Faculty of Engineering

CSE115: Digital Design

Lecture 11:
Simplifying Product of Sums

Suggested Reading

- Sections 4.3.6

Simplifying the Product of Sums

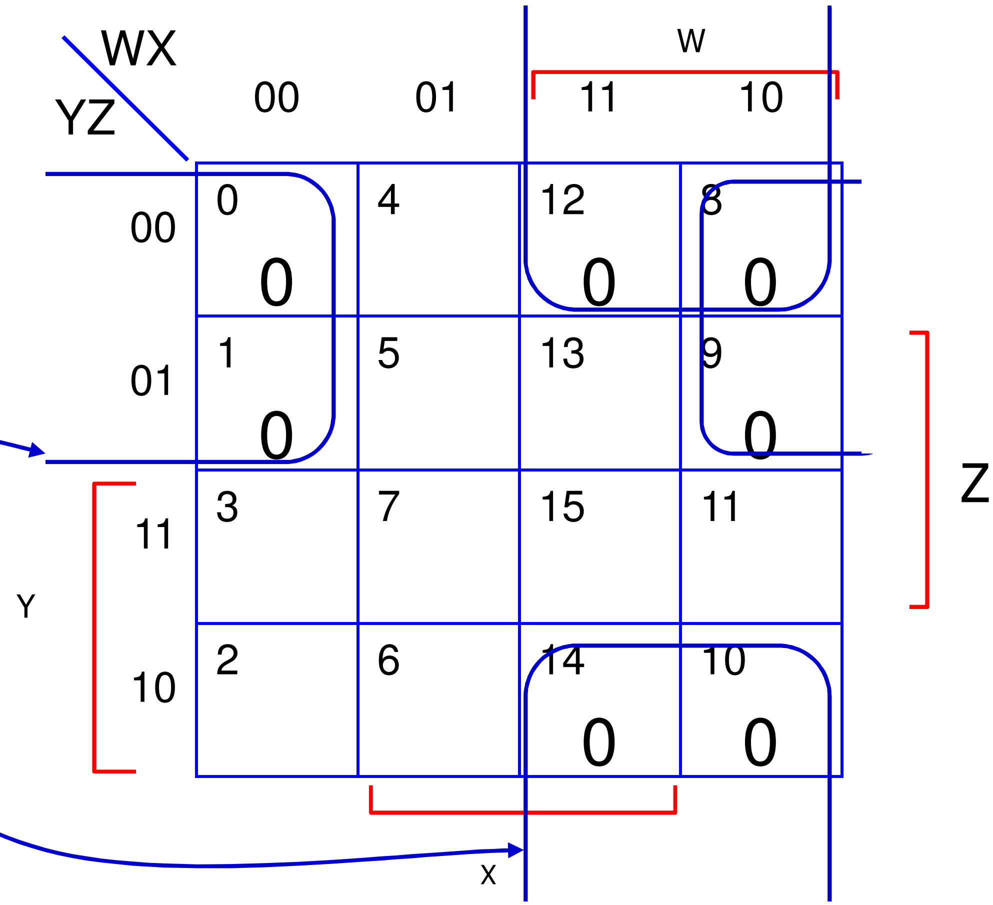
1. **Plot 0s** corresponding to **maxterms** of function.
2. **Circle largest** possible rectangular **sets of 0s**.
   - # of 0s in set must be power of 2
   - OK to cross edges
3. Read off product terms, one per circled set.
   - Variable is 0 → include variable
   - Variable is 1 → include complement of variable
   - Variable is both 1 and 0 → variable not included
4. Circled sets and corresponding product terms are called ‘**prime implicants**’
5. Minimum number of gates and gate inputs

Example 1

The prime implicants:

Cells (0,1,8,9): X=0, Y=0
The sum term: **X+Y**

Cells (8,10,12,14): W=1, Z=0
The sum term: **W'+Z**



The two prime implicants are essential prime implicants and cover all zeros

The minimal product of sums: **F = (X + Y) . (W' + Z)**

Exercise

Row	W	X	Y	Z	F
0	0	0	0	0	**1**
1	0	0	0	1	**1**
2	0	0	1	0	**1**
3	0	0	1	1	**0**
4	0	1	0	0	**1**
5	0	1	0	1	**1**
6	0	1	1	0	**1**
7	0	1	1	1	**0**
8	1	0	0	0	**1**
9	1	0	0	1	**1**
10	1	0	1	0	**0**
11	1	0	1	1	**0**
12	1	1	0	0	**1**
13	1	1	0	1	**1**
14	1	1	1	0	**1**
15	1	1	1	1	**0**

