



Faculty of Engineering

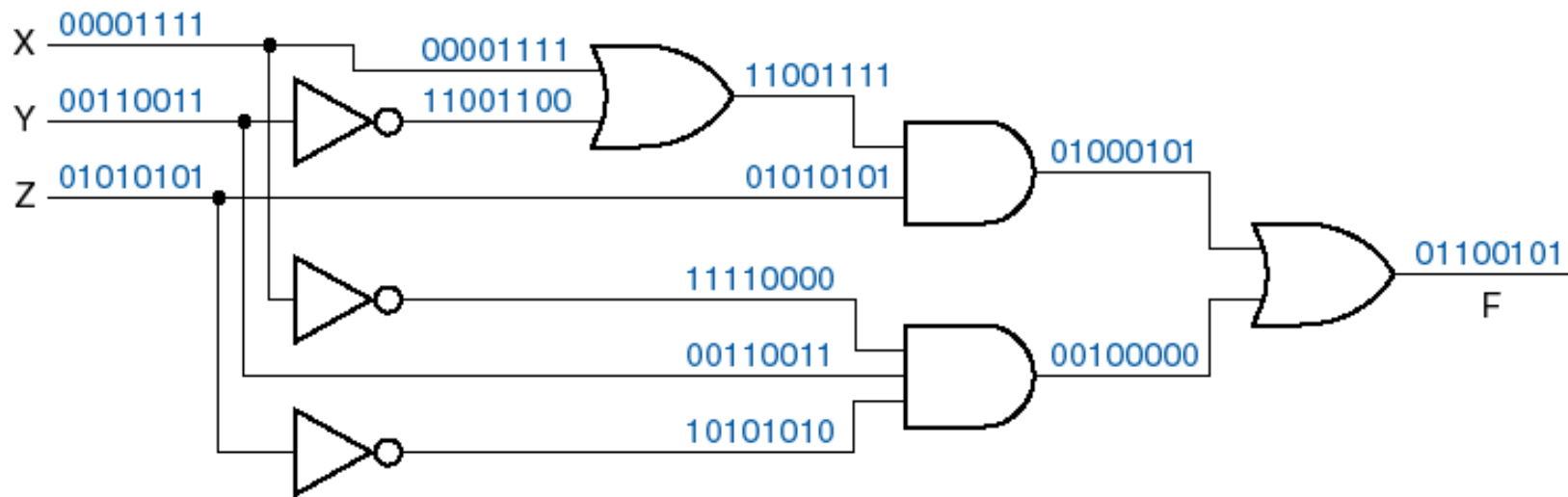
CSE115: Digital Design

Lecture 6:
Combinational Circuit Analysis

Suggested Reading

- [Section 4.2](#)

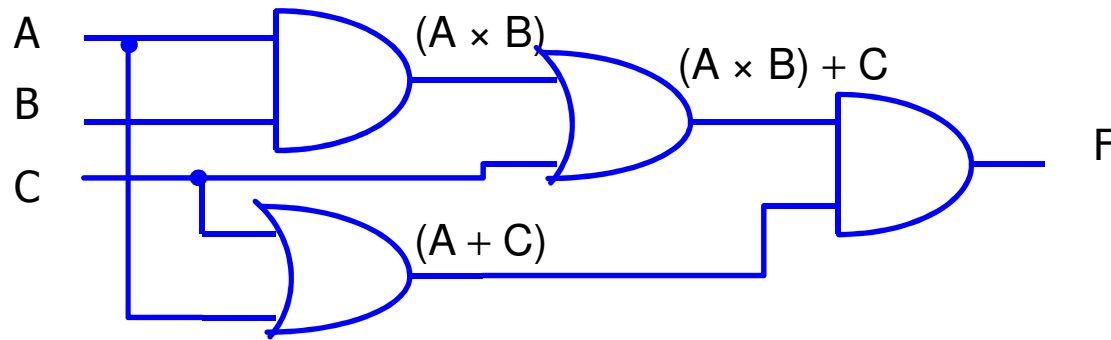
Combinational Circuit Analysis



Logic Circuit → Logic Function → Truth Table

Example 1:

