Faculty of Engineering

CSE115: Digital Design

Lecture 8:
Karnaugh Maps

Suggested Reading

- Sections 4.3.3-4.3.4

Visualizing T10 – Karnaugh Maps

A karnaugh map is a representation of the truth table by a matrix of cells, where each cell corresponds to a minterm (or a maxterm) of the logic function.

For n-variable function, we need 2^n rows truth table and 2^n cells.

The cell number is equivalent to the row number in the truth table.

The truth table values are copied into their corresponding cells.

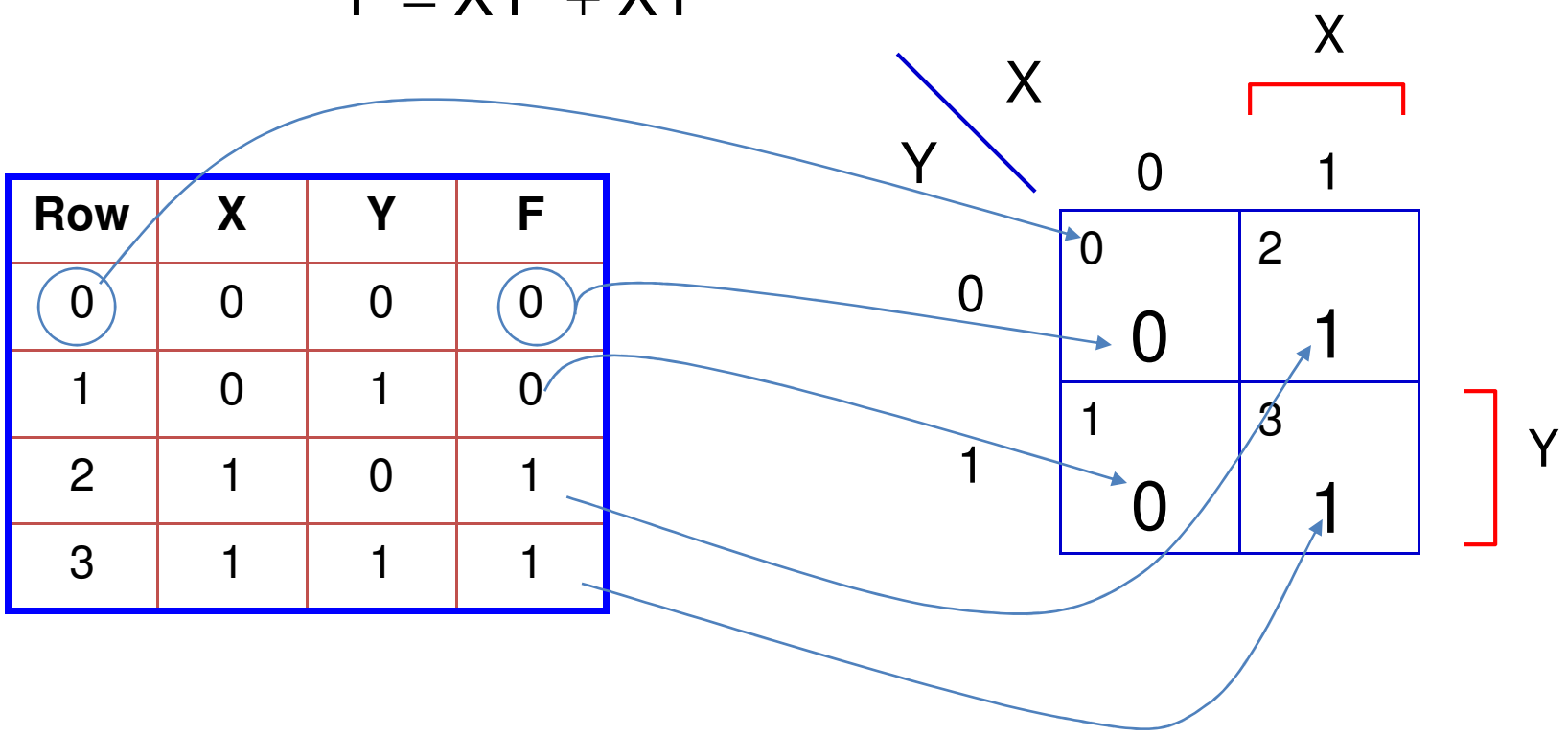
The cell arrangements help to identify the input variable redundancy $X \times Y + X \times Y' = X$

