Faculty of Engineering

CSE115: Digital Design

Lecture 7:
Combinational Circuit Synthesis

Suggested Reading

- Sections 4.3.1-4.3.2

Combinational Circuit Synthesis

Truth Table → Logic Function → Logic Circuit

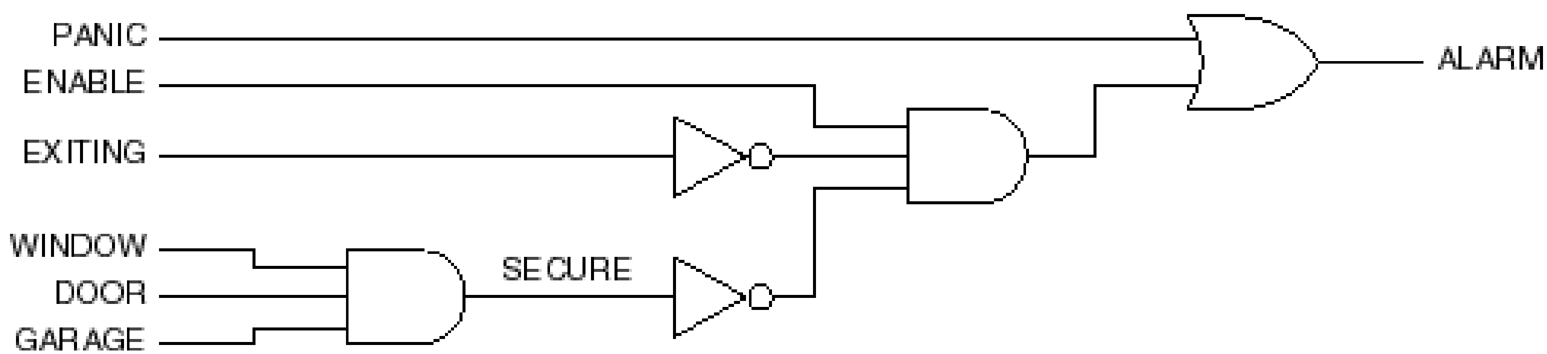
1. **Logic Function:**
   - The canonical sum expression.
   - The canonical product expression.
2. **The canonical Implementations:**
   - AND-OR and its equivalent NAND-NAND
   - OR-AND and its equivalent NOR-NOR
3. **Logic Function minimization:**
   - Simplifying the logic function to reduce the number of gates.
4. **Minimization methods:**
   - Using theorems
   - Karnaugh map

Combinational Circuit Design

- Sometimes you can write an equation or equations directly using 'logic'.

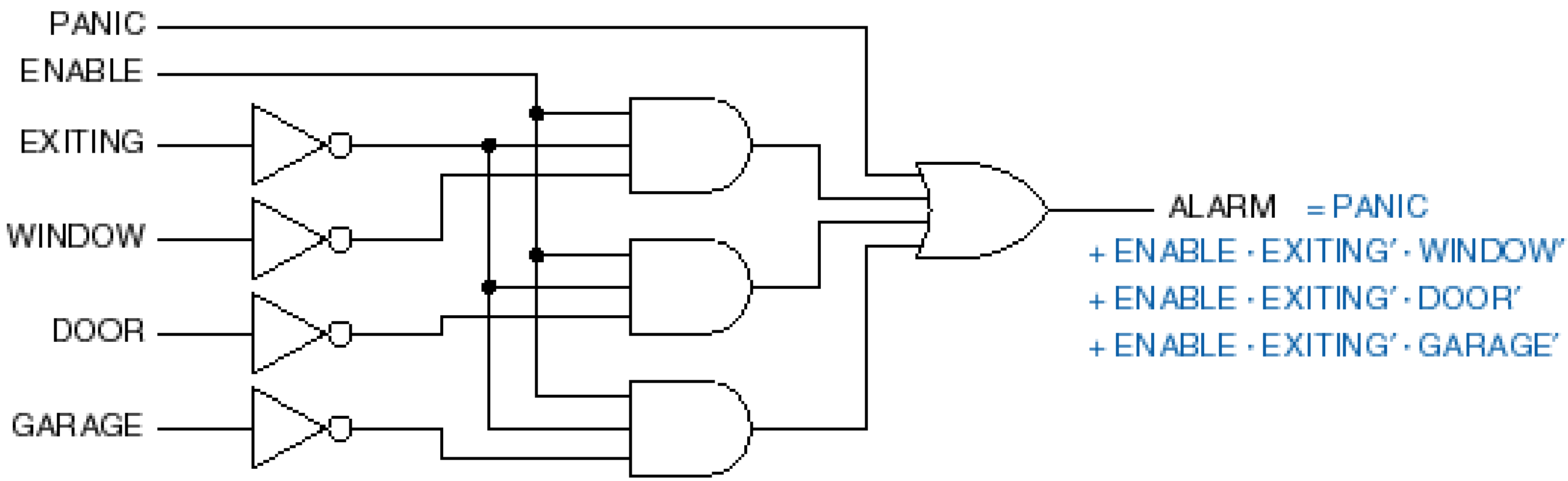
Example (alarm circuit):