Logic Circuit \rightarrow Logic Function \rightarrow Truth Table



$$F = (A + C) \times ((A \times B) + C)$$

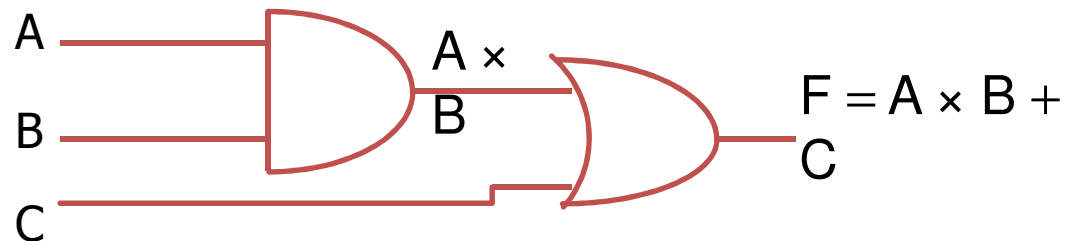
A	B	C	AB	AB+C	A+C	F
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	1	1	1
1	0	0	0	0	1	0
1	0	1	0	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Example 1:

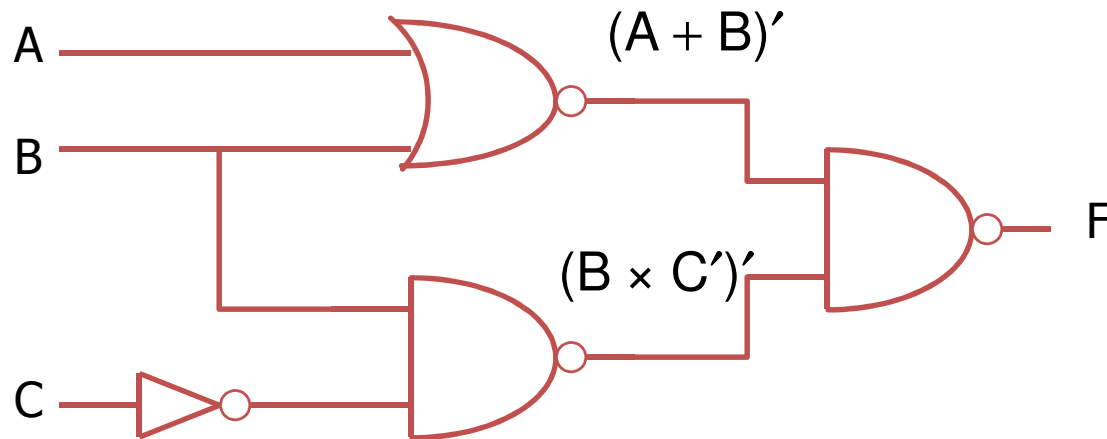
Logic Function Simplification

$$\begin{aligned} F &= (A + C) \times ((A \times B) + C) \\ &= A \times (A \times B + C) + C \times (A \times B + C) \\ &= A \times A \times B + A \times C + C \times A \times B + C \times C \\ &= A \times B + A \times C + A \times B \times C + C \\ &= (A \times B + A \times B \times C) + (A \times C + C) \\ &= A \times B \times (1 + C) + ((A + 1) \times C) \\ &= A \times B + C \end{aligned}$$

Same function,
different circuit



Example 2:



$$F = [(A + B)' \times (B \times C)']'$$

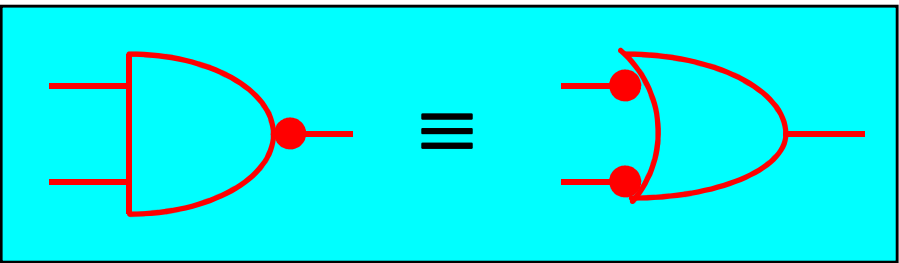
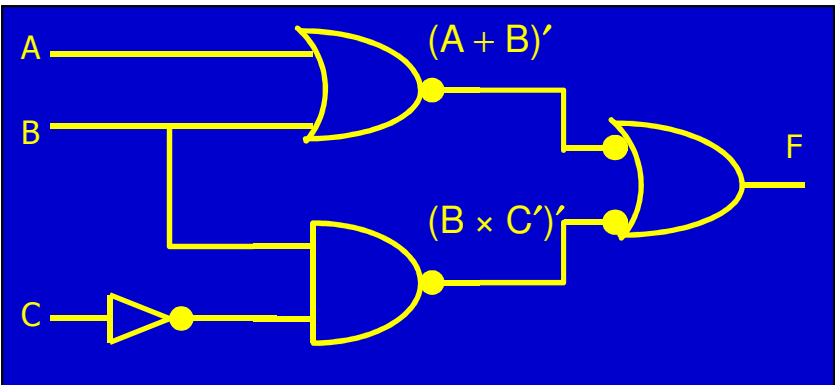
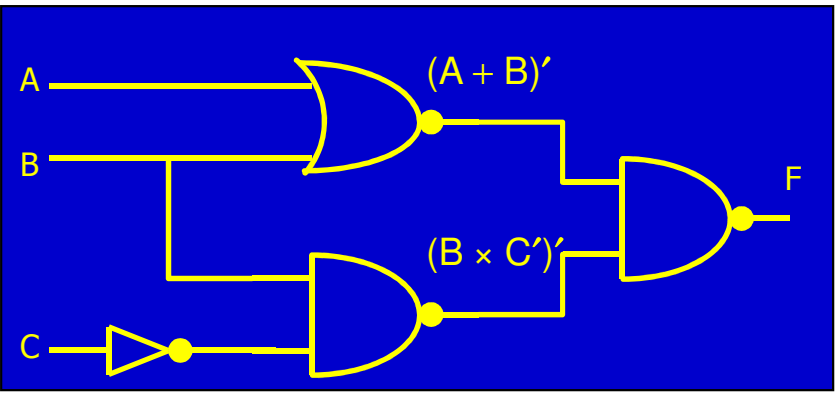
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$F_{A,B,C} = \Sigma (2,3,4,5,6,7)$$