YZ \ WX	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

W (columns 11, 10); X (columns 01, 11); Z (rows 01, 11); Y (rows 11, 10)

Essential prime implicants:

(3,7,15,11): Z=1, Y=1; The sum term: Z'+Y'

(10,11): W=1, X=0,Y=1; The sum term: W'+X+Y'

F=(Z'+Y').(W'+X+Y')

Simplifying POS - Another method

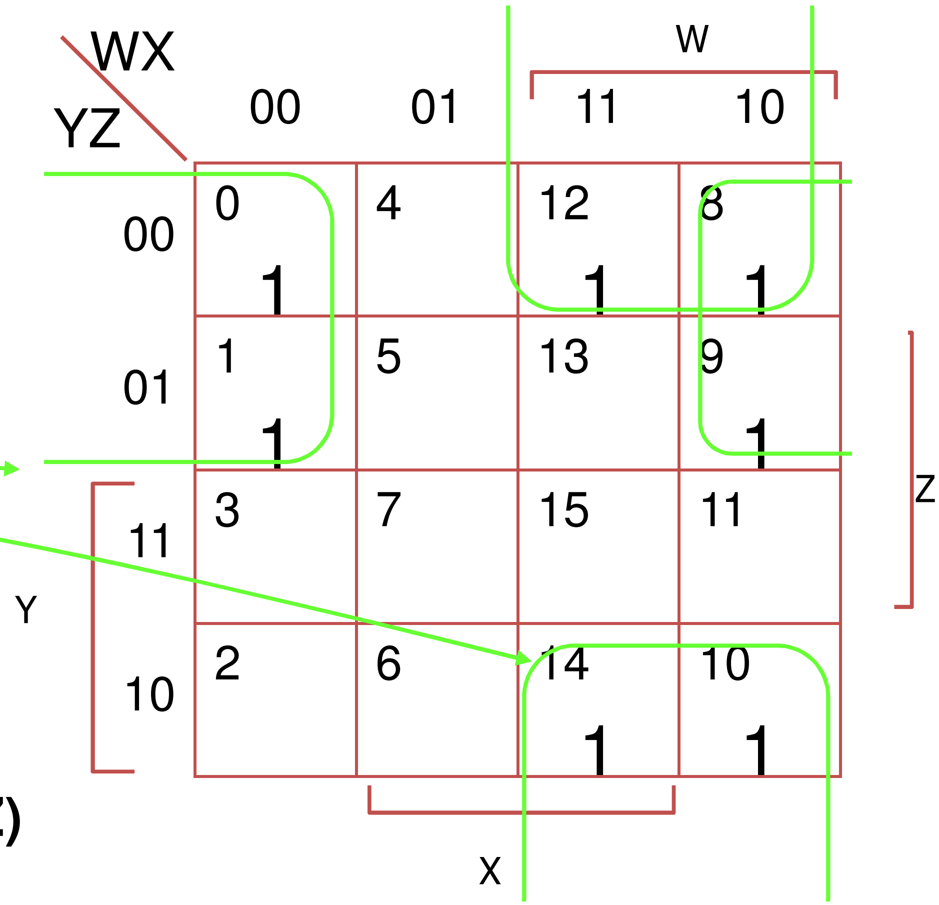
Complement the function. Use K-map to get the **minimal sum** of the complement function. **Complement the minimal sum** to get the minimal product

1. The function is complemented and represented using K-map:

2. The essential prime implicants are:
   WZ', X'Y

3. The minimal Sum: M = X'Y' + WZ'

4. F = M' = (X'Y'+WZ')' = **(X+Y).(W'+Z)**



Minimal Product of Sums vs. Minimal Sum of Products:

Compare the minimal product and the minimal sum designs to find the best (least # of gates) realization.

