Faculty of Engineering

CSE115: Digital Design

Lecture 9:
Prime Implicants

Suggested Reading

- Sections 4.3.5

Definitions

1. A logic function P ***implies*** a logic function F if for every input combination for which P=1, then F=1 also. (P is an ***implicant*** of F)

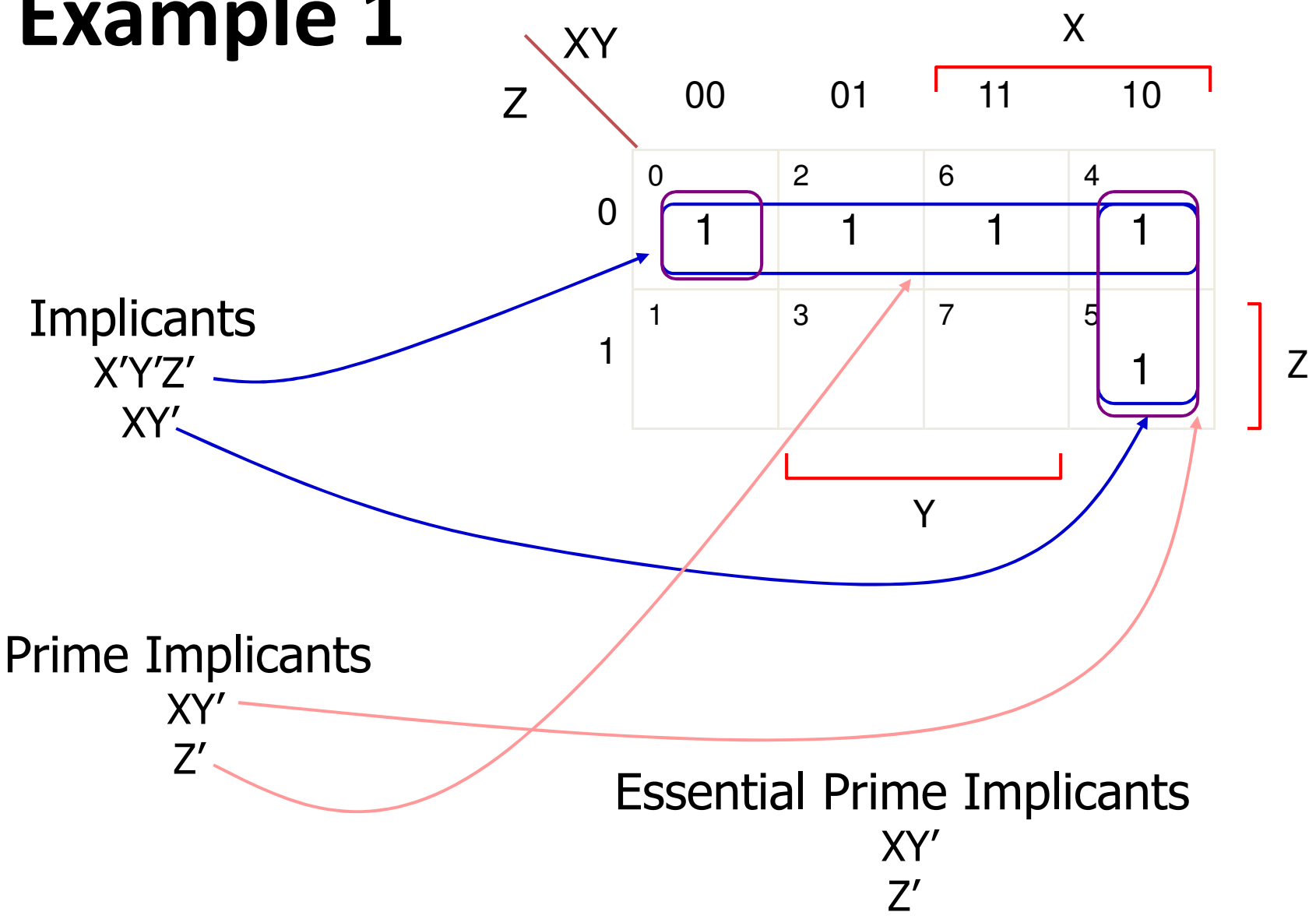
2. Any minterm or combination of minterms in the canonical sum expression is an ***implicant*** of the output function

3. A ***prime implicant*** is a group of combined minterms that can't be combined with any other minterm or group of minterms

4. ***Essential prime implicant*** is a prime implicant in which one or more minterms are unique (i.e at least one minterm is not contained in any other prime implicant. A unique minterm is called an ***Essential 1-Cell***

5. A ***minimal sum*** of a logic function is a sum-of-products expression for F such that no sum-of-products expression for F has fewer product terms.