ALARM = PANIC + ENABLE . EXITING' . SECURE'
SECURE = WINDOW . DOOR . GARAGE
ALARM = PANIC + ENABLE . EXITING' . (WINDOW . DOOR . GARAGE)'

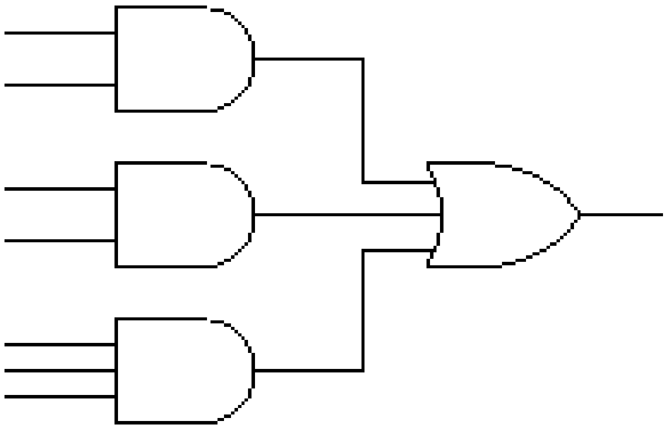


Example (Contd): Alarm Circuit Transformation

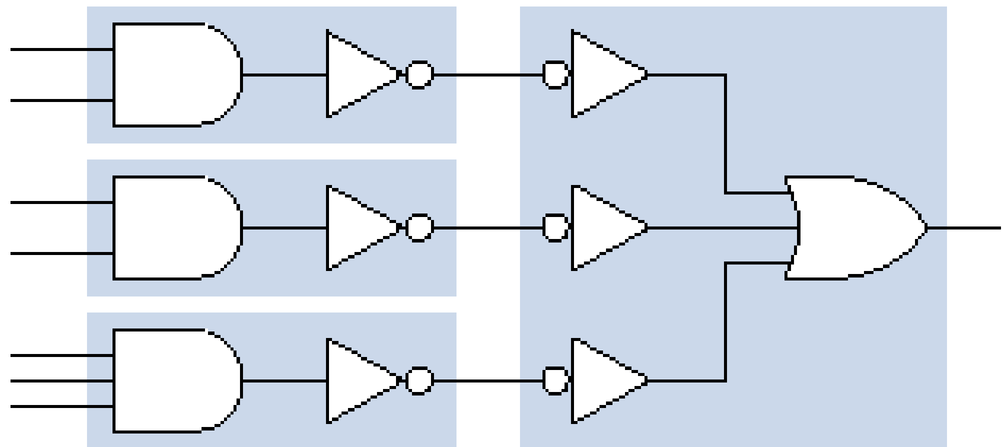
- Sum-of-products form
  - Useful for programmable logic devices
  - Multiply out