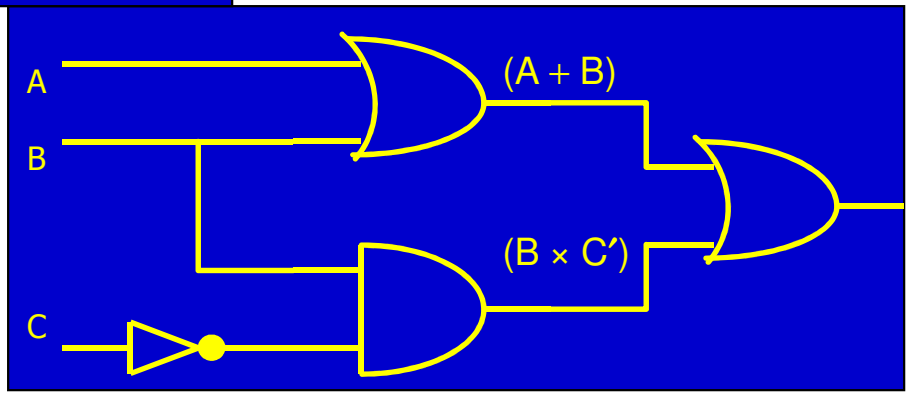
OR

$$F_{A,B,C} = \Pi (0,1)$$

Example 2 (Contd.):



$$F = A + B + B \times C'$$



Example 2 (Contd.):

$$F = [(A + B)' \times (B \times C')']'$$

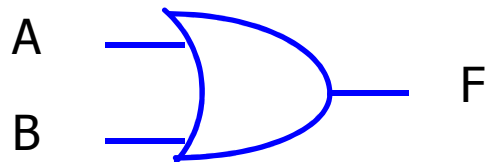
$$= (A + B)'' + (B \times C')'' \quad \text{using Demorgan's theorem}$$