Example 1


XY
X
Z
00
01
11
10
0
1
0
2
6
4
1
1
1
1
1
3
7
5
1
Z
Y
Implicants
X′Y′Z′
XY′
Prime Implicants
XY′
Z′
Essential Prime Implicants
XY′
Z′

Example 2

Combining (0,2)

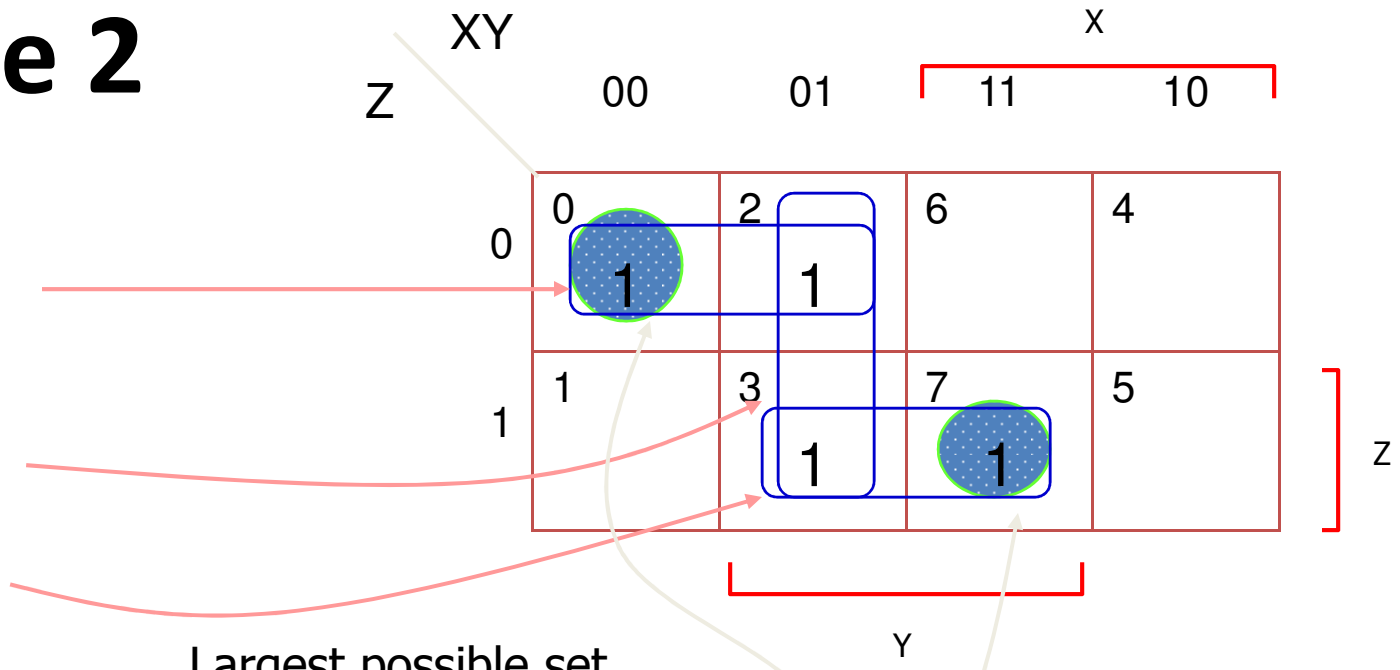
Product term: **X'Z'**

Combining (2,3)

Product term: **X'Y**

Combining (3,7)

Product term: **YZ**



Largest possible set

Unique minterm

X'Z', **X'Y**, and **YZ** are prime implicants

X'Z', **YZ** are **essential prime implicants**

X'Y is non-essential prime implicant (redundant) because all its minterms are covered in the other essential prime implicants

F= X'Z'+X'Y+YZ OR: **F = X'Z'+YZ** (the minimal sum of F)

Example 3

The prime implicants:

1- (0,2) **X'Z'**

2- (0,4) **Y'Z'**

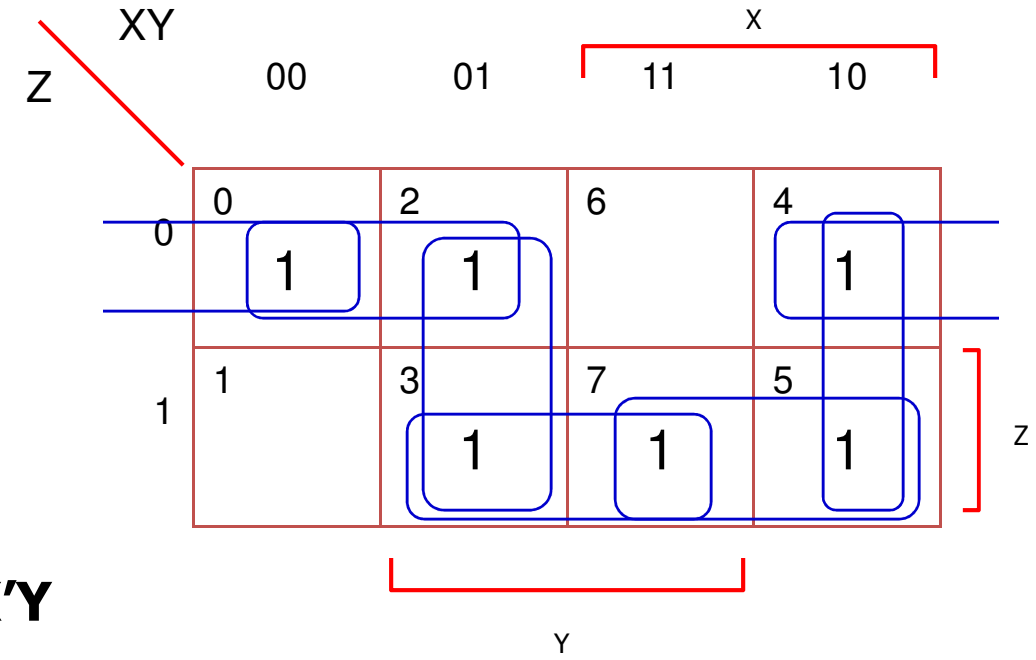
No essential prime implicant!

