Faculty of Engineering

CSE115: Digital Design

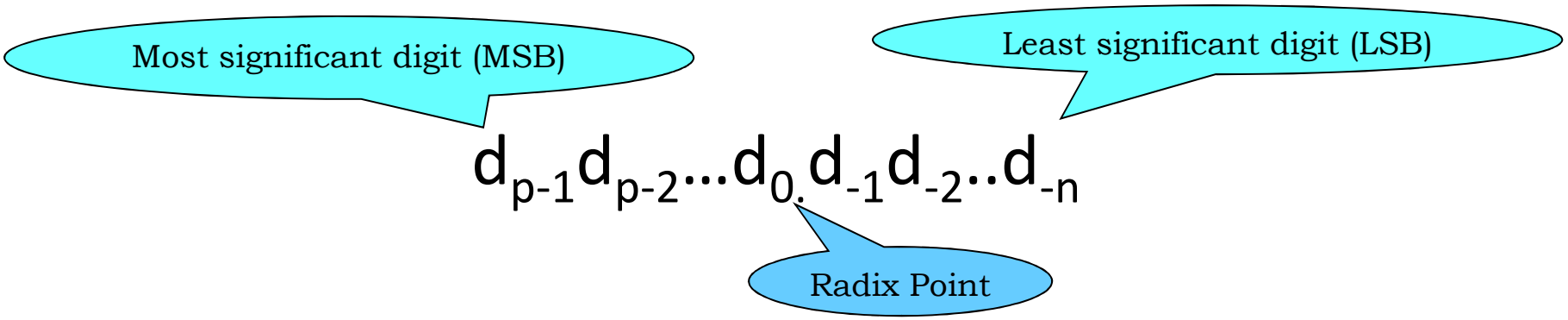
Lecture 2:
Number Systems

Suggested Reading

- Sections 2.1-2.3

Positional Number System

- General form of a number:



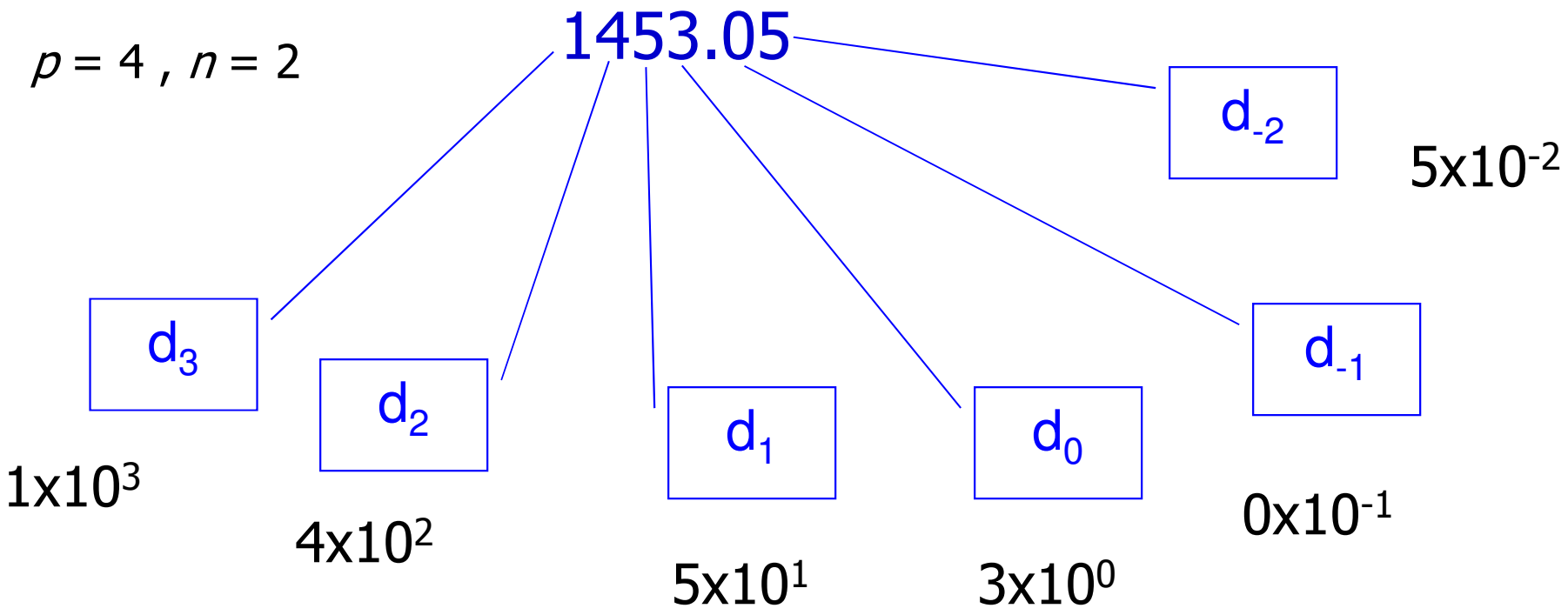
- The value of the number:

$$D = \sum_{i=-n}^{p-1} d_i \cdot r^i$$

Radix or Base

Decimal System

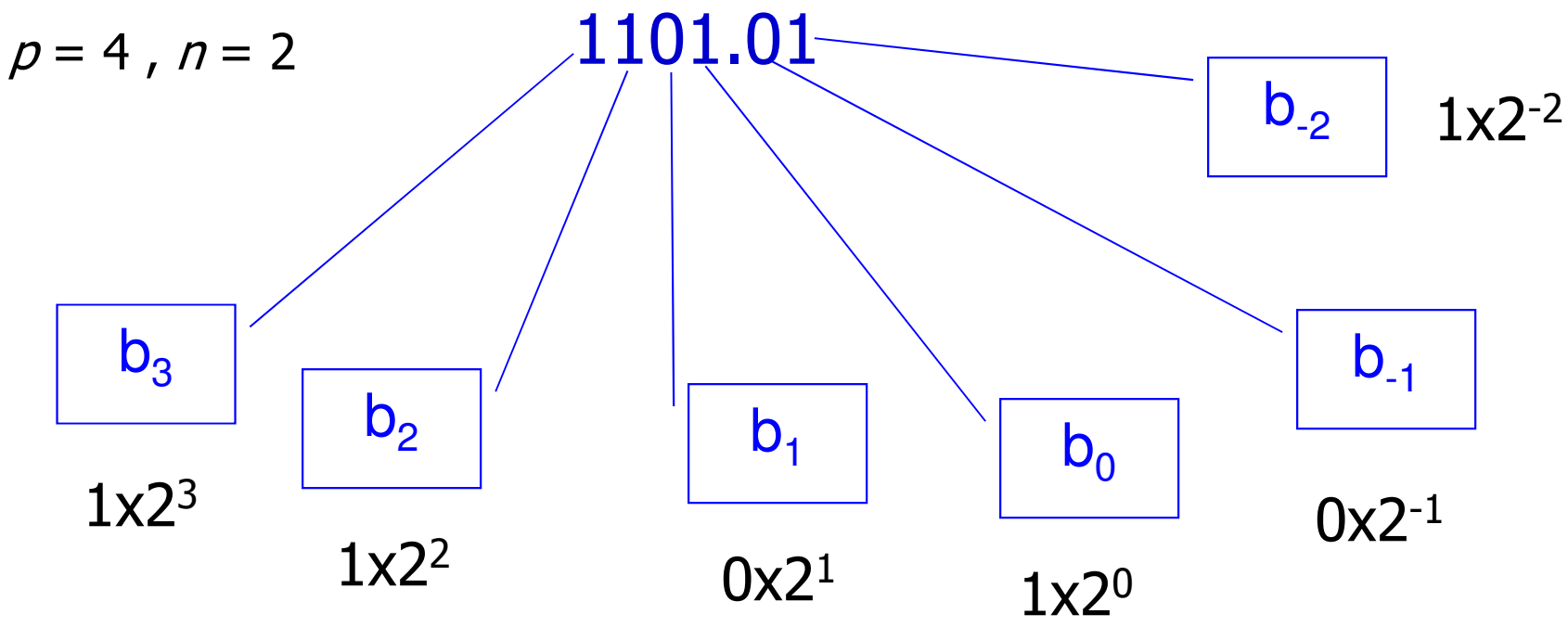
- ***d*** **:** 0, 1, 2, . . . , 9 ***r* = 10**



$1x10^3 + 4x10^2 + 5x10^1 + 3x10^0 + 0x10^{-1} + 5x10^{-2} = 1453.05$

Binary System

- ***b*** **:** 0 , 1 ***r* = 2**



$1x2^3 + 1x2^2 + 0x2^1 + 1x2^0 + 0x2^{-1} + 1x2^{-2} = 13.25$

Exercise

- Calculate the equivalent decimal numbers:

1_2 $1 \times 2^0 = 1$

10_2 $1 \times 2^1 + 0 \times 2^0 = 2 + 0 = 2$

11_2 $1 \times 2^1 + 1 \times 2^0 = 2 + 1 = 3$

100_2 $1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 4 + 0 + 0 = 4$

1011_2 $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 0 + 2 + 1 = 11$

Octal and Hexadecimal Numbers

- **Octal** number System:

  $r = 8$ d : 0, 1, 2, … , 7

- **Hexadecimal** number System:

  $r = 16$ d : 0, 1, 2, …, 9, A, B, C, D, E, F

- Used for representations of long binary numbers

Binary, Decimal, Octal and Hex #s

Decimal	Binary	Octal	Hexadecimal
0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Binary – Octal Conversion

- Starting from the decimal point:
- Separate the bits into groups of three
- Replace each group with its corresponding Octal digit

$$100011110 . 10101_2 =$$

$$100\ 011\ 110 . 101\ 010_2 =$$

$$436 . 52_8$$

Binary - Hexadecimal Conversion

- Starting from the decimal point:
- Separate the bits into groups of four
- Replace each group with its corresponding Hexadecimal digit

$$100011110\ .\ 10101_2 =$$

$$0001\ 0001\ 1110\ .\ 1010\ 1000_2 =$$

$$11E\ .\ A8_{16}$$

Octal - Binary Conversion

- Convert each Octal digit into its corresponding three bit string

$$436\ .\ 52_8 =$$

$$100\ 011\ 110\ .\ 101\ 010_2$$

Hexadecimal - Binary Conversion

- Convert each Hexadecimal digit into its corresponding four bit string

$$A5E\ .\ C8_{16} =$$

$$1010\ 0101\ 1110\ .\ 1100\ 1000_2$$

Radix-r to Decimal Conversion

$$D = \sum_{i=-n}^{p-1} d_i \cdot r^i$$

Decimal to Radix-r Conversion

- Successive division of **D** by ***r***
- The remainder of the long division will give the digits starting from the *least significant digit*

Decimal to Binary Conversion

45_{10}

45/2 = 22	The remainder is	1	**LSB**	
22/2 = 11		0		
11/2 = 5		1		
5/2 = 2	1			
2/2 = 1	0			
1/2 = 0	1	**MSB**		

MSB					**LSB**
32	**16**	**8**	**4**	**2**	**1**
1	**0**	**1**	**1**	**0**	**1**

Exercises

Convert 5 into binary: **1 0 1**

hexadecimal: **5**

Convert 17 into binary: **1 0 0 0 1**

hexadecimal: **11**

Convert 34 into binary: **1 0 0 0 1 0**

hexadecimal: **22**