Sum-of-Products Form



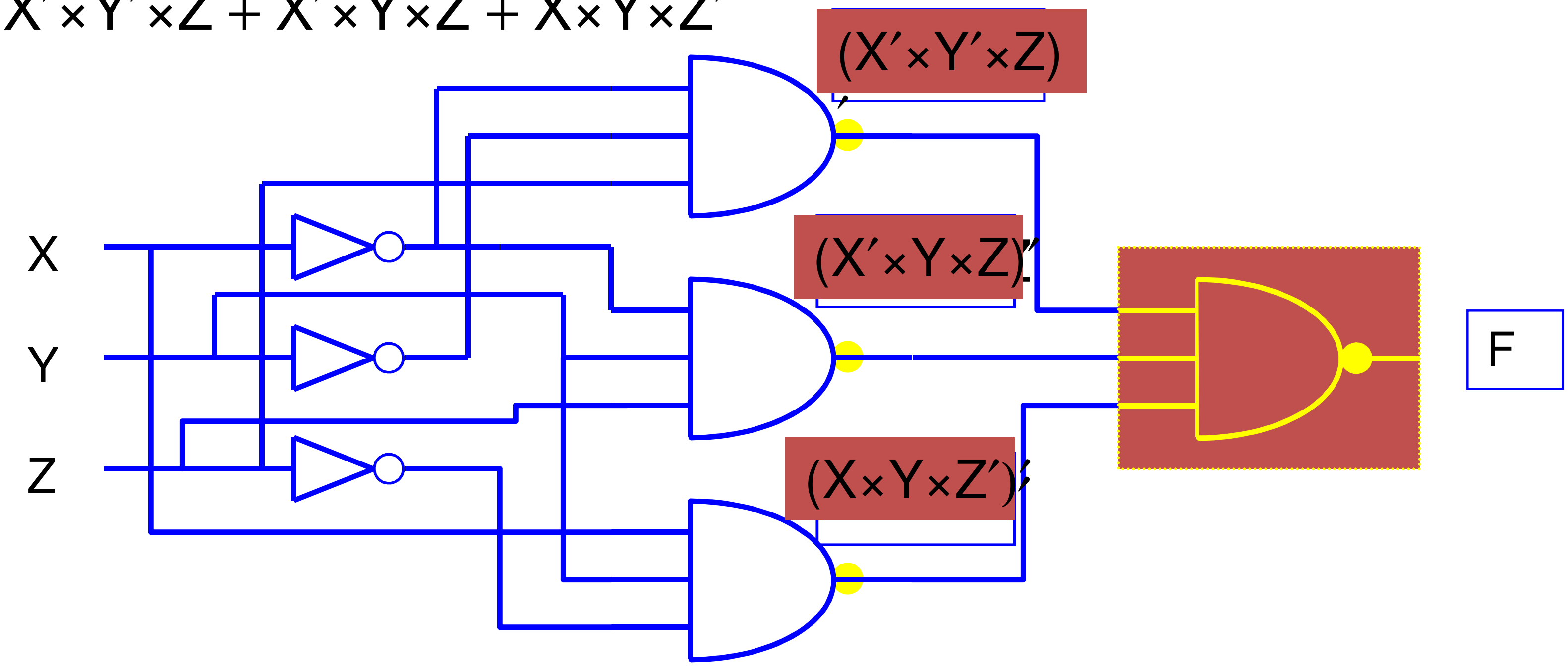
AND-OR

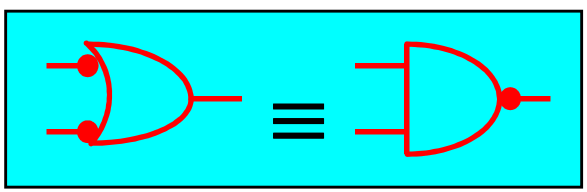


NAND-NAND

NAND-NAND Implementation

$F = X' \times Y' \times Z + X' \times Y \times Z + X \times Y \times Z'$