$$= A + B + B \times C'$$

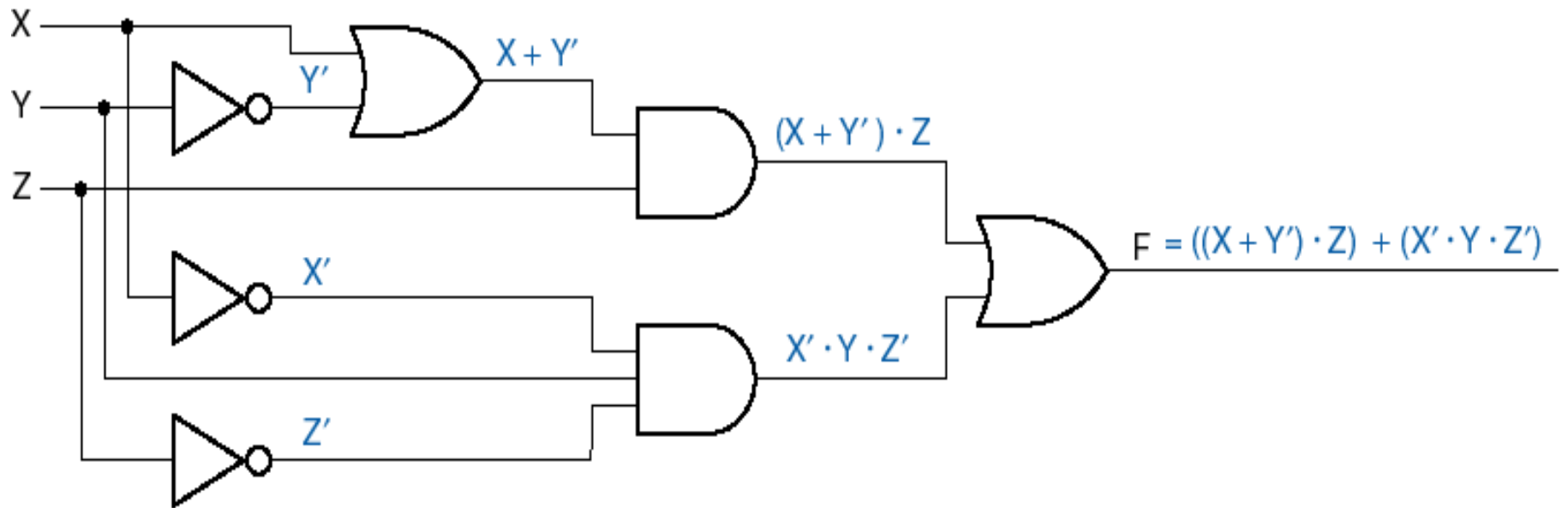
$$= A + B \times (1 + C')$$

$$= A + B \times 1$$

$$= A + B$$



Example 3:



(Multiply out)

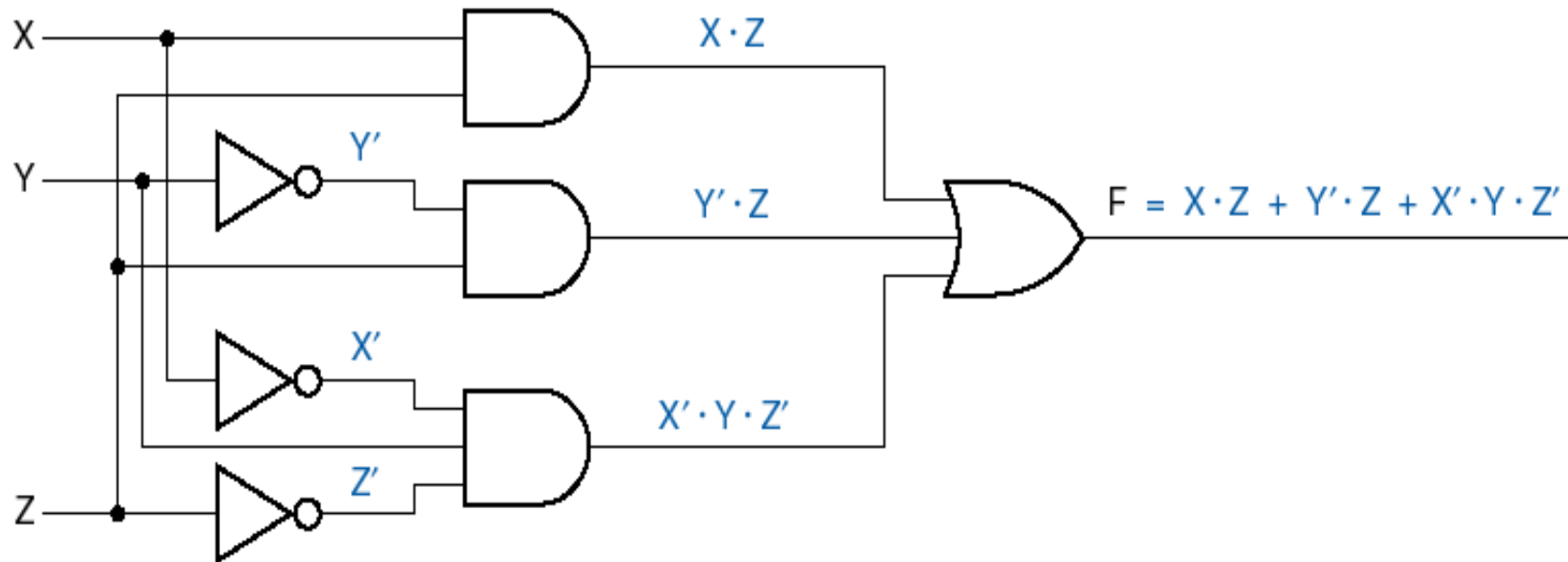
$$F = ((X + Y') \times Z) + (X' \times Y \times Z')$$

$$F = (X \times Z) + (Y' \times Z) + (X' \times Y \times Z')$$

Example 3 (Contd.):