3- (2,3) **X'Y**

4- (3,7) **YZ**

5- (4,5) **XY'**

6 -(5,7) **XZ**



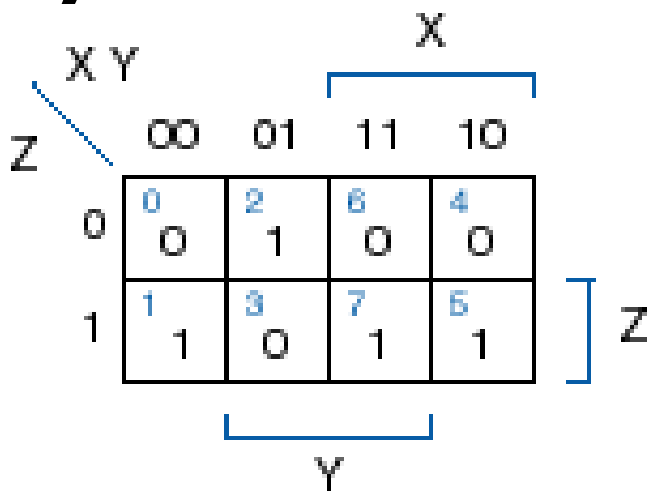
Two possible minimal sums:

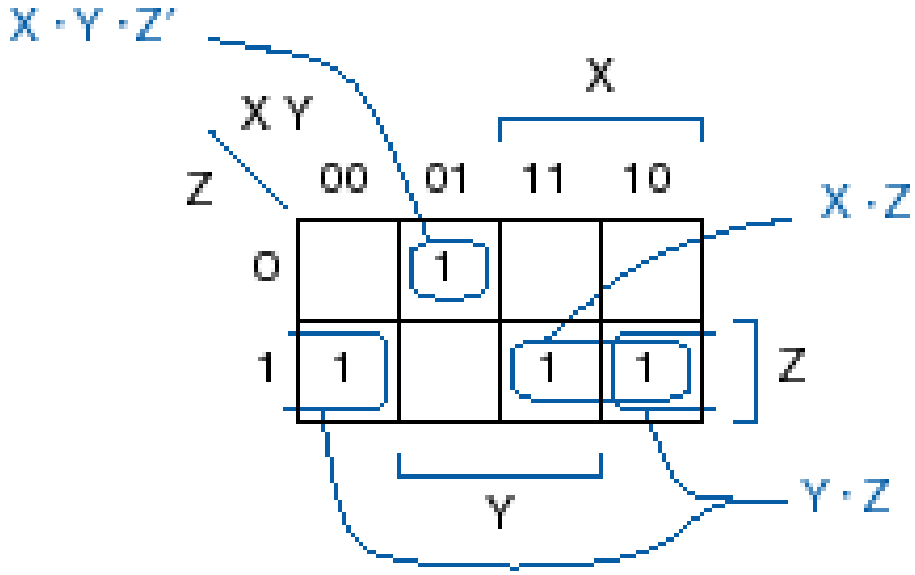
1- Using the prime implicants 1,4,and 5; **F= X'Z'+YZ+XY'**

2- Using the prime implicants 2,3,and 6; **F= Y'Z'+ X'Y+XZ**

Example 4: F = S(1,2,5,7)

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1





Summary

1. Load the **minterms** and maxterms into the K-map by placing the 1's and 0's in the appropriate cells.
2. Look for **groups of minterms** and write the corresponding product terms (the prime implicants):
   - The group size should be a power of 2.
   - Find the largest groups of minterms first then find smaller groups of minterms until all groups are found and all 1-cells are covered.
3. Determine the **essential prime implicants**.
4. Select all essential prime implicants and the **minimal set** of the remaining prime implicants that cover the remaining 1's.
5. Its possible to get more that one equally simplified expression if more than one set of the remaining prime implicants contains the same number of minterms.