000$.
- **Small structures can be accelerated faster than big ones.**



Scaling of Transient Time

- The transient time t_{tr} to move from point A to B can be calculated as:

$$x = \frac{1}{2} a t_{tr}^2 \qquad t_{tr} = \sqrt{\frac{2x}{a}} = \sqrt{2} \cdot (x)^{0.5} \cdot (a)^{-0.5}$$

- In matrix form:

$$t_{tr} = \begin{bmatrix} s^0 \\ s^0 \\ s^0 \\ s^0 \end{bmatrix} \begin{bmatrix} s^1 \\ s^1 \\ s^1 \\ s^1 \end{bmatrix}^{0.5} \begin{bmatrix} s^{-2} \\ s^{-1} \\ s^0 \\ s^1 \end{bmatrix}^{-0.5} = \begin{bmatrix} s^0 \\ s^0 \\ s^0 \\ s^0 \end{bmatrix} \begin{bmatrix} s^{0.5} \\ s^{0.5} \\ s^{0.5} \\ s^{0.5} \end{bmatrix} \begin{bmatrix} s^1 \\ s^{0.5} \\ s^0 \\ s^{-0.5} \end{bmatrix} = \begin{bmatrix} s^{1.5} \\ s^1 \\ s^{0.5} \\ s^0 \end{bmatrix}$$

- As the force scales down (s^1 to s^4), the transient time increases.
- This is an interesting result. Even in the worst case, where $F=[s^4]$, the time required to perform a task is constant when the scale goes down.
- when $F=[s^2]$, the time required to perform a task decreases as the scale “s” goes down.
- **Small things are quick.**



Scaling of Power Density

- The power, P , or work done on the object per unit time is:

$$P = \frac{Fx}{t_{tr}}$$

- In matrix form:

$$P = \begin{bmatrix} s^1 \\ s^2 \\ s^3 \\ s^4 \end{bmatrix} \begin{bmatrix} s^1 \\ s^1 \\ s^1 \\ s^1 \end{bmatrix} \begin{bmatrix} s^{-1.5} \\ s^{-1} \\ s^{-0.5} \\ s^0 \end{bmatrix} = \begin{bmatrix} s^{0.5} \\ s^2 \\ s^{3.5} \\ s^5 \end{bmatrix}$$

- The power that can be produced per unit volume is:

$$\frac{P}{\text{volume}} = \begin{bmatrix} s^{0.5} \\ s^2 \\ s^{3.5} \\ s^5 \end{bmatrix} \begin{bmatrix} s^3 \\ s^3 \\ s^3 \\ s^3 \end{bmatrix}^{-1} = \begin{bmatrix} s^{-2.5} \\ s^{-1} \\ s^{0.5} \\ s^2 \end{bmatrix}$$



Scaling of Power Density

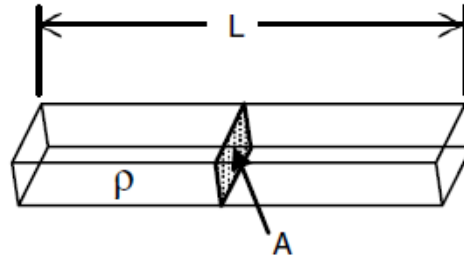
$$\frac{P}{\text{volume}} = \begin{bmatrix} s^{0.5} \\ s^2 \\ s^{3.5} \\ s^5 \end{bmatrix} \begin{bmatrix} s^3 \\ s^3 \\ s^3 \\ s^3 \end{bmatrix}^{-1} = \begin{bmatrix} s^{-2.5} \\ s^{-1} \\ s^{0.5} \\ s^2 \end{bmatrix}$$

- When the force scales as $[s^2]$, then the power per unit volume scales as $[s^{-1}]$. For example, when the scale decreases by a factor of 10, the force reduces by a factor of 100 but the power that can be generated per unit volume increases by a factor of 10. Thus, for force laws with a power higher than $[s^2]$, the power generated per unit volume degrades as the scale decreases.



