MECHANICS OF MACHINES (1)

Dr. Hossam Doghiem
Syllabus

1. Mechanisms
2. Velocity and Acceleration
3. Equilibrium of Machines & Turning Moment Diagram (Flywheel)
4. Cams
5. Gear (Geometry and Train)
6. Balancing
References

1. The theory of machines, T. Bevan
2. The theory of machines, P. L. Ballaney
3. The theory of machines, R. S. khurmi & J. K. Gupta
4. The theory of machines (worked example), Ryder
5. The theory of machines (solved example), Onvoner
6. The theory of machines, W. Grean
7. Mechanics of machine, Ham & Crane
8. Mechanics for engineering, Duncan & Macmillan
CHAPTER 1
MECHANISMS
Definitions

Theory of machines

This branch of engineering-science is very essential for an engineer in designing various parts of a machine.
1. **Kinematics**
Study of the relative motion between the various parts of a machine

2. **Dynamics**
Study of the forces which acts on the machine parts

   2.1. **Statics**
Deals with the forces assuming the machine parts to be massless

   2.2. **Kinetics**
Deals with the inertia forces arising from the combined effect of the mass and the motion of the parts
Definitions

Example: Reciprocating engine

Rotary speed of the crank shaft relative to the reciprocating speed of the piston form a **kinematic problem**

The thrust exerted by the steam or gas on the piston and force produced on the connecting rod form a **static problem**
**Definitions**

**Link or element**

A link may be defined as a resistant (rigid or non rigid) body fixed or in motion which transmits force with negligible deformation.

It has 2 or more pairing elements by which it may be connected to other bodies for transmitting force or motion.
Definitions

Examples of links which are resistant but not rigid:

A) Liquids
Resistant to compressive forces and used as links in hydraulic presses

B) Chains & Belts
which are resistant to tensile forces and transmitting motion and forces
Definitions

Kinematic pair
Two links which are connected together in such a way that their relative motion is completely constrained

Complete constrain pair
The relative motion is limited to a definite direction

Turning Pair
Sliding Pair
Screw Pair

There is a relation between the rotation of A and the axial displacement of A relative to B
Definitions

Incomplete pair

As an example of this pair

The relative motion may be slide-rotate-sliding and rotation

So there is nothing in connection A & B to determine which of the motions take place
Definitions

Pairs

Lower

Higher

When relative motion takes place, there is a **contact surface** between the two links (turning pair- sliding pair- screw pair)

The two links have **line or point contact** while they are in motion(Cams- Gears- Bearings)

The pair must be force-closed in order to provide completely constrained motion
Definitions

Kinematic chain

when a number of links are connected by means of pairs the resulting assemblage is called kinematic chain
Definitions

<table>
<thead>
<tr>
<th>Kinematic chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locked</td>
</tr>
<tr>
<td>n = 0</td>
</tr>
<tr>
<td>No relative motion is possible (Structure)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kinematic chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constrained</td>
</tr>
<tr>
<td>n = 1</td>
</tr>
<tr>
<td>Definite relative motion is possible</td>
</tr>
<tr>
<td>The basis of all machine</td>
</tr>
<tr>
<td>Single input – single output</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kinematic chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
</tr>
<tr>
<td>n &gt; 1</td>
</tr>
<tr>
<td>the relative motion is possible but not definite</td>
</tr>
</tbody>
</table>

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Definitions

Mechanism

If one of the links of the kinematic chain is fixed, the chain became mechanism (inversions different fixed links)
Definitions

Machine

Is a mechanism which receive energy in some available form and uses it to do some particular kind of work
Definitions

Degrees of freedom \( n \)

The link have 3 degrees of freedom

Two links have 6 degrees of freedom

If two links jointed together by turning pair the degree of freedom become 4

i.e. one lower pair removes 2 degree of freedom from the system
Definitions

\[ n = 3L - 2P_l - 3 \]
\[ = 12 - 8 - 3 = 1 \]

Where

\( n \): is the degrees of freedom
\( P_l \): number of lower pairs
\( P_h \): number of higher pairs

\[ n = 3L - 2P_l + cP_h - 3 \]
\[ 1 = 9 - 4 + cP_h - 3 \]
\[ c = -1 \]

\[ n = 3L - 2P_l - P_h - 3 \]