New circuit, same function

$$F = (X \times Z) + (Y' \times Z) + (X' \times Y \times Z')$$

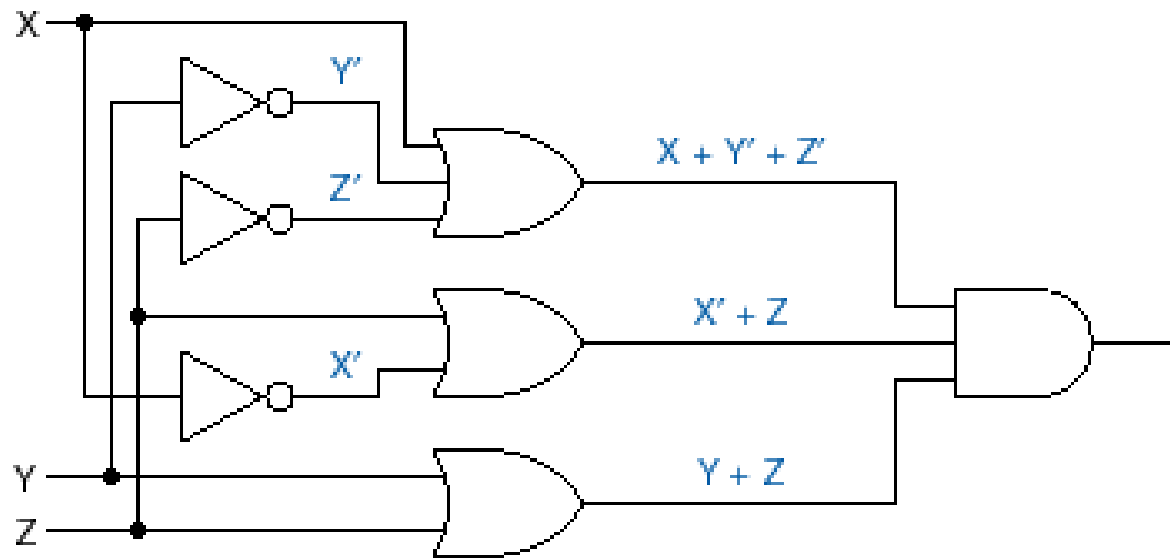


Example 3 (Contd.):