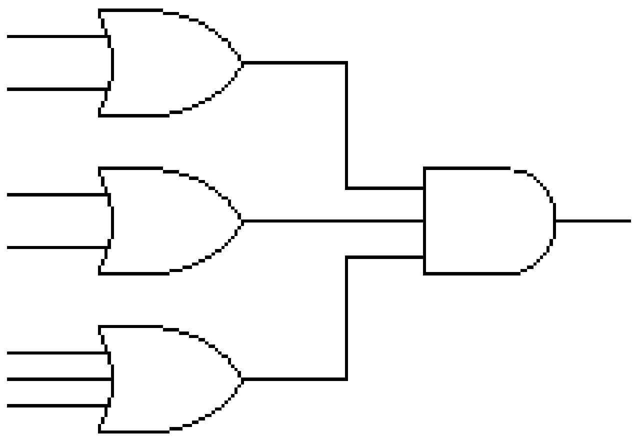




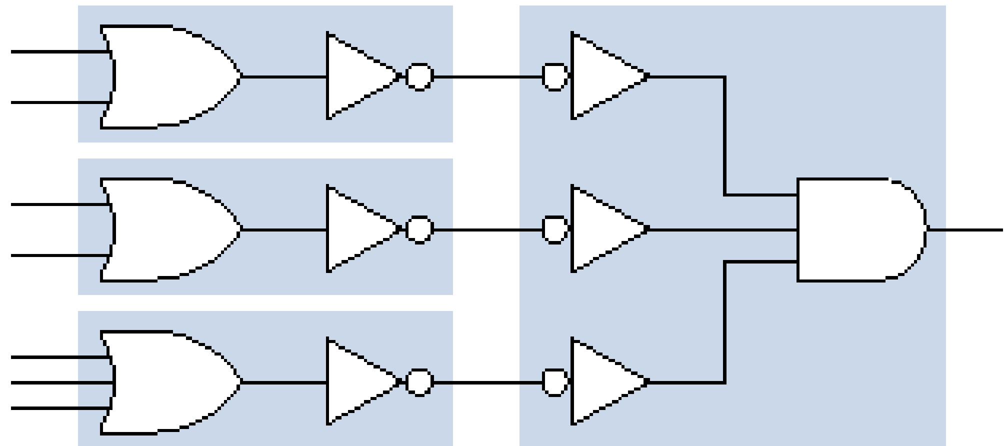
$F = [(X' \times Y' \times Z)' \times (X' \times Y \times Z)' \times (X \times Y \times Z')']'$