Two-Variable Karnaugh Map

Example 1:

$$F = XY' + XY$$

Row	X	Y	F
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1

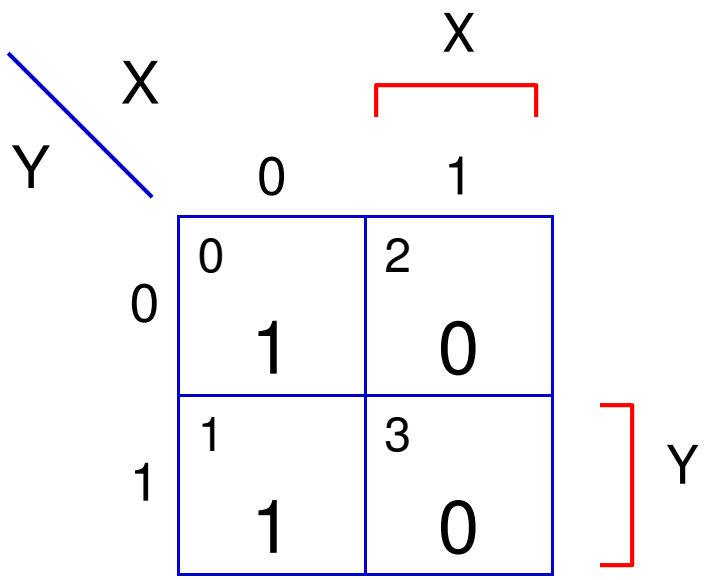


Simplification: $F = X(Y' + Y) = X \cdot 1 = X$

Example 1

$$F = X'Y' + X'Y$$

Row	X	Y	F
0	0	0	1
1	0	1	1
2	1	0	0
3	1	1	0

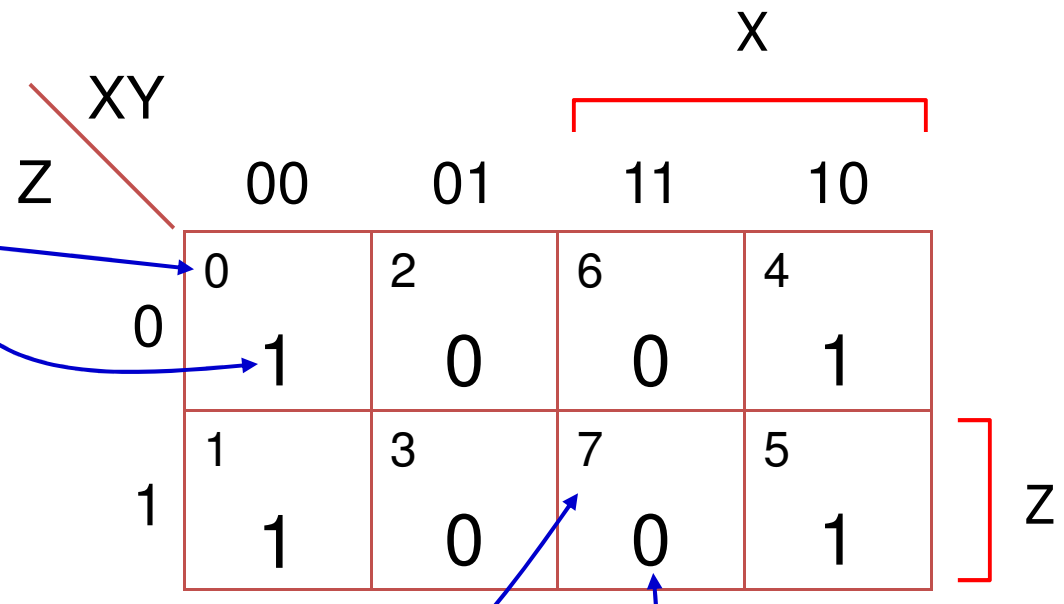


Simplification: $\mathbf{F} = X'(Y' + Y) = X' \cdot 1 = \mathbf{X'}$

Three-Variable Karnaugh Map

Example 3: $F = X'Y'Z' + X'Y'Z + XY'Z' + XY'Z$

Row	X	Y	Z	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0



Simplification: $\mathbf{F} = X'Y'(Z' + Z) + XY'(Z' + Z) = X' \cdot Y' + XY' = (X' + X)Y' = \mathbf{Y'}$