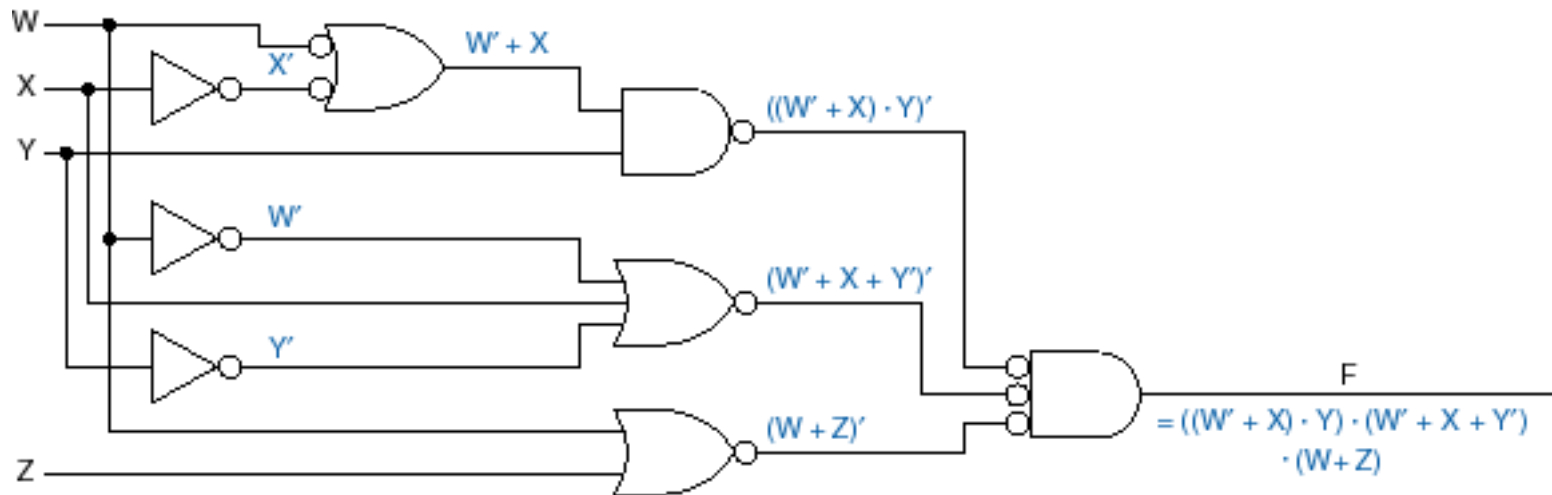
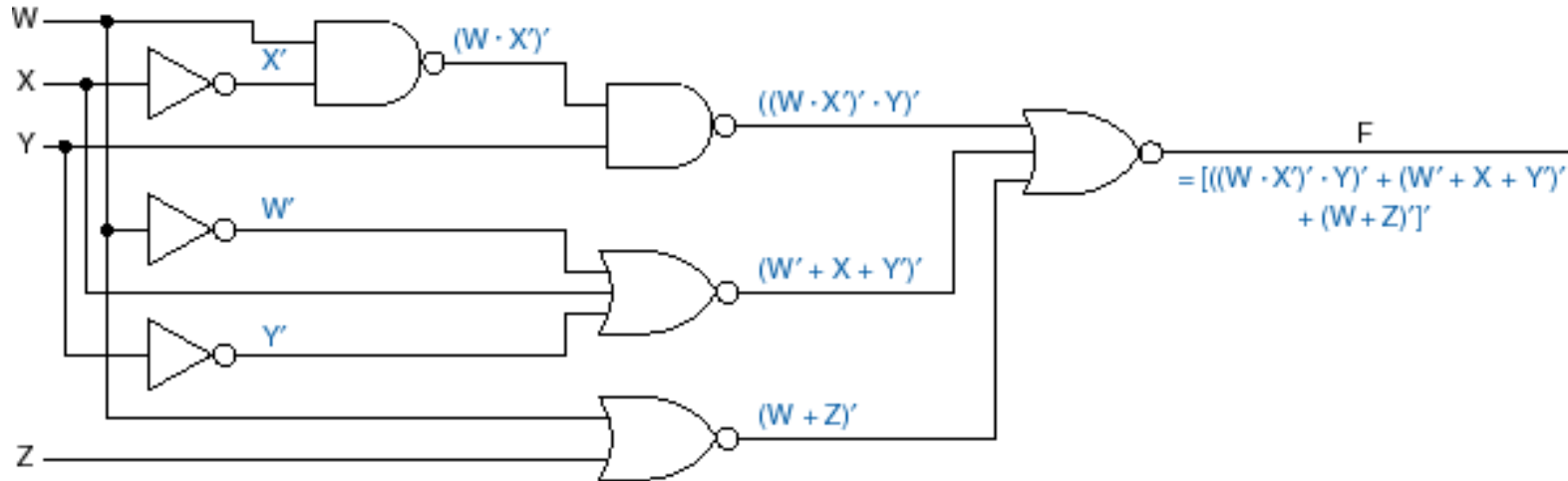
$$F = ((X + Y') \cdot Z) + (X' \cdot Y \cdot Z') \quad \textbf{(Add out)}$$

$$((X + Y' + X') \cdot (X + Y' + Y) \cdot (X + Y' + Z) \cdot (Z + X') \cdot (Z + Y) \cdot (Z + Z'))$$
$$1 \quad \cdot \quad 1 \quad \cdot (X + Y' + Z) \cdot (X' + Z) \cdot (Y + Z) \cdot 1$$