\( P_h \): number of higher pairs
Inversions

Different mechanisms can be obtained by fixing in turn different links in a kinematic chain

It is important to note that inverting a mechanism doesn’t change the motions of its links relative to each other, but does change their absolute motions
Inversions

Example 1: original gear train, epicyclic gear train

1\textsuperscript{st} inversion: Original Train

2\textsuperscript{nd} inversion: Epicyclic Gear Train
Inversions

Example 2: Inversions of slider crank chain

1st Inversion: the cylinder is fixed: reciprocating engine mechanism
Inversions

Example 2: Inversions of slider crank chain

2\textsuperscript{nd} inversion: PC becomes fixed: oscillating cylinder engine

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**Inversions**

**Example 2: Inversions of slider crank chain**

3rd Inversion: fixing the link OC: Whitworth or quick return motion mechanism (slotting and shaping machines)

CP rotates at uniform speed

\[
\frac{t_c}{t_R} = \frac{180 - \theta}{\theta}
\]

\[
\omega = \frac{d\theta}{dt} = k
\]

\[
\theta \propto t
\]
Example 2: Inversions of slider crank chain

4\textsuperscript{th} inversion: fixing the piston: pendulum pump

CP will oscillate, QO will reciprocate
Inversions

Example 3: Inversions of double slider crank chain

1\textsuperscript{st} inversion: If the slotted frame is fixed: ellipse trammels

\[ x = a \cos \theta \]
\[ y = b \sin \theta \]

\[ \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = \cos^2 \theta + \sin^2 \theta = 1 \]

i.e. \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

a=semi-minor axis,  b= semi- major axis
Inversions

Example 3: Inversions of double slider crank chain

1st inversion: If the slotted frame is fixed: ellipse trammels

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Inversions

Example 3: Inversions of double slider crank chain

2\textsuperscript{nd} inversion: If one of the two blocks is fixed: scotch yoke
it is used for converting rotary into reciprocating motion

Scotch yoke mechanism
Inversions

Example 3: Inversions of double slider crank chain

3rd inversion: Coupling link AB is fixed: Oldham’s coupling

If one block is turned through a definite angle, the frame and the other block must turn through the same angle.
Inversions

Example 3: Inversions of double slider crank chain

3rd inversion: Coupling link AB is fixed: Oldham’s coupling

If the two shafts remain parallel the distance h may vary while the shafts are in motion without affecting the transmission of uniform motion from one shaft to the other

The centre of the disc will describe a circular path with h as a diameter
Hooke’s joint (Universal Joint)

To transmit the motion between two intersecting shafts

Where a shaft drive has to be fitted to a flexible frame (tractors)

The centre of the cross must lies on the axis of each shaft
Hooke’s joint (Universal Joint)

Gear box to back axle

Proper Geometry for Conventional Two Joint Drive Shaft
(Centerline of pinion on differential & trans are parallel)
Hooke’s joint

Relation between the angular velocities

\[ \theta: \text{Angular displacement of the driver} \quad \omega = \frac{d\theta}{dt} \]

\[ \beta: \text{Angular displacement of the driven} \quad \omega_1 = \frac{d\beta}{dt} \]

\[ \tan \beta = \frac{ON}{NC_2} \quad \tan \theta = \frac{OM}{MC_1} = \frac{OM}{NC_2} \]

\[ \frac{\tan \beta}{\tan \theta} = \frac{ON}{OM} = \frac{1}{\cos \alpha} \]

\[ \tan \theta = \tan \beta \cdot \cos \alpha \]
Hooke’s joint (Universal Joint)

\[
\tan \theta = \tan \beta \cdot \cos \alpha
\]

Differentiating this equation

\[
\sec^2\theta \frac{d\theta}{dt} = \cos \alpha \cdot \sec^2\beta \cdot \frac{d\beta}{dt}
\]

\[
\frac{\omega}{\omega_1} = \cos \alpha \cdot \cos^2\theta \sec^2\beta
\]

\[
\sec^2\beta = 1 + \tan^2\beta = 1 + \frac{\tan^2\theta}{\cos^2\alpha} = 1 + \frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}
\]

\[
= \frac{\cos^2\theta \cdot \cos^2 \alpha + \sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha} = \frac{\cos^2\theta \cdot \cos^2 \alpha + 1 - \cos^2\theta}{\cos^2\theta \cdot \cos^2\alpha} = \frac{1 - \cos^2\theta \cdot \sin^2\alpha}{\cos^2\theta \cdot \cos^2\alpha}
\]
Hooke’s joint (Universal Joint)

