## ECE 334: Electronic Circuits

## Lecture 4: BJT Small Signal Analysis

## Example: Graphical Analysis



## Graphical construction of Ib and Vbe



$$
\begin{aligned}
& \mathrm{I}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{BE}}\right) / \mathrm{R}_{\mathrm{B}} \\
& \text { If } \mathrm{V}_{\mathrm{BE}}=0, \mathrm{I}_{\mathrm{B}}=\mathrm{V}_{\mathrm{BB}} / \mathrm{R}_{\mathrm{B}} \\
& \text { If } \mathrm{I}_{\mathrm{B}}=0, \mathrm{~V}_{\mathrm{BE}}=\mathrm{V}_{\mathrm{BB}}
\end{aligned}
$$

## Load Line



## Graphical Construction of Ic and Vce



## Graphical Analysis




## Bias Point Location Effects


-Load-line A results in bias point $Q_{A}$ which is too close to $V_{C C}$ and thus limits the positive swing of $v_{C E}$.
-Load-line B results in an operating point too close to the saturation region, thus limiting the negative swing of $v_{C E}$.

## A Practical BJT Amplifier using Coupling and Bypass Capacitors



- AC coupling through capacitors is used to inject an ac input signal and extract the ac output signal without disturbing the DC Q-point
- Capacitors provide negligible impedance at frequencies of interest and provide open circuits at dc.

In a practical amplifier design, $C_{1}$ and $C_{3}$ are large coupling capacitors or dc blocking capacitors, their reactance ( $\mathrm{X}_{\mathrm{C}}$ $\left.=\left|Z_{C}\right|=1 / \omega C\right)$ at signal frequency is negligible. They are effective open circuits for the circuit when DC bias is considered.
$C_{2}$ is a bypass capacitor. It provides a low impedance path for ac current from emitter to ground. It effectively removes $R_{\mathrm{E}}$ (required for good Q-point stability) from the circuit when ac signals are considered.

## DC and AC Analysis -- Application of Superposition

- DC analysis:
- Find the DC equivalent circuit by replacing all capacitors by open circuits and inductors (if any) by short circuits.
- Find the DC Q-point from the equivalent circuit by using the appropriate large-signal transistor model.
- AC analysis:
- Find the AC equivalent circuit by replacing all capacitors by short circuits, inductors (if any) by open circuits, dc voltage sources by ground connections and dc current sources by open circuits.
- Replace the transistor by its small-signal model (to be developed).
- Use this equivalent circuit to analyze the AC characteristics of the amplifier.
- Combine the results of dc and ac analysis (superposition) to yield the total voltages and currents in the circuit.


## DC Equivalent for the BJT Amplifier



Disconnecled

DC Equivalent Circuit

- All capacitors in the original amplifier circuit are replaced by open circuits, disconnecting $v_{I}, R_{I}$, and $R_{3}$ from the circuit and leaving $R_{E}$ intact. The the transistor $Q$ will be replaced by its DC model.


## AC Equivalent for the BJT Amplifier



- The coupling and bypass capacitors are replaced by short circuits. The DC voltage supplies are replaced with short circuits, which in this case connect to ground.


## AC Equivalent for the BJT Amplifier (contd.)



$$
\begin{aligned}
& R_{B}=R_{1}\left\|R_{2}=10 \mathrm{k} \Omega\right\| 30 \mathrm{k} \Omega \\
& R=R_{C}\left\|R_{3}=4.3 \mathrm{k} \Omega\right\| 100 \mathrm{k} \Omega
\end{aligned}
$$

- By combining parallel resistors into equivalent $R_{B}$ and $R$, the equivalent AC circuit above is constructed. Here, the transistor will be replaced by its equivalent small-signal AC model (to be developed).


## Hybrid-Pi Small-Signal AC Model for the BJT



- The hybrid-pi small-signal model is the intrinsic lowfrequency representation of the BJT.
- The small-signal parameters are controlled by the Q-point and are independent of the geometry of the BJT.

Transconductance:

$$
g_{m}=\frac{I_{C}}{V_{T}} \cong 40 I_{C}
$$

Input resistance:

$$
r_{\pi}=\frac{\beta_{o} V_{T}}{I_{C}}=\frac{\beta_{o}}{g_{m}}
$$

Output resistance:

$$
r_{o}=\frac{V_{A}+V_{C E}}{I_{C}}
$$

## Equivalent Forms of the Small-signal Model for the BJT


(a)

(b)

- The voltage-controlled current source $g_{m} v_{b e}$ can be transformed into a current-controlled current source,

$$
\begin{aligned}
& v_{b e}=i_{b} r^{r}=i_{b} \frac{\beta_{o}}{g_{m}} \\
& \therefore g_{m} v_{b e}=g_{m} i_{b} r_{\pi}=\beta_{o} i_{b} \\
& i_{c}=g_{m} v_{b e}+\frac{v_{c e}}{r_{o}} \cong g_{m} v_{b e}=\beta_{o} i_{b}
\end{aligned}
$$

- The basic relationship $i_{c}=\beta i_{b}$ is useful in both dc and ac analysis when the BJT is biased in the forward-active region.


## Examples

- On the Board