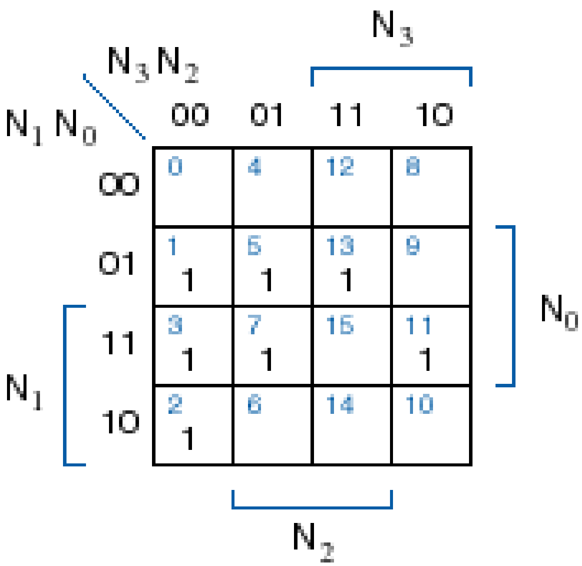
In the previous example:

The minimal sum is: $F = W'X + W'Y + XZ + YZ$

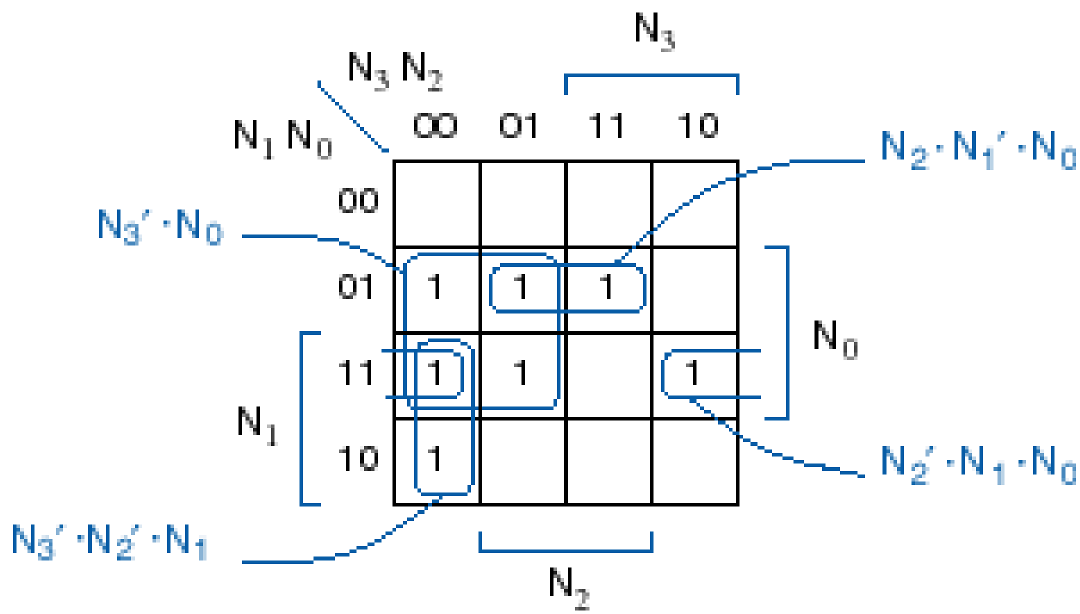
The minimal product: $F = (X + Y) . (W' + Z)$

$\therefore$ The **minimal product** implementation is cheaper.

Prime-Number Detector (again)