Karnaugh-Map Usage

1. **Plot 1s** corresponding to **minterms** of function.
2. **Circle largest** possible rectangular **sets of 1s**.
   - # of 1s in set must be power of 2
   - OK to cross edges
3. Read off product terms, one per circled set.
   - Variable is 1 → include variable
   - Variable is 0 → include complement of variable
   - Variable is both 0 and 1 → variable not included
4. Circled sets and corresponding product terms are called '**prime implicants**'
5. Minimum number of gates and gate inputs

Example 2

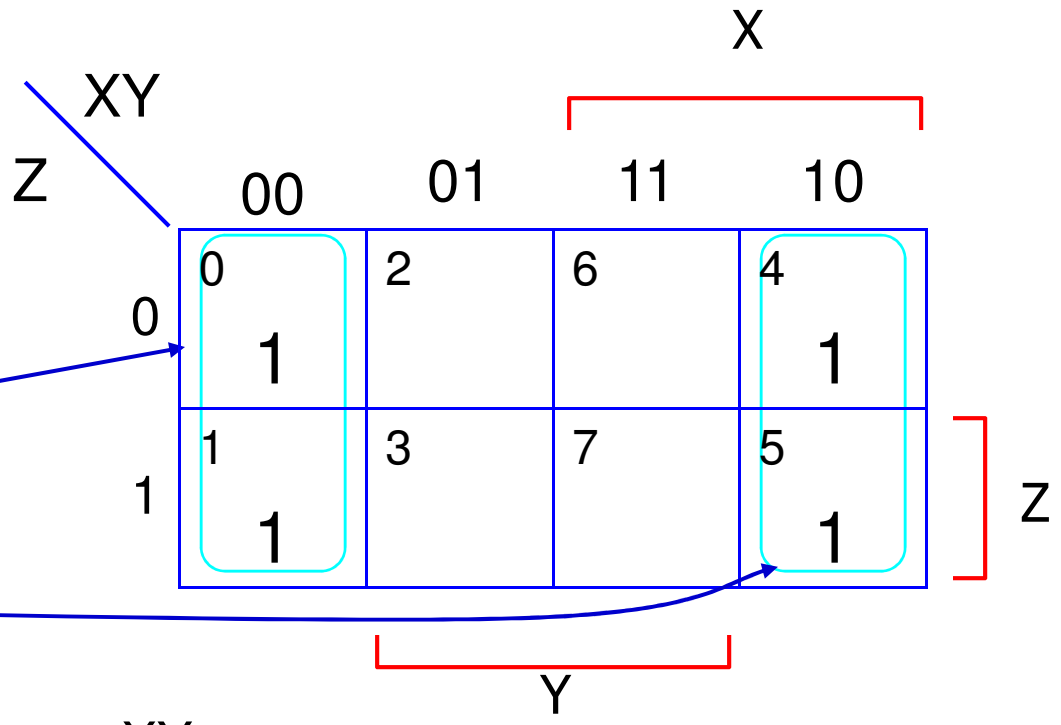
Combining Two cells:

I) Cells (0,1): X=Y=0; Z=0,1

Product term: **X'Y'**

Cells (4,5): X=1; Y=0; Z=0,1

Product term : **XY'**



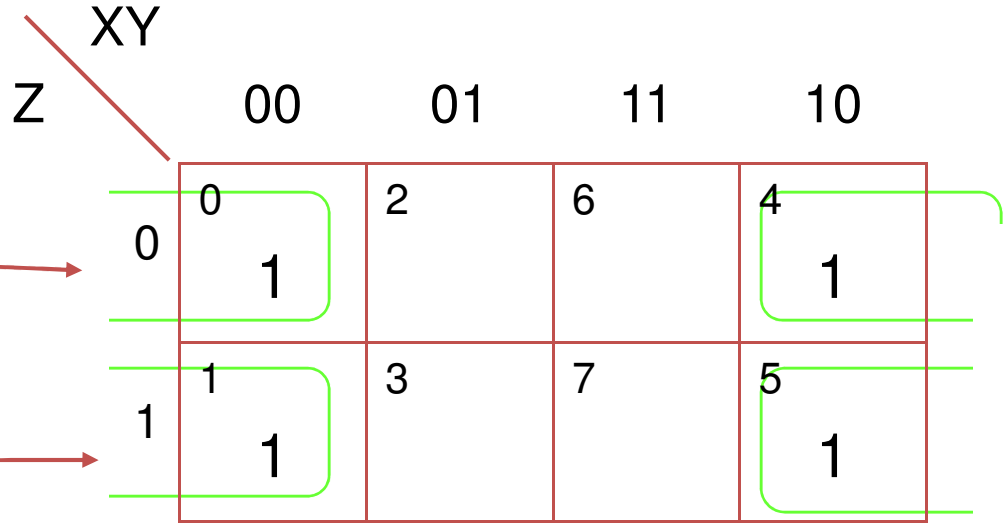
OR:

II) Cells (0,4): Y=0; Z=0; X=0,1

Product term: **Y'Z'**

Cells (1,5): Y=0; Z=1; X=0,1

Product term : **Y'Z**



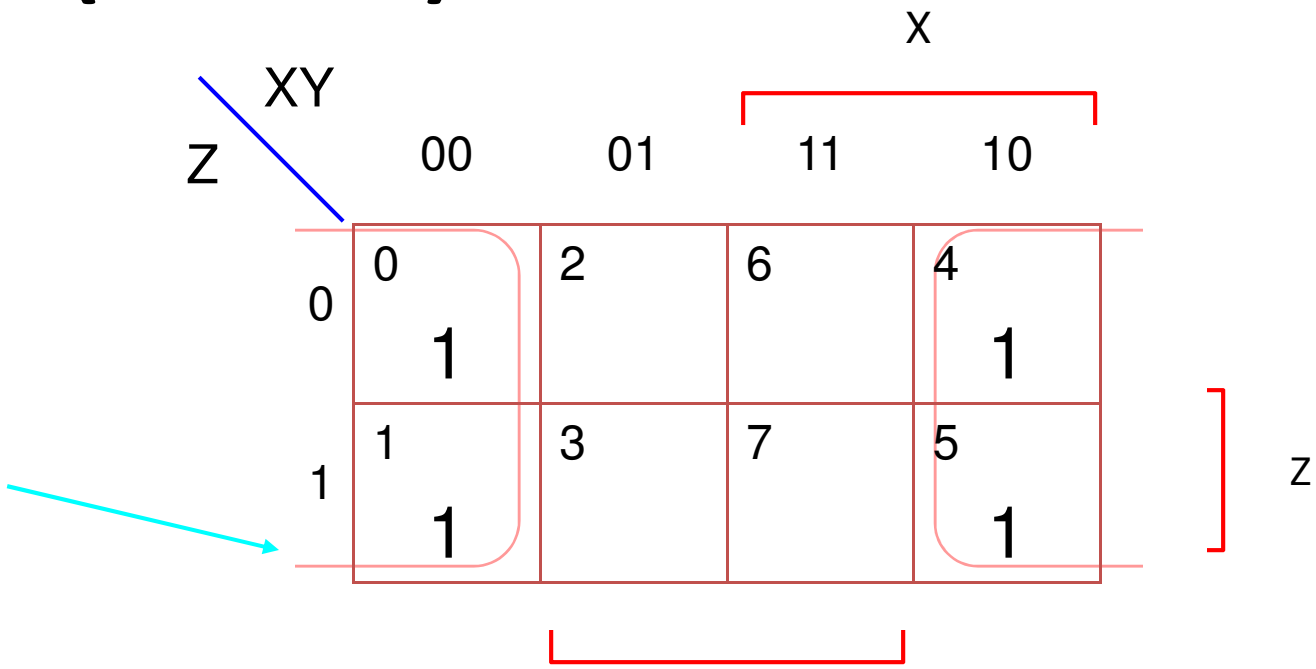
Example 2 (Contd.)