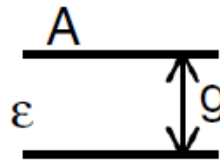
Scaling of electrical components

Resistance - R



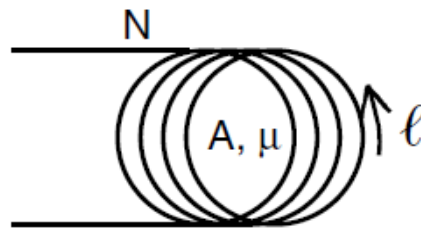
$$R = \frac{\rho L}{A} \propto \frac{1}{S}$$

Capacitance - C



$$C = \frac{\epsilon A}{g} \propto S$$

Inductance - L



$$L = \frac{\mu N^2 A}{\ell} \propto S$$



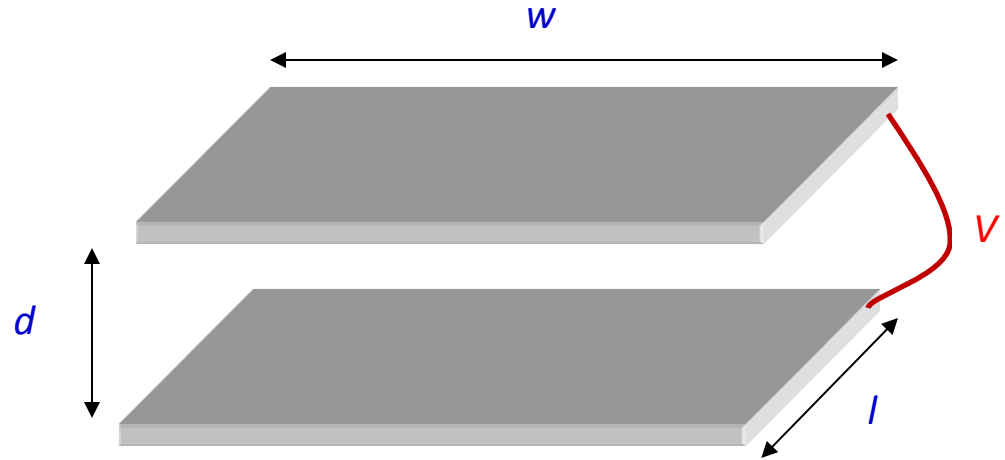
Scaling of electrostatic force

- Stored potential energy is:

$$U = \frac{1}{2} CV^2$$

- The capacitance is:

$$C = \frac{\epsilon_o A}{d} = \frac{\epsilon_o wl}{d}$$



- The electrostatic force is expressed as:

$$F = \frac{\partial U}{\partial x} = \frac{\partial}{\partial x} \left[\frac{1}{2} \epsilon_o \frac{wl}{d} V^2 \right]$$

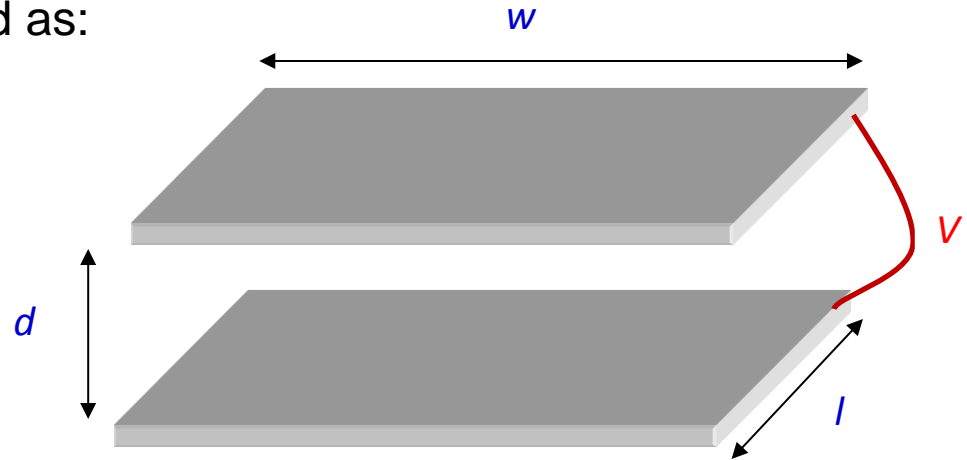
- The applied voltage can be expressed as $V = E d$, where E is the electric field strength (Volt/m).