Hence

\[ \frac{\omega}{\omega_{1\text{max}}} = \cos \alpha \]

at \( \cos \theta = \pm 1 \)

i.e. at \( \theta = 0, \pi, 2\pi \ldots \) etc.

\[ \omega_{1\text{max}} = \omega \cos \alpha \]

\[ \frac{\omega}{\omega_{1\text{min}}} = \frac{1}{\cos \alpha} \]

at \( \cos \theta = 0 \)

i.e. at \( \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots \) etc.

\[ \omega_{1\text{min}} = \omega \cos \alpha \]
Hooke’s joint (Universal Joint)

\[ \omega_{1\text{max}} = \frac{\omega}{\cos \alpha} \]
\[ \omega_{1\text{min}} = \omega \cos \alpha \]

\[ \Delta \omega_1 = \frac{1}{\cos \alpha} - \cos \alpha = \frac{1 - \cos^2 \alpha}{\cos \alpha} = \sin \alpha \tan \alpha \]

\[ \Delta \omega_1 \propto \alpha^2 \]
Hooke's joint (Universal Joint)

Conditions of equal speeds

Put \( \frac{\omega}{\omega_1} = 1 \)

\[
1 - \cos^2 \theta \cdot \sin^2 \alpha = \cos \alpha
\]

\[1 - \cos \alpha = \cos^2 \theta \cdot \sin^2 \alpha\]

\[
\cos^2 \theta = \frac{1 - \cos \alpha}{\sin^2 \alpha} = \frac{1 - \cos \alpha}{1 - \cos^2 \alpha} = \frac{1 - \cos \alpha}{(1 - \cos \alpha)(1 + \cos \alpha)} = \frac{1}{1 + \cos \alpha}
\]

\[
\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{1}{1 + \cos \alpha} = \frac{\cos \alpha}{1 + \cos \alpha}
\]

\[
\frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos \alpha}{(1 + \cos \alpha)} \cdot (1 + \cos \alpha) = \cos \alpha
\]

\[\tan \theta = \pm \sqrt{\cos \alpha}\]
Hooke’s joint (Universal Joint)

Angular acceleration of the driven shaft

\[ \omega_1 = \omega \frac{\cos \alpha}{1 - \cos^2 \theta \sin^2 \alpha} = \omega \cos \alpha (1 - \cos^2 \theta \sin^2 \alpha)^{-1} \]

\[ \frac{d\omega_1}{dt} = \omega \cos \alpha \left[-(1 - \cos^2 \theta \sin^2 \alpha)^{-2} \cdot (\sin^2 \alpha \cdot \frac{2 \cos \theta \sin \theta}{\sin 2\theta}) \right] \frac{d\theta}{dt} \]

\[ \alpha_1 = \frac{-\omega^2 \cos \alpha \sin^2 \alpha \sin 2\theta}{(1 - \cos^2 \theta \sin^2 \alpha)^2} \]

\( \alpha_1 \) will increase by increasing \( \alpha \), in normal practice such \( \alpha \) don’t exceed 10°

Maximum acceleration occur when

\[ \cos^2 \theta \approx \frac{2 \sin^2 \alpha}{2 - \sin^2 \alpha} \]

This relation is valid if \( \alpha < 30^\circ \)
Hooke’s joint

If the driving and the driven shafts are equally inclined to the intermediate shaft and the 2 forks on the intermediate shaft lie in the same plane, it is evident that speeds of driving and driven shafts are identical and the fluctuation of speed are confined to intermediate shaft, which may be made short and light.
Hooke’s joint

If the forks of the intermediate shaft lie in planes perpendicular to each other, the fluctuation of the driven shaft shall vary between $\cos^2 \alpha$ and $\frac{1}{\cos^2 \alpha}$.