**Fourth Year, Mechatronics Department.**
CSE-473: Digital Control
Sheet #02: The z-Transform
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**Ain Shams University**
**Faculty of Engineering**

1- Find the z-transform of the following continuous-time-signals (You Need To Discretize the signal First) in a closed form:-
   a. $e^{-t}$
   b. $e^{-(t-T)}u(t-T)$ where u is the unit-step function.
   c. $e^{-(t-5T)}u(t-5T)$ where u is the unit-step function.

2- Find the z-transform, in a closed form, of the number sequence generated by sampling the time function $e(t)$ every $T$ seconds starting at $t=0$. The function $e(t)$ is specified by the Laplace Transform,

$$E(s) = \frac{2(1 - e^{-5Ts})}{s(s+2)}$$

3- Find the z-transform, in a closed form, of the number sequence generated by, $f(k) = 8k(2^{k-2}) - 2^k + 5$, where $k=0,1,2,\ldots$

4- Find the z-transform, in a closed form, of the number sequence generated by, $f(k) = \sum_{h=0}^{k} 2^{3h}$, where $k=0,1,2,\ldots$

5- Find $f(0)$, $f(1)$, and $f(10)$ for $F(z) = \frac{0.1}{z(z-0.9)}$ using direct division. Check $f(0)$ using the initial value theorem.

6- Find $x(k)$ for $k=0, 1, 2, 3, 4,$ and $5$ if:-

$$X(z) = \frac{1}{(z-0.9)(z^2 - 0.9z + 0.92)(z^2 - 0.87z + 0.99)}.$$

7- A function x(t) is sampled and the resulting sequence has the following z-transform

$$X(z) = \frac{z^3 - 2z}{(z^4 - 0.9z^2 + 0.8)},$$

   a. Find the z-transform of $x(t-2T)u(t-2T)$.
   b. Find the z-transform of $x(t+2T)u(t)$.
   c. Find the z-transform of $x(t-2T)u(t-2T)$.
   d. Find the z-transform of $x(t-T)u(t-2T)$.

8- Determine the initial and final values of x(k) where:-

$$X(z) = \frac{z^2}{(z-1)(z^2+1.3z+0.4)}.$$

Also, find the x(k) using three different methods.

9- Find the inverse z-transform of X(z) considering all possible combinations of the parameters a and b:-

$$X(z) = \frac{1}{(z^2+az+b)}.$$

10- Solve the following difference equation using the sequential technique:-

$$y(k) - 3y(k-1) + 2y(k-2) = x(k).$$

Consider the following: $x(k) = \delta(k) + \delta(k-1)$, y(-1)=y(-2)=0. Determine the final value of the signal y(k). Will the final value give the correct value of x(k) as $k \to \infty$? Justify your answer.

11- Given the following difference equation:-

$$y(k+2) - 0.75y(k+1) + 0.125y(k-2) = x(k).$$

Consider the following: $x(k) = u(k-1)$, $y(0)=y(1)=0$. Solve the difference equation to get y(k) using the inverse z-transform. Plot the resulting signal.

12- Write a digital computer program to solve the difference equations of problems 11 and 12. In your program implementation, plot the resulting signals.

13- \We can use z-transforms to find the sum of integers raised to various powers. This is accomplished by first recognizing that the sum is the solution of the difference equation f (k) = f (k −1) + a(k) where a(k) is the $k^{th}$ term in the summation. Evaluate the following summations using z-transforms.

a. $f(k) = \sum_{h=1}^{k} h$
b. $f(k) = \sum_{h=1}^{k} h^2$.