Scaling of electrostatic force

- The electrostatic force is expressed as:

$$F = \frac{\partial U}{\partial x} = \frac{\partial}{\partial x} \left[\frac{1}{2} \epsilon_0 \frac{wl}{d} E^2 d^2 \right]$$
$$= \frac{\partial}{\partial x} \left[\frac{1}{2} \epsilon_0 wld E^2 \right]$$



- The Electrostatic force will scale as

$$F = s^2 E^2 \text{ or, } F \propto s^2$$

- In this case, scaling of the force depends on the scaling of the electric field E .
- If the field strength, E , does not change with the scale, then $E \propto s^0$ and F will scale as s^2 .



Scaling of electrostatic force

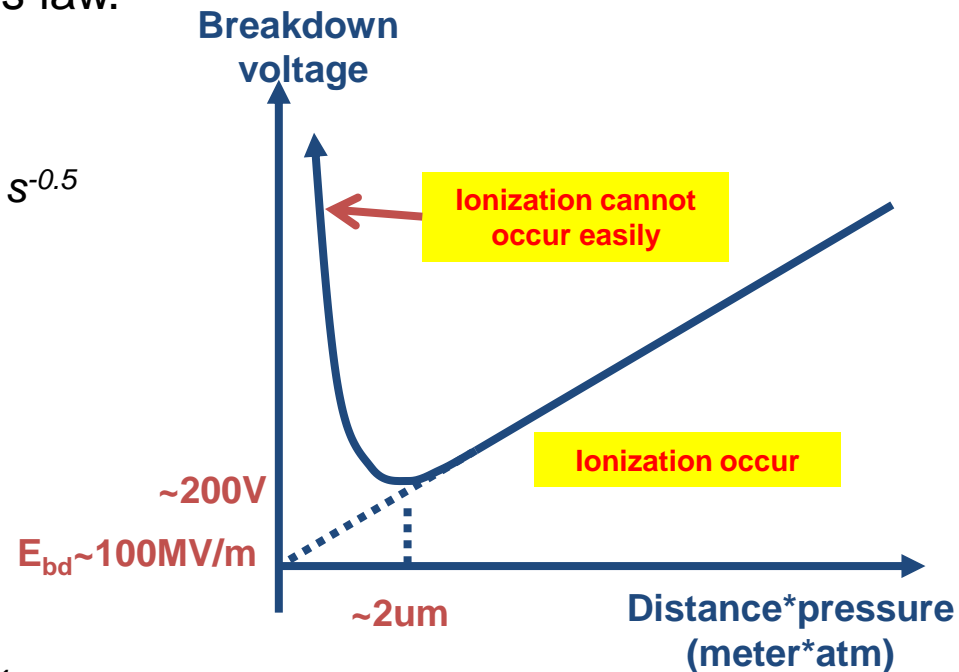
- For micro-scale, where the separation between electrodes and surfaces is reduced dramatically, the voltage can be increased without breaking down the air gap. This is explained by Paschen's law.

- For micro domain, the breakdown electric field can be scaled as $s^{-0.5}$

- Then the force can be scaled even better as

$$F \propto s^2 (s^{-0.5})^2 \propto s^1$$

- The electrostatic force scales as s^2 or s^1 .
- The electrostatic force is surface force, so it scales well in the micro domain.



Scaling of electromagnetic force

□ Force between two wires:

a- Constant current density: $F=[s^4]$.

b- Constant heat flow through the surface of the wire: $F=[s^3]$.

c- Constant temperature rise of the wire: $F=[s^2]$.

□ Force between a wire and a permanent magnet:

a- Constant current density: $F=[s^3]$.

b- Constant heat flow through the surface of the wire: $F=[s^{2.5}]$.

c- Constant temperature rise of the wire: $F=[s^2]$.

▪ The electromagnetic force scales as s^2 for best case scenario.

▪ The electromagnetic force is body, or volume, force, so it does not scale well in the micro domain.

▪ In micro domain, electrostatic forces are more favorable.



Summary

- Geometrical and physical parameters of a system change according to the change in its scale.
- Small structures can be accelerated faster than big ones.
- Small things are quick.
- MEMS designer should be aware of which forces are favorable and which are not.
- As the force scales down, the power density reduces.
- Electrostatic force is a surface force.
- Electromagnetic force is a body force, or volume force.
- Electrostatic force scales better than electromagnetic force in micro domain.
- In macro world Electromagnetic force is favorable more than electrostatic force.