$F = X' \times Y' \times Z + X' \times Y \times Z + X \times Y \times Z$

Product-of-Sums Form



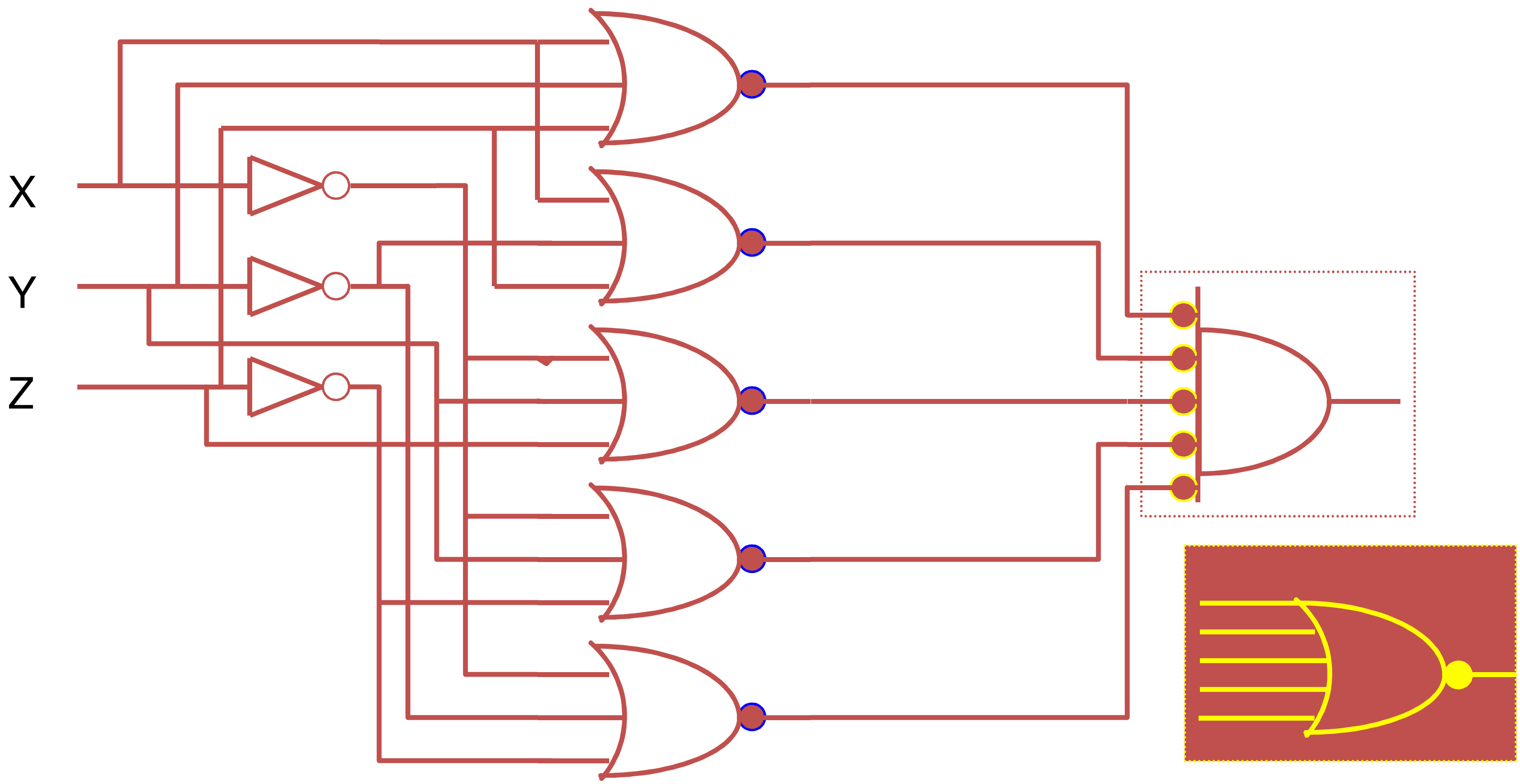
OR - AND



NOR - NOR

NOR-NOR Implementation

- F = (X + Y + Z)·(X + Y' + Z)·(X' + Y + Z)·(X' + Y + Z')·(X' + Y' + Z')