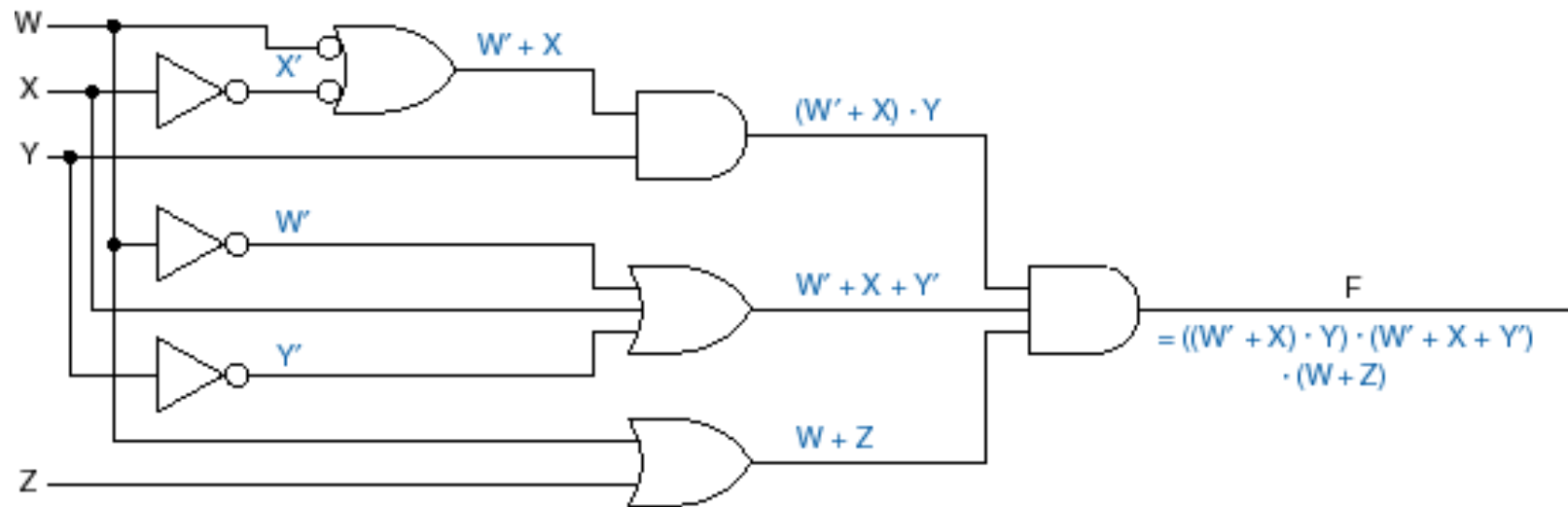
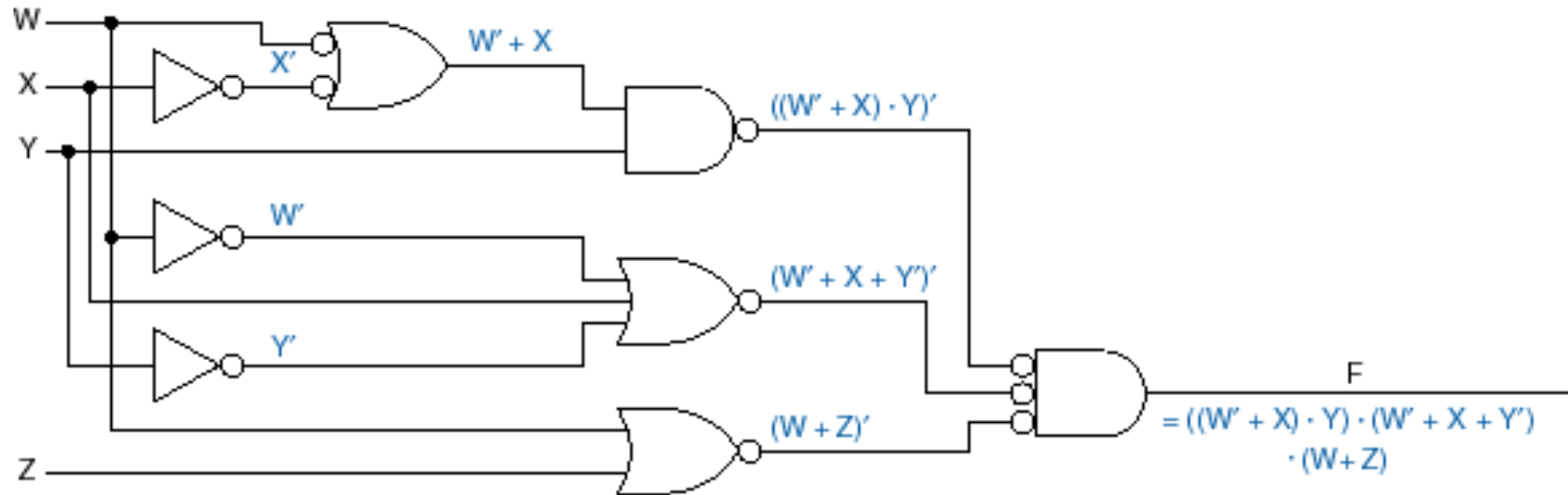
$$(X + Y' + Z) \cdot (X' + Z) \cdot (Y + Z)$$