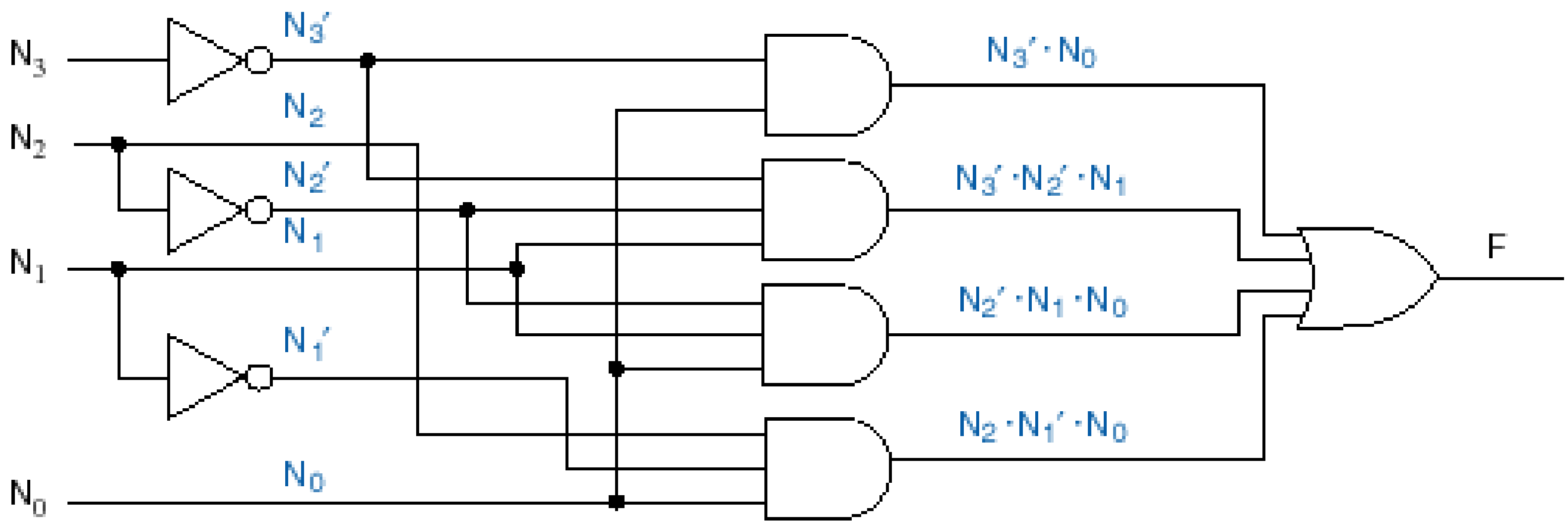
$F = \Sigma_{N3,N2,N1,N0}(1,2,3,5,7,11,13)$



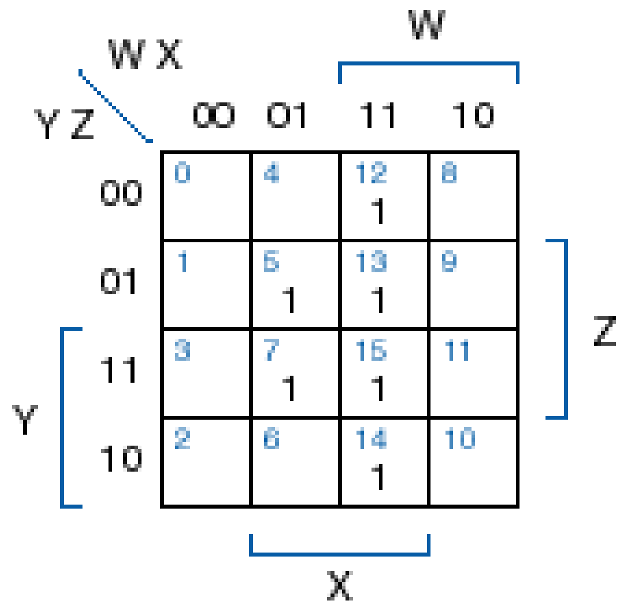
$F = N_3' \cdot N_0 + N_3' \cdot N_2' \cdot N_1 + N_2' \cdot N_1 \cdot N_0 + N_2 \cdot N_1' \cdot N_0$

Prime-Number detector (contd.)