Minimization Example: Prime Number Detector

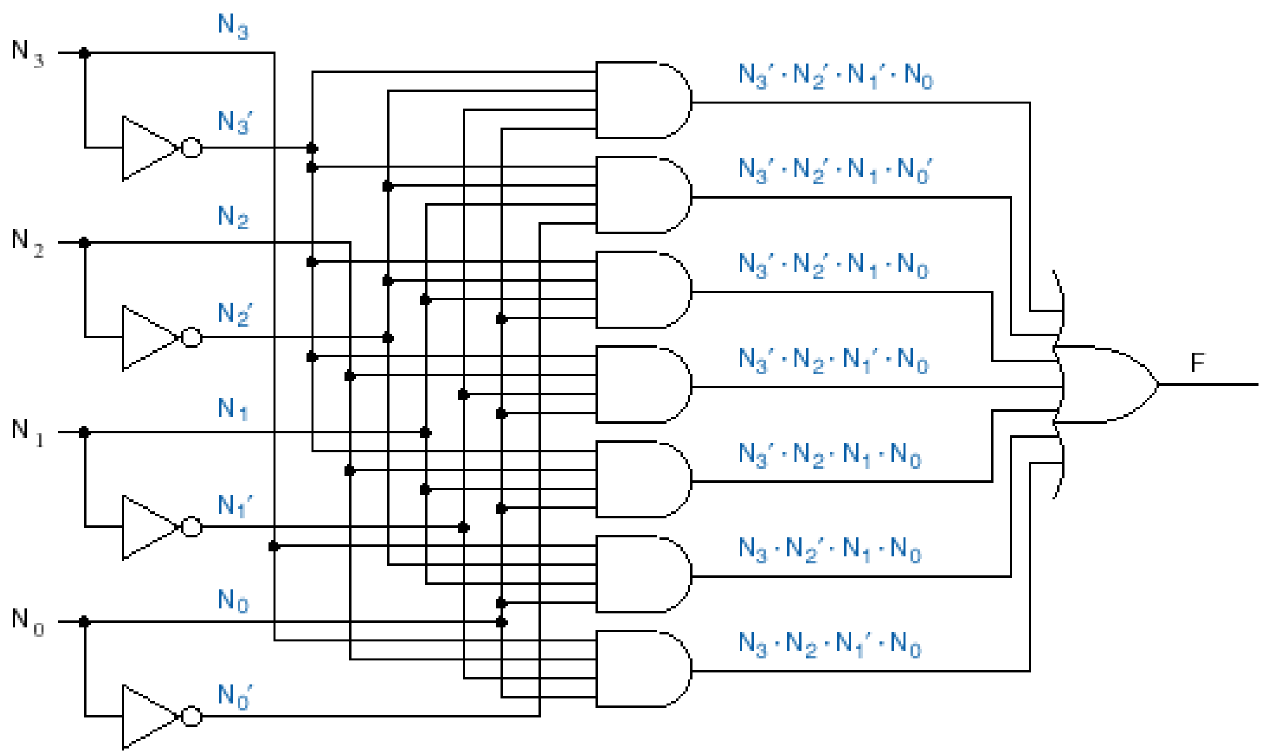
Truth table → canonical sum (sum of minterms)

$$F = \Sigma_{N3N2N1N0}(1,2,3,5,7,11,13)$$

Row	N3	N2	N1	N0	F
0	0	0	0	0	**0**
1	0	0	0	1	**1**
2	0	0	1	0	**1**
3	0	0	1	1	**1**
4	0	1	0	0	**0**
5	0	1	0	1	**1**
6	0	1	1	0	**0**
7	0	1	1	1	**1**
8	1	0	0	0	**0**
9	1	0	0	1	**0**
10	1	0	1	0	**0**
11	1	0	1	1	**1**
12	1	1	0	0	**0**
13	1	1	0	1	**1**
14	1	1	1	0	**0**
15	1	1	1	1	**0**

Example (contd.): Minterm list → Canonical Sum

$F = \Sigma_{N3N2N1N0}(1,2,3,5,7,11,13)$
$= N_3'.\ N_2'.\ N_1'.\ N_0 + N_3'.\ N_2'.\ N_1.\ N_0' + N_3'.\ N_2'.\ N_1.\ N_0 + N_3'.\ N_2.\ N_1'.\ N_0$
$+ N_3'.\ N_2.\ N_1.\ N_0 + N_3.\ N_2'.\ N_1.\ N_0 + N_3.\ N_2.\ N_1'.\ N_0$



Example (Contd.): Algebraic Simplification

(T10) $\mathbf{X} \times \mathbf{Y} + \mathbf{X} \times \mathbf{Y}' = \mathbf{X}$

$$F = \Sigma_{N3N2N1N0}(1,2,3,5,7,11,13)$$
$$= N_3'.\ N_2'.\ N_1'.\ N_0 + N_3'.\ N_2'.\ N_1.\ N_0 + N_3'.\ N_2.\ N_1'.\ N_0 + N_3'.\ N_2.\ N_1.\ N_0 + \ldots$$
$$= (N_3'.\ N_2'.\ N_1'.\ N_0 + N_3'.\ N_2'.\ N_1.\ N_0) + (N_3'.\ N_2.\ N_1'.\ N_0 + N_3'.\ N_2.\ N_1'.\ N_0) + \ldots$$
$$= N_3'.\ N_2'.\ N_0 + N_3'.\ N_2.\ N_0 + \ldots$$

Reduce number of gates and gate inputs