Combining four cells

III) Cells (0,1,4,5)

X=0,1; Z=0,1; Y=0

The product term: **Y'**



The sum of products expression of F:

From I) F =X'Y'+XY' or

From II) F=Y'Z'+Y'Z or:

From III) **F =Y'**

Example 2

Combine cells (0,2,6,4)

X=0,1; Y= 0,1; Z=0

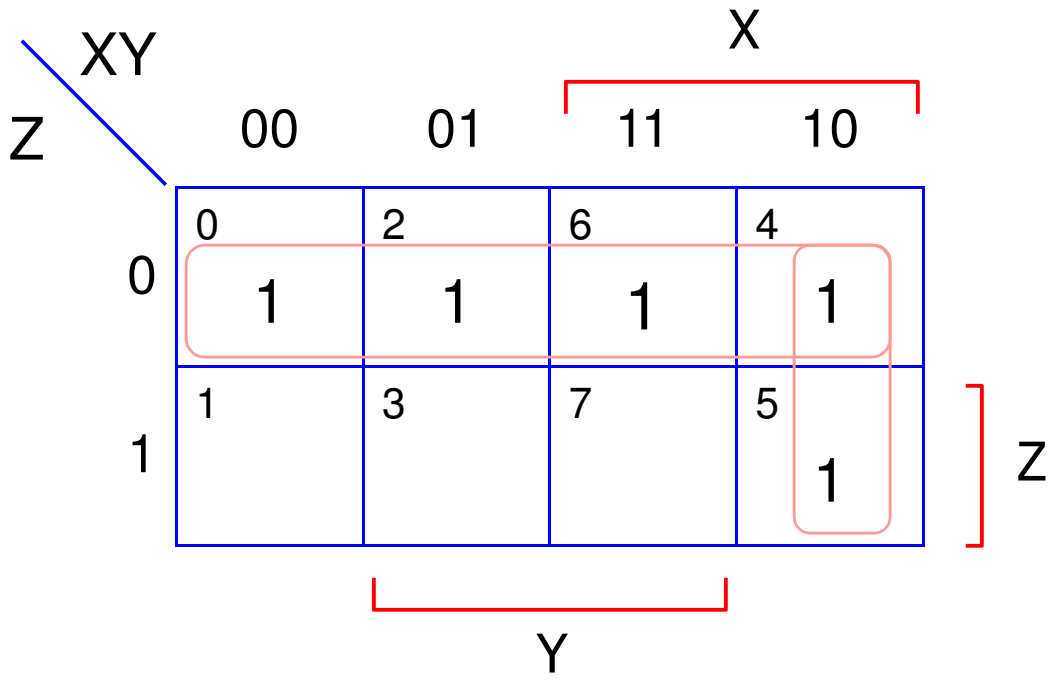
Product Term: **Z'**

Combine cells (4,5)

X=1; Y=0; Z=0,1

Product Term: **XY'**

F = XY'+Z'



The canonical sum is: F=X'Y'Z'+XYZ'+XY'Z'+XY'Z+XYZ'

Exercise