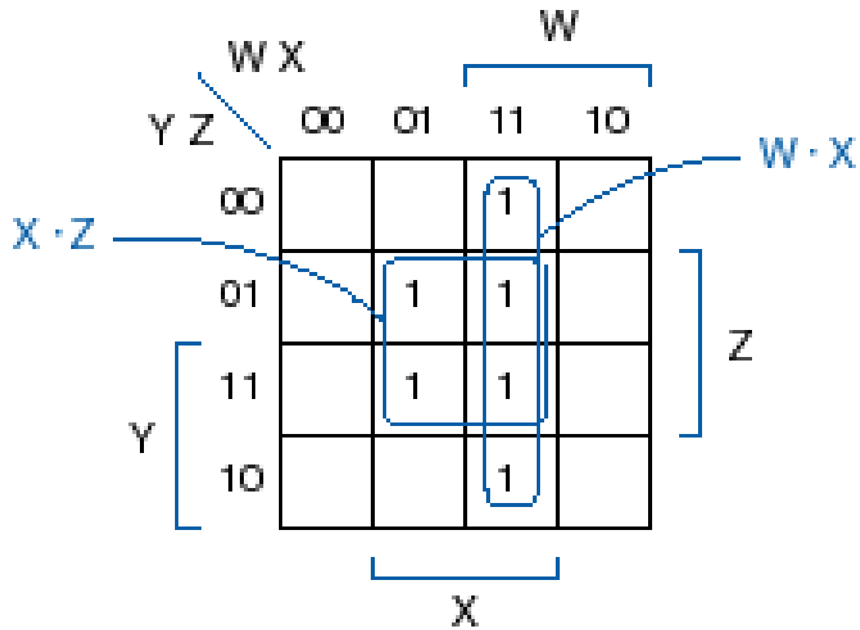
When we solved algebraically, we missed one simplification -- the circuit below has three less gate inputs.



Example 2

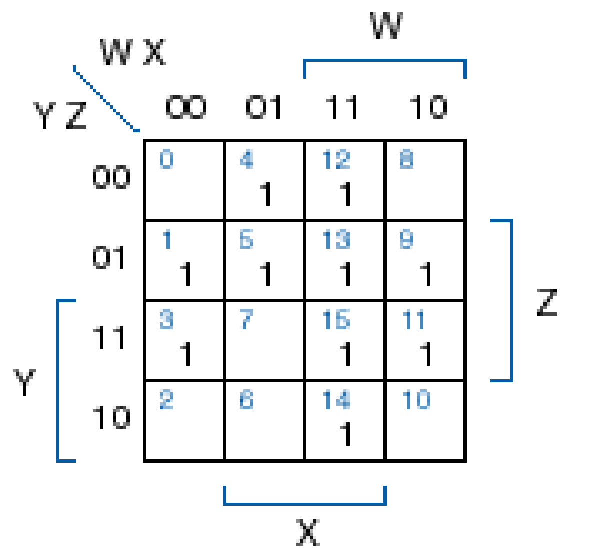


$F = \Sigma_{W,X,Y,Z}(5,7,12,13,14,15)$

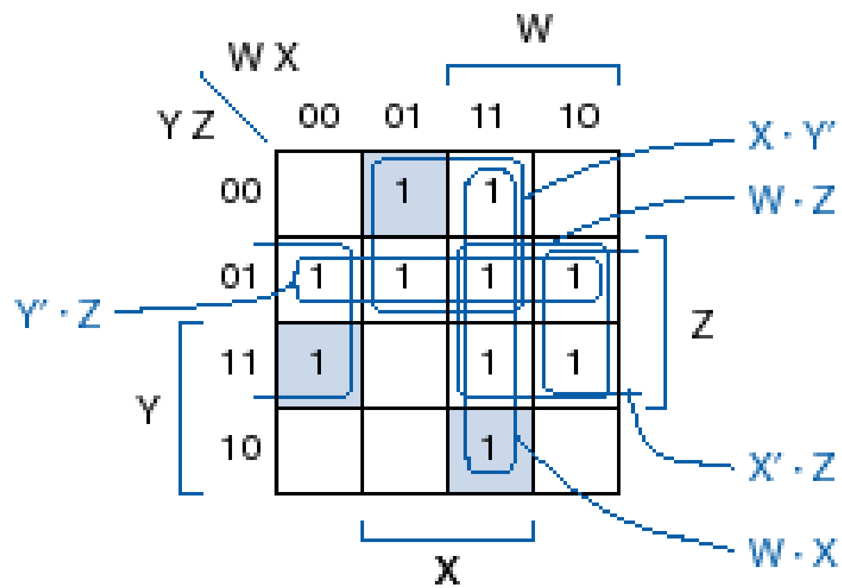


$F = X \cdot Z + W \cdot X$

Example 3



$F = \Sigma_{W,X,Y,Z}(1,3,4,5,9,11,12,13,14,15)$



$F = X \cdot Y' + X' \cdot Z + W \cdot X$