Shortcut: Symbol Substitution

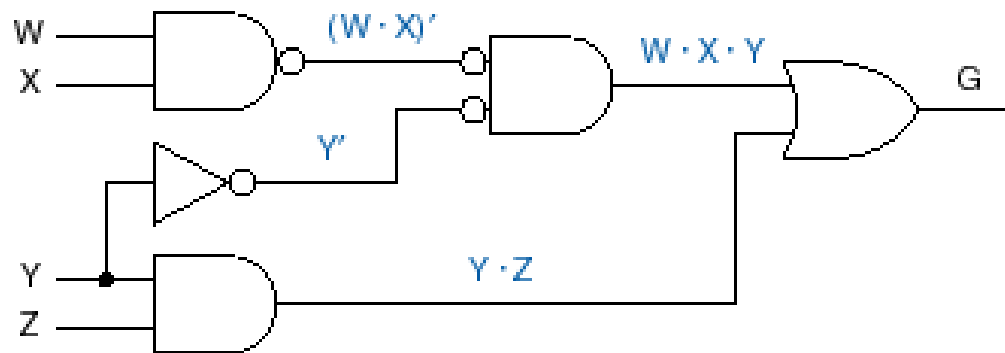
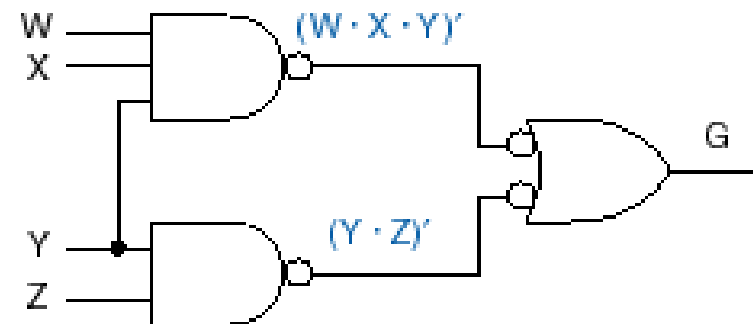
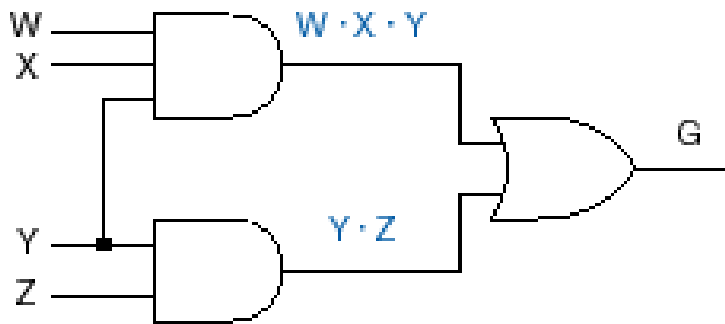


Different Circuit, Same Function



Example 4

$$G_{W,X,Y,Z} = W \cdot X \cdot Y + Y \cdot Z$$



Combinational Analysis Review

Combinational Analysis Review:

Logic Circuit → Logic Function → Truth Table

Analyze a combinational logic circuit by obtaining a formal description of its logic function

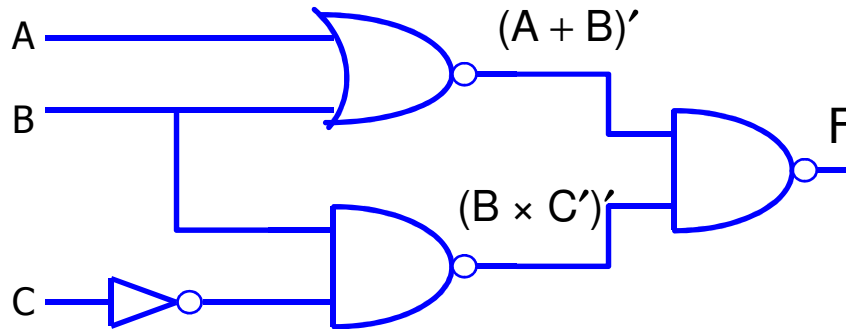
Outputs of a combinational logic circuit depends only on its current inputs (not on history)

Kinds of combinational analysis:

- exhaustive (truth table)
- algebraic (expressions)

Example:

I)



II)

$$F = [(A + B)' \times (B \times C')']'$$

III)

	A	B	C	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$F_{A,B,C} = \Sigma (2,3,4,5,6,7)$ **IV)**

$$F = A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC$$

$F_{A,B,C} = \Pi (0,1)$ **V)**

$$F = (A+B+C)(A+B+C')$$

VI)

