CSE468: IMAGE PROCESSING

Image Enhancement in the Spatial Domain (Part 4)

Lecturer: Dr. Hossam Hassan

Email : hossameldin.hassan@eng.asu.edu.eg

Computers and Systems Engineering

Spatial Filtering

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

- use filter (can also be called as mask/kernel/template or window)
- the values in a filter subimage are referred to as coefficients, rather than pixel.
- our focus will be on masks of odd sizes, e.g. 3x3, 5x5,…

Spatial Filtering Process

- simply move the filter mask from point to point in an image.
- at each point (x,y), the response of the filter at that point is calculated using a predefined relationship.

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$
$$= \sum_{i=i}^{mn} w_i z_i$$

Linear Filtering

- Linear Filtering of an image f of size MxN filter mask of size mxn is given by the expression

$$g(x,y)=\sum_{t=-a}^{a}[\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)]$$

where a = (m-1)/2 and b = (n-1)/2

To generate a complete filtered image this equation must be applied on all pixels of the image.

Smoothing Spatial Filters

- used for blurring and for noise reduction
- blurring is used in preprocessing steps, such as
  - removal of small details from an image prior to object extraction
  - bridging of small gaps in lines or curves
- noise reduction can be accomplished by blurring with a linear filter and also by a nonlinear filter

Smoothing Linear Filters

- output is simply the average of the pixels contained in the neighborhood of the filter mask.
- called averaging filters or lowpass filters.

Smoothing Linear Filters

- replacing the value of every pixel in an image by the average of the gray levels in the neighborhood will reduce the "sharp" transitions in gray levels.
- sharp transitions
  - random noise in the image
  - edges of objects in the image
- thus, smoothing can reduce noises (desirable) and blur edges (undesirable)

3x3 Smoothing Linear Filters

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

box filter

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

weighted average

the center is the most important and other pixels are inversely weighted as a function of their distance from the center of the mask

Weighted average filter

- the basic strategy behind weighting the center point the highest and then reducing the value of the coefficients as a function of increasing distance from the origin is simply an attempt to reduce blurring in the smoothing process.

General form : smoothing mask

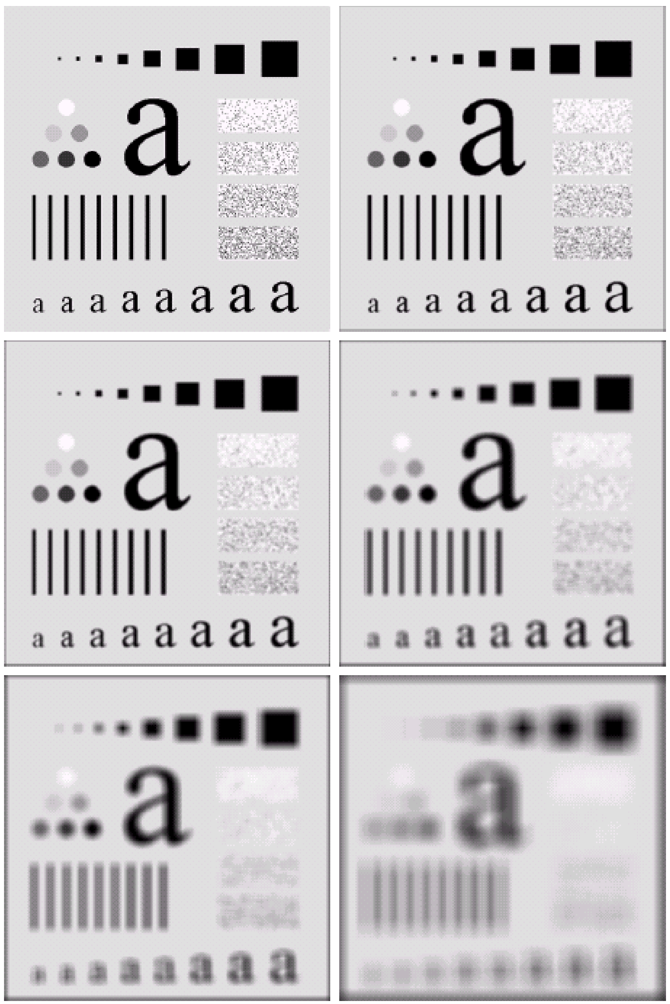
- filter of size mxn (m and n odd)

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

summation of all coefficient of the mask

a	b
c	d
e	f

Example



- a). original image 500x500 pixel
- b). - f). results of smoothing with square averaging filter masks of size n = 3, 5, 9, 15 and 35, respectively.
- Note:
  - big mask is used to eliminate small objects from an image.
  - the size of the mask establishes the relative size of the objects that will be blended with the background.

Example

Noisy Image

A 3X3 Smoothing Filter

A 5X5 Smoothing Filter

A 7X7 Smoothing Filter

Order-Statistics Filters (Nonlinear Filters)

- the response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter
- example
  - median filter : $R = \text{median}\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - max filter : $R = \max\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - min filter : $R = \min\{z_k \mid k = 1,2,\ldots,n \times n\}$
- note: n x nis the size of the mask

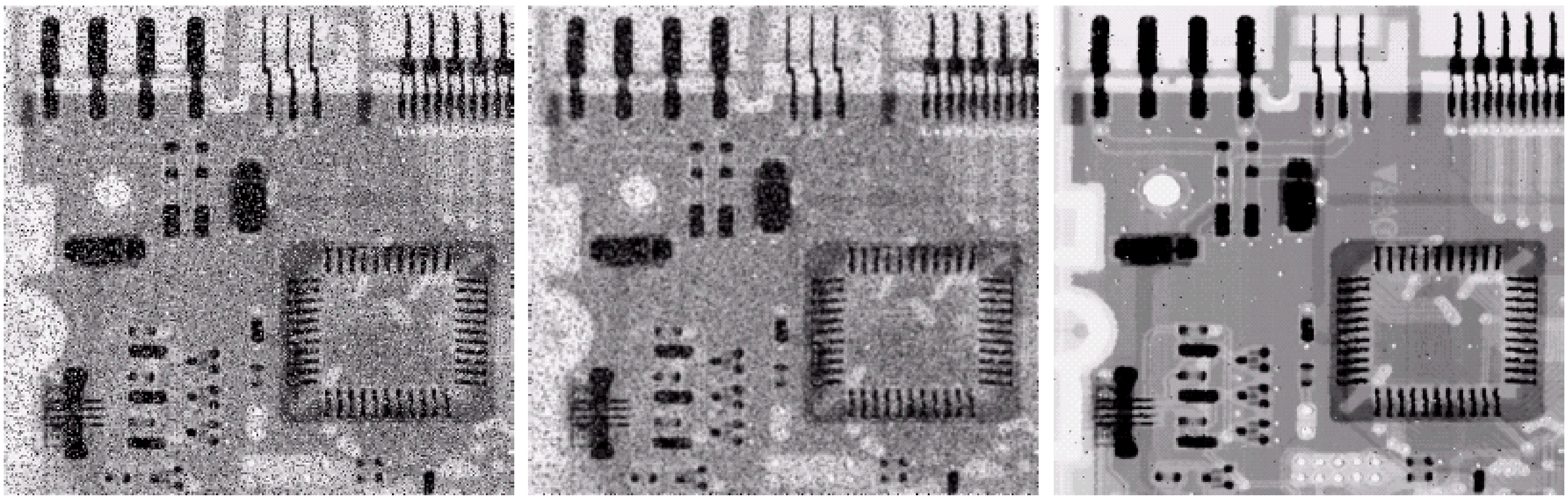
Median Filters

- replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel (the original value of the pixel is included in the computation of the median)
- quite popular because for certain types of random noise (impulse noise ⇨ salt and pepper noise) , they provide excellent noise-reduction capabilities, with considering less blurring than linear smoothing filters of similar size.

Median Filters

- forces the points with distinct gray levels to be more like their neighbors.
- isolated clusters of pixels that are light or dark with respect to their neighbors, and whose area is less than $n^2/2$ (one-half the filter area), are eliminated by an n x n median filter.
- eliminated = forced to have the value equal the median intensity of the neighbors.
- larger clusters are affected considerably less

Example : Median Filters



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Example : Salt & Pepper Noise- Weighted Average Filter

Noisy Image

A 3X3 Smoothing Filter

A 5X5 Smoothing Filter

A 7X7 Smoothing Filter

Example : Salt & Pepper Noise- Median Filter (Note the Difference)

Noisy Image

A 3X3 Median Filter

A 5X5 Median Filter

A 7X7 Median Filter

Sharpening Spatial Filters

- to highlight fine detail in an image
- or to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.

Blurring vs. Sharpening

- as we know that blurring can be done in spatial domain by pixel averaging in a neighbors
- since averaging is analogous to integration
- thus, we can guess that the sharpening must be accomplished by spatial differentiation.

Derivative operator

- the strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.
- thus, image differentiation
  - enhances edges and other discontinuities (noise)
  - deemphasizes area with slowly varying gray-level values.

First-order derivative

- a basic definition of the first-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Second-order derivative

- similarly, we define the second-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

First and Second-order derivative of f(x,y)

- when we consider an image function of two variables, f(x,y), at which time we will dealing with partial derivatives along the two spatial axes.

Gradient operator

$$\nabla f = [\frac{\partial f(x,y)}{\partial x} \frac{\partial f(x,y)}{\partial y}]^T$$

Laplacian operator

(linear operator)

$$\nabla^2 f = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

Discrete Form of Laplacian

from

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

yield,

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

Result Laplacian mask

0	1	0
1	−4	1
0	1	0

Laplacian mask implemented an extension of diagonal neighbors

1	1	1
1	−8	1
1	1	1

Other implementation of Laplacian masks

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

give the same result, but we have to keep in mind that when combining (add / subtract) a Laplacian-filtered image with another image.

Effect of Laplacian Operator

- as it is a derivative operator,
  - it highlights gray-level discontinuities in an image
  - it deemphasizes regions with slowly varying gray levels
- tends to produce images that have
  - grayish edge lines and other discontinuities, all superimposed on a dark,
  - featureless background.

Correct the effect of featureless background

- easily by adding the original and Laplacian image.
- be careful with the Laplacian filter used

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) \\ f(x,y) + \nabla^2 f(x,y) \end{cases}$$

if the center coefficient of the Laplacian mask is negative

if the center coefficient of the Laplacian mask is positive

Example

Input Image

Laplacian

Result

Mask of Laplacian + addition

- to simplify the computation, we can create a mask which does both operations, Laplacian Filter and Addition